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Analysis of queuing systems with threshold renovation mechanism and inverse service discipline

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Abstract. The paper presents a study of three queuing systems with a threshold renovation mechanism and an inverse service discipline. In the model of the first type, the threshold value is only responsible for activating the renovation mechanism (the mechanism for probabilistic reset of claims). In the second model, the threshold value not only turns on the renovation mechanism, but also determines the boundaries of the area in the queue from which claims that have entered the system cannot be dropped. In the model of the third type (generalizing the previous two models), two threshold values are used: one to activate the mechanism for dropping requests, the second — to set a safe zone in the queue. Based on the results obtained earlier, the main time-probabilistic characteristics of these models are presented. With the help of simulation modeling, the analysis and comparison of the behavior of the considered models were carried out.

Key words and phrases: queuing system, active queue management, renovation mechanism, threshold, time-probabilistic characteristics, GPSS modelling

1. Introduction

According to [1] the problem of congestion avoidance for communication networks does not have a satisfying solution, so the development and the analysis of new active queue management (AQM) algorithms appears to be the actual task for researches [2]–[13] and practitioners [14]–[24].

In this paper we will consider queuing systems with probabilistic renovation mechanism, which allows to adjust the number of packets in the system by dropping (resetting) them from the queue depending on the ratio of a certain control parameter with specified thresholds [25], [26] at the moment of the end of service on the device (server) [27]–[29] in contrast to standard RED algorithm, when a possible reset occurs at the time of the next packet arrival and the control parameter is an exponentially weighted average queue

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length [30]–[34]. In our models the renovation mechanism uses one or two thresholds (which determine as the place in the buffer from which the packets are dropped, but also the place to which the reset of packets occurs).

The previous works devoted to the analysis of queuing systems with threshold based renovation are [35]–[38]. In [35], [36] some aspects of using the renovation mechanism (different types of renovation, definitions and brief overview were also given) with one or several thresholds as the mathematical models of active queue management mechanisms were considered. Some results of comparing classic RED algorithm with renovation mechanism were presented. In [37] two queuing models with threshold based renovation mechanism were presented: in the first model the threshold value is only responsible for activating the renovation mechanism (the mechanism for probabilistic reset of claims), in the second model the threshold value not only turns on the renovation mechanism, but also determines the boundaries of the area in the queue from which claims that have entered the system cannot be dropped. In [38] the queuing system with two threshold values (one to activate the mechanism for dropping requests, the second — to set a safe zone in the queue) for renovation mechanism was investigated. All three queuing systems have been studied for the service discipline FCFS (First Come First Served), and in this article we will present some results for the discipline LCFS (Last Come First Served). The study will again be carried out using embedded Markov chains. We will not consider in detail the derivation of the stationary distribution of the number of customers (which does not depend on the service discipline and presented in [37], [38]) and will focus only on the service (reset) probabilities and on time characteristics.

The structure of the article is following. In the section 2 the results for the queuing model, where the threshold value is only responsible for activating the renovation mechanism, are presented; the section 3 is devoted to the queuing model, in which the threshold value not only turns on the renovation mechanism, but also determines the boundaries of the area in the queue from which claims that have entered the system cannot be dropped. In section 4 the characteristics for the queuing system with two threshold values (one to activate the mechanism for dropping requests, the second — to set a safe zone in the queue) for renovation mechanism are presented. In section 5 the results of GPSS simulation are considered. The last section 6 concludes the paper with the short discussion.

2. The first model

Consider the $GI/M/1/\infty$ queuing system, shown in the figure 1, with the implemented renovation mechanism and a threshold value Q_1, which determines the boundary in the queue, starting from which the dropping of customers begins. If the current number of packets in the system $i \leqslant Q_1 + 1$ (the threshold value Q_1 is not been overcome), then none of the packets will be dropped from the queue. If the current number of packets in the system $i \geqslant Q_1 + 1$, then with probability q the packet finishing the service can drop all packets from the queue and leave the system, or with probability $p = 1 - q$ the serviced packet simply leaves the system.

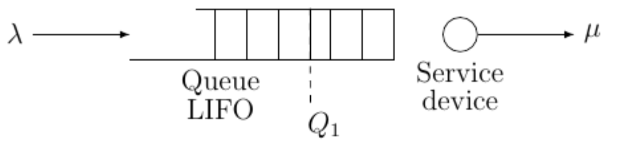

Figure 1. Queuing system model

2.1. The service probability and the loss probability for a received packet

Let $p^{(\text{loss})}$ be the probability that the packet received in the system will be dropped by renovation mechanism and let $p_i^{(\text{loss})}$ be the probability that a packet arriving and finding in the system exactly i packets will be dropped.

