SHAKEDOWN ANALYSIS OF A TRUSS COLUMN WITH LARGE DISPLACEMENTS BY DIRECT INCREMENTAL METHOD

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The deformation and the stability of steel structures are affected by both geometric and physical nonlinearity. If the displacements of a structure are large, the equilibrium conditions and strain-displacement relations can differ significantly from the corresponding expressions for small displacements. If the steel in parts of the structure yields, the stress-strain relations are modified so that the distribution of the stiffness in the structure changes and mechanisms can be formed. This paper describes the example of shakedown analysis of a truss under cyclic loads by direct incremental method developed by the author. The method accounts for geometrical nonlinearity and allows for large displacements of a structure.

KEY WORDS: steel trusses, cyclic load, shakedown, large displacements.

The behavior of steel structures under cyclic loads differs significantly from their behavior under monotonously increasing loads. If the loads on a structure increase monotonously, parts of the structure yield when a sufficiently high load level has been reached. The load bearing capacity reaches a limit value when a mechanism is formed so that the deformation of the structure can increase without further increase in the load. The structure has reached its limit load. If the loads on a structure vary cyclically and all parts of the structure remain elastic, the behavior of the structure will be the same in all load cycles. If some parts of the structure yield in the first few load cycles, two types of behavior can be distinguished. In some structures, the behavior becomes elastic after the first few load cycles have been completed and remains elastic in all subsequent load cycles. This type of behavior is called shakedown. In other structures, the extent of the plastic zones becomes ever larger as the number of load cycles increases until the structure reaches a limit state or becomes unserviceable due to excessive deformations. This type of behavior is called ratcheting. In this paper the example of the shakedown analysis of the truss is presented that was worked out using the software developed by the author.

1. Workflow of Shakedown Analysis

Let the following attributes of a space truss be given: node identifiers and global node location coordinates; bar identifiers, section properties and bar end node identifiers; node load pattern; prescribed displacement pattern; time histories for a set of load cases; period of the cyclic load; number of time increments per period; participation factors of the load cases in the load combination; maximum number of periods to be computed.

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An elastic-plastic analysis of the space truss is performed which accounts for large displacements of the nodes and yields the following results: load factor increment for each load step; node displacement coordinates for each station on the load path; reaction coordinates for each station on the load path; axial bar forces for each station on the load path; accumulated plastic strains for each station of the load path; limit state of the truss if it occurs; notification of shakedown in the computed periods if it occurs. The workflow of the algorithm is shown in Figure 1. The algorithm has been implemented in the software that was used for the analysis of the truss column described in the following sections. The software is based on the program SpaceTruss developed by V. Galishnikova and described in [1] and [2]. The detailed description of the algorithm is given in [3].

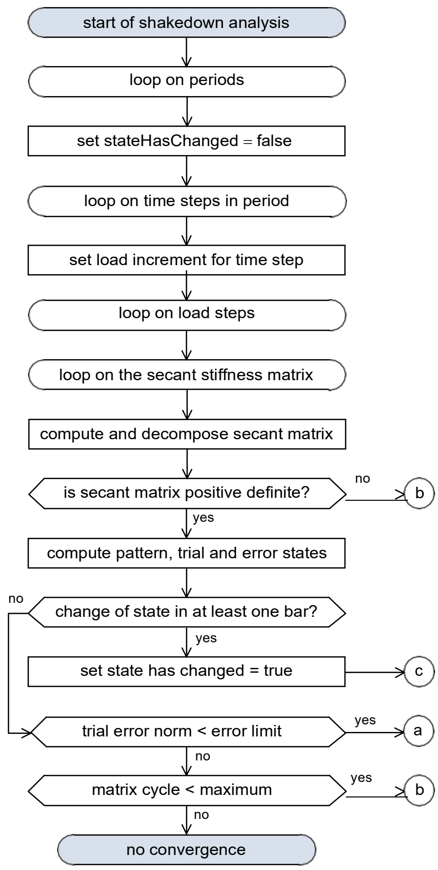

Fig. 1. Work flow of the shakedown analysis

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2. Shakedown Analysis of a Rectangular Column

Plans and elevations of the structure are shown in figure 2. The plan of the truss is a square with a side length of 2.0 m. The elevation of the truss consists of 6 stories with a height of 4.0 m each. The load bearing structure consists of 4 vertical chords at the corners of the plan, 4 horizontal ties at each level, 8 crossed diagonals in the 4 vertical planes of each story and 2 crossed diagonals in the horizontal plane at each level. The nodes and their identification numbers are shown in figure 2. The prescribed displacements of the pinned nodes at the lowest level are null in the 3 coordinate directions. The bars are identified with the numbers of their two end nodes. All bars have an area of 0.002 m^2, yield strength 2.4x105 kN/m^2 and modulus of elasticity 2.1x108 kN/m^2.

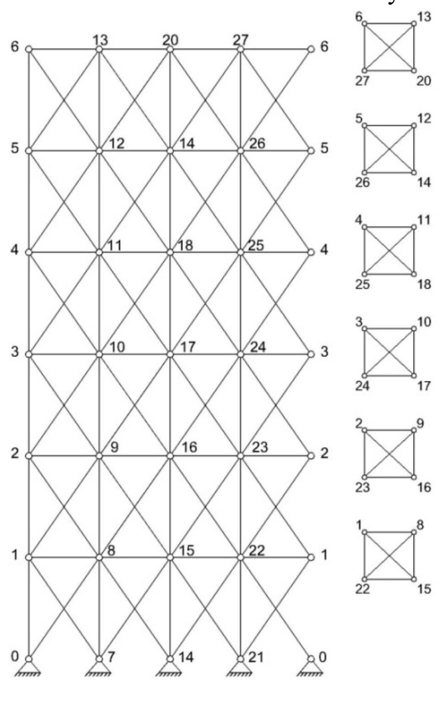

Fig. 2. Nodes and Bars in the Model of the Column Truss

The rectangular truss column is subjected to a cyclic load. The load pattern consists of four equal loads applied at the topmost nodes in the downward direction. The pseudo time diagram for a load cycle is a triangle whose time base is subdivided into eight equal intervals as shown in figure 1. The time stations are numbered 0 to 8. The results for the four chords at a level of a column are equal. The results for the eight diagonals in the vertical planes at a level are also equal. The results are therefore presented for a typical column and a typical chord at each level of the column.

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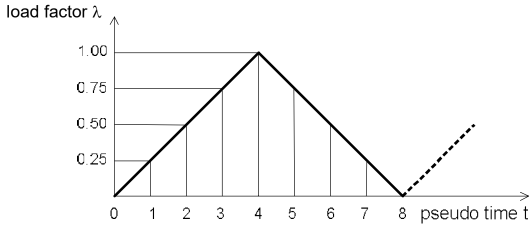

Fig. 2.Variation of the load factor during a period

The first analysis for the cyclic load acting on the column is performed with a safety factor 1.0. The column remains elastic at all time stations. The load factor is then increased to 1.5, 2.75, 5.375 and 10.6875. The column remains elastic at all time stations. This shows that the column shakes down in the range 1.0 to 10.6875 of the safety factor. For the analyses with higher safety factors, the following states of the truss are defined. Bars which are not listed as plastic in a state remain elastic.

state A: column 1-2 is plastic
state B: columns 1-2, 4-5 are plastic
state C: columns 1-2, 3-4, 4-5 are plastic
state D: columns 1-2, 2-3, 3-4, 4-5 are plastic
state E: columns 1-2, 2-3, 3-4, 4-5, 5-6 are plastic
state F: columns 0-1, 1-2, 2-3, 3-4, 4-5, 5-6 are plastic
state G: columns 0-1, 1-2, 2-3, 3-4, 4-5, 5-6, diagonal 2-10 are plastic

In state F, all columns are plastic but all diagonals are elastic. The load can therefore be increased further. In state G, all columns are plastic as well as all diagonals in one level. The truss column has therefore reached its limit state.

Safety factor 21.34375. The truss is elastic at time stations 1 and 2. In the load step to station 3, the columns become plastic for various load levels so that the load step is subdivided as follows:

load -1101.441 state A
load -1101.606 state B
load -1101.766821 state C
load -1101.766842 state D
load -1116.870 state E
load -1132.723 state F
load -1138.257 state G

There is no shakedown since G is the limit state of the truss.

Safety factor 16.015625. The safety factor is reduced to 0.5 ∗(10.6875 + 21.34375) = 16.015625. The truss remains elastic at stations 1 and 2. In the load step to station 3, the columns become plastic for various load levels so that the load step is subdivided as follows:

load -1101.441 state A
load -1101.606 state B
load -1101.766821 state C
load -1101.766842 state D
load -1116.870 state E
load -1201.172 station 3

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In the load step to station 4, the limit load of -1183.257 is reached so that the truss does not shake down.