Let $p_i^{(\text{loss})}(x)$ be the probability that in a time less than x a packet that finds other i packets in the system will be dropped. Then:

$$p_i^{(\text{loss})} = \int_0^\infty p_{i,0}^{(\text{loss})}(x)dx,$$

where $p_{i,j}^{(\text{loss})}(x)$ is the probability that in time less than x the packet, before which there are i other packets in the queue and after which there are other j packets, will be dropped, $i, j \geqslant 0$.

Let $\tau_{i,j}^{(\text{loss})}(x)$ be the probability density functions and let $\rho_{i,j}^{(\text{loss})}(s)$ be the Laplace–Stieltjes transforms. Then:

$$\tau_{i,j}^{(\text{loss})}(x) = \left(p_{i,j}^{(\text{loss})}(x)\right)'_x, \quad \rho_{i,j}^{(\text{loss})}(s) = \int_0^\infty \tau_{i,j}^{(\text{loss})}(x)dx.$$

a) If $i + j + 1 \leqslant Q_1$ the threshold is not crossed, then:

$$\tau_{i,j}^{(\text{loss})}(x) = \int_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} dA(y) \tau_{i,j-k+1}^{(\text{loss})}(x-y).$$

b.1) If $i + j + 1 > Q_1$, $i + 1 \leqslant Q_1$, then:

$$\tau_{i,j}^{(\text{loss})}(x) = \sum_{k=1}^{\min(j, i+j+1-Q_1)} \frac{\mu^k x^{k-1}}{(k-1)!} e^{-\mu x} \cdot p^{k-1} \cdot q \cdot \overline{A}(x) + \\ + \int_0^y \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} \cdot p^{\min(k, i+1+j-Q_1)} dA(y) \tau_{i,j}^{(\text{loss})}(x-y).$$

b.2) If $i+j+1 > Q_1$, $i+1 > Q_1$, then:

$$\tau^{(\text{loss})}_{i,j}(x) = \sum_{k=1}^{j} \frac{\mu^k x^{k-1}}{(k-1)!} e^{-\mu x} \cdot p^{k-1} \cdot q \cdot \overline{A}(x) + \\ + \int_0^y \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} \cdot p^k dA(y) \tau^{(\text{loss})}_{i,j-k+1}(x-y).$$

Then for the Laplace–Stieltjes transforms $\rho^{(\text{loss})}_{i,j}(s)$ we have:
a) If $i+j+1 \leqslant Q_1$, then:

$$\rho^{(\text{loss})}_{i,j}(s) = \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} \alpha^{(k)}(\mu+s) \cdot \rho^{(\text{loss})}_{i,j-k+1}(s).$$

b.1) If $i+j+1 > Q_1$, $i+1 \leqslant Q_1$, then:

$$\rho^{(\text{loss})}_{i,j}(s) = \sum_{k=1}^{\min(j,i+1+j-Q_1)} \frac{(-1)^{k-1} \mu^k}{(k-1)!} \overline{\alpha}^{(k-1)}(\mu+s) \cdot p^{k-1} \cdot q + \\ + \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} p^{\min(k,i+j+1-Q_1)} \alpha^{(k)}(\mu+s) \cdot \rho^{(\text{loss})}_{i,j-k+1}(s).$$

b.2) If $i+j+1 > Q_1, i+1 > Q_1$, then:

$$\rho^{(\text{loss})}_{i,j}(s) = \sum_{k=1}^{j} \frac{(-1)^{k-1} \mu^k}{(k-1)!} \overline{\alpha}^{(k-1)}(\mu+s) \cdot p^{k-1} \cdot q + \\ + \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} p^k \alpha^{(k)}(\mu+s) \cdot \rho^{(\text{loss})}_{i,j-k+1}(s).$$

2.2. Time characteristics of the system

Let $W^{(\text{serv})}(x)$ and $W^{(\text{loss})}(x)$ be the distribution functions of the time spent in the queue by the served and dropped packets.