Safety factor 13.351562. The safety factor is reduced to $0.5 * (10.6875 + 16.015625) = 13.351562$ The truss remains elastic at stations 1 to **3.** In the load step to station 4, the columns become plastic for various load levels so that the load step is subdivided as follows:

load -1101.441 state A
load -1101.606 state B
load -1101.766821 state C
load -1101.766842 state D
load -1116.870 state E
load -1132.723 state F
load -1138.257 station 4

The truss does not reach its limit state in load step 4. During the unloading at stations 5 to 8 the truss is elastic. In the second load cycle, the truss remains elastic at all time stations so that there is shakedown. The following residual stresses act in the truss at time station 8 in the second load cycle. The residual state of stress is self-equilibrating. The vertical equilibrium check at the nodes shows the resultant of the internal forces vanishes.

Residual forces for safety factor 13.351						
column	force	ε_t	ε_p	diagonal	force	ε_t
0-1	85.818	-0.000151	-0.000355	0-8	-47.976	-0.000114
1-2	102.104	-0.000187	-0.000430	1-9	-57.081	-0.000136
2-3	101.930	-0.000187	-0.000430	2-10	-56.984	-0.000136
3-4	101.931	-0.000187	-0.000430	3-11	-56.984	-0.000136
4-5	102.017	-0.000187	-0.,000430	4-12	-57.032	-0.000136
5-6	93.959	-0.000169	-0.000393	5-13	-52.537	-0.000125

Column ε_t of the table contains the total strain, column ε_p the plastic strain. The columns have yielded and their residual stresses are tensile. The diagonals have not yielded and their residual stresses are compressive. The vertical equilibrium conditions at the nodes without external loads are satisfied.

The results of the analyses for the other factors of safety have been analyzed in a similar manner. The results are summarized in the following table:

Summary of the Shakedown Analyses					
safety factor	shakedown	safety factor	shakedown	safety factor	shakedown
21.343750	no	13.393188	no	13.382132	no
16.015625	no	13.372375	yes	13.381806	no
13.351562	yes	13.382782	no	13.381644	no
14.683594	no	13.377579	yes	13.381562	yes
13.518066	no	13.380180	yes	13.381603	yes
13.434814	no	13.381481	yes	13.381623	yes

The safety factor of the truss for shakedown is contained in the range 13.381623 to 13.681644. The ratio between the load for which the truss column

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remains elastic in all load cycles and the shakedown load for which the truss column remains elastic in all load cycles after shakedown has occurred is 1388.16 / 1101.44 = 1.260. The increase due to shakedown is thus 26.0 percent.

The residual forces at the maximum safety factor 13.382 for shakedown are larger that the residual forces which are shown above for safety factor 13.351, as would be expected.

3. Conclusion

The example shows that the implementation of the shakedown theory on the Java platform converges properly. The bisection method reduces the uncertainty interval for the maximum safety factor in 25 bisections to 13.381644 - 13.381623 = 0.000021. The width of the interval is $1.57 * 10^{-6}$ of the value of the factor.

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РАСЧЕТ РЕШЕТЧАТОЙ СТОЙКИ НА ПРИСПОСОБЛЯЕМОСТЬ В УСЛОВИЯХ БОЛЬШИХ ПЕРЕМЕЩЕНИЙ ПРИ ПОМОЩИ ПРЯМОГО ИТЕРАЦИОННОГО МЕТОДА

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На перемещения и устойчивость стальных конструкций оказывают влияние как геометрическая, так и физическая нелинейность. При больших перемещениях условия равновесия и геометрические соотношения существенно отличаются от соответствующих уравнений при малых перемещениях. Если сталь в элементах конструкции достигает текучести, то физические уравнения модифицируются так, что распределение жесткостей в конструкции изменяется и появляется возможность формирования пластического механизма разрушения. В настоящей статье рассмотрен пример расчета на приспособляемость фермы-стойки под действием циклической нагрузки при помощи прямого инкрементального метода, разработанного автором. Метод позволяет учесть геометрическую и физическую нелинейность и допускает большие перемещения конструкции.

КЛЮЧЕВЫЕ СЛОВА: стальные фермы, циклическая нагрузка, приспособляемость, большие перемещения.

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