2.2.1. Time characteristics for a served packet

$W^{(\text{serv})}_{i,j}(x)$ — the intermediary distribution function of the time spent by the served packet in the queue, if there are i other packets in the queue before the considered one and there are j others after it. Then

$$W^{(\text{serv})}(x) = \left(\sum_{i=0}^{\infty} \pi_i W^{(\text{serv})}_{i,0}(x) \right) \cdot \frac{1}{p^{(\text{serv})}},$$

where steady-state probabilities π_i $(i \geqslant 0f)$ are defined in [37], [38]. For densities $w_{i,j}^{(\text{serv})}(x) = \left(W_{i,j}^{(\text{serv})}(x)\right)'$, we will consider several cases.

a) Consider the case when $i + j + 1 > Q_1$, $0 \leqslant i < Q_1$

$$w_{i,j}^{(\text{serv})}(x) = \frac{\mu^{j+1}x^j}{j!}e^{-\mu x}p_{i+1,j}^{(\text{serv})}\overline{A}(x) + \\ + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}p^{\min(k,j+i+1-Q_1)}dA(y)w_{i,j-k+1}^{(\text{serv})}(x-y),$$

$$p^{\min(k,j+i+1-Q_1)} = \begin{cases} p^k, k \leqslant j+i+1-Q_1, \\ p^{j+1+i-Q_1, k>j+i-Q_1}. \end{cases}$$

b) Let's move on to the case when $i \geqslant Q_1$

$$w_{i,j}^{(\text{serv})}(x) = \frac{\mu^{j+1}x^j}{j!}e^{-\mu x}p^j\overline{A}(x) + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}p^k dA(y)w_{i,j}^{(\text{serv})}(x-y).$$

If $i + j + 1 \geqslant Q_1$ the threshold is not crossed, then:

$$w_{i,j}^{(\text{serv})}(x) = \frac{\mu^{j+1}x^j}{j!}e^{-\mu x}\overline{A}(x) + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}dA(y)w_{i,j}^{(\text{serv})}(x-y).$$

The Laplase–Stieltjes transforms for derived densities.

If $i + j + 1 \leqslant Q_1$, then:

$$\omega_{i,j}^{(\text{serv})}(s) = \frac{(-1)^j\mu^{j+1}}{j!}\overline{\alpha}^{(j)}(\mu+s) + \sum_{k=0}^{j} \frac{(-1)^k\mu^k}{k!}\alpha^{(k)}(\mu+s)\omega_{i,j-k+1}^{(\text{serv})}(s),$$

$$\omega_{i,j}^{(\text{serv})}(s) = \int\limits_0^\infty w_{i,j}^{(\text{serv})}(x)e^{-sx}dx \text{ — Laplace–Stieltjes transform.}$$

If $0 \leqslant i < Q_1$, but $i + j + 1 > Q_1$, then:

$$\omega_{i,j}^{(\text{serv})}(s) = \frac{(-1)^j\mu^{j+1}}{j!}\overline{\alpha}^{(j)}(\mu+s) \cdot p^{j+i+1-Q_1} + \\ + \sum_{k=0}^{j} \frac{(-1)^k\mu^k}{k!}\alpha^{(k)}(\mu+s) \cdot p^{\min(k,j+i+1-Q_1)} \cdot \omega_{i,j-k+1}^{(\text{serv})}(s).$$

If $i \geqslant Q_1$, then:

$$\omega_{i,j}^{(\text{serv})}(s) = \frac{(-1)^j \mu^{j+1}}{j!} \overline{\alpha}^{(j)}(\mu+s) \cdot p^j + \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} \alpha^{(k)}(\mu+s) \cdot p^k \cdot \omega_{i,j-k+1}^{(\text{serv})}(s).$$

2.2.2. Time characteristics for a dropped packet

$W_{i,j}^{(\text{loss})}(x)$ — the intermediary distribution function of the time spent by the dropped packet in the queue, if there are i other packets in the queue before the considered one and there are j others after it. Then

$$W^{(\text{loss})}(x) = \left(\sum_{i=0}^{\infty} \pi_i W_{i,0}^{(\text{loss})}(x) \right) \cdot \frac{1}{p^{(\text{loss})}}.$$

For densities $w_{i,j}^{(\text{loss})}(x) = \left(W_{i,j}^{(\text{loss})}(x) \right)'$, we also will consider several cases.

a) The first case is when $i+1+j \leqslant Q_1$, so the selected packet can be dropped only due to the reception of new packets in the system and overcoming the threshold value

$$w_{i,j}^{(\text{loss})}(x) = \int_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} dA(y) w_{i,j-k+1}^{(\text{loss})}(x-y).$$

b) for the second case, when $i+1+j > Q_1$, $(i+1 \leqslant Q_1)$, several subcases should be considered:

b.1)

$$w_{i,j}^{(\text{loss})}(x) = \sum_{k=1}^{\min(i,i+1+j-Q_1)} \frac{\mu^k x^{k-1}}{(k-1)!} e^{-\mu x} \cdot p^{k-1} \cdot q \cdot \overline{A}(x) + \\ + \int_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} \cdot p^{\min(k,i+1+j-Q_1)} dA(y) w_{i,j-k+1}^{(\text{loss})}(x-y).$$

b.2) If $i+1 > Q_1$, then:

$$w_{i,j}^{(\text{loss})}(x) = \sum_{k=1}^{j} \frac{\mu^k x^{k-1}}{(k-1)!} e^{-\mu x} \cdot p^{k-1} \cdot q \cdot \overline{A}(x) + \\ + \int_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} \cdot p^k dA(y) w_{i,j-k+1}^{(\text{loss})}(x-y).$$

The Laplase–Stieltjes transforms for derived densities.

a) For the case when $i+j+1 \leqslant Q_1$ we have

$$\omega_{i,j}^{(\mathrm{loss})}(s) = \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} \alpha^{(k)}(\mu+s) \cdot \omega_{i,j-k+1}^{(\mathrm{loss})}(s).$$

b) For the case when $i+j+1 > Q_1$, $i+1 \leqslant Q_1$ we obtain:
b.1)

$$\omega_{i,j}^{(\mathrm{loss})}(s) = \sum_{k=1}^{\min(j,i+1+j-Q_1)} \frac{(-1)^{k-1}\mu^k}{(k-1)!} \overline{\alpha}^{(k-1)}(\mu+s) \cdot p^{k-1} \cdot q + \\ + \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} p^{\min(k,i+j+1-Q_1)} \alpha^{(k)}(\mu+s) \cdot \omega_{i,j-k+1}^{(\mathrm{loss})}(s).$$

b.2)

$$\omega_{i,j}^{(\mathrm{loss})}(s) = \sum_{k=1}^{j} \frac{(-1)^{k-1}\mu^k}{(k-1)!} \overline{\alpha}^{(k-1)}(\mu+s) \cdot p^{k-1} \cdot q + \\ + \sum_{k=0}^{j} \frac{(-1)^k \mu^k}{k!} p^{k} \alpha^{(k)}(\mu+s) \cdot \omega_{i,j-k+1}^{(\mathrm{loss})}(s).$$

3. The second model

The second queuing model is also $GI/M/1/\infty$ queuing system, shown in the figure 2, with the implemented renovation mechanism, but the threshold value Q_1 determines the boundary in the queue, starting from which the dropping of customers begins and also determines the safe zone from where packets cannot be dropped.

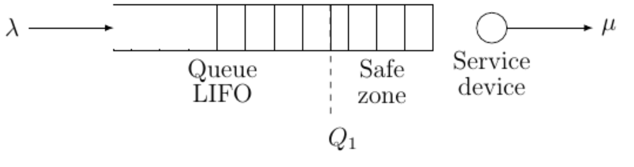

Figure 2. Queuing system model 2

If the current number of packets in the system i is less or equal to $Q_1 + 1$ (the threshold value Q_1 has not been overcome), then none of the packets will be dropped from the queue. If the current number of packets in the system i is greater then $Q_1 + 1$, then with probability q the packet, finishing the service and leaving the system, will drop all packets from the queue (outside the safe zone), or with probability $p = 1 - q$ the serviced packet simply leaves the system.

Let π_i be the steady-state probability distribution of the embedded Markov chain that the packet comming into the system will find in it i other packets ($i \geqslant 0$) [37], [38].

Let $p^{(\text{loss})}$ and $p^{(\text{serv})}$ be the probability that the received packet in the system will be dropped from the queue or will be transferred to service device.

The $p_i^{(\text{serv})}$ is the auxiliary probability that the packet will be served if it finds other i packets in the system.

$$p^{(\text{serv})} = \sum_{i=0}^{\infty} p_i^{(\text{serv})} \cdot \pi_i = 1 - \pi_{Q_1+1} \cdot \frac{q}{(1-g)(1-pg)}.$$

$$p^{(\text{loss})} = 1 - p^{(\text{serv})} = 1 - \left(1 - \pi_{Q_1+1} \cdot \frac{q}{(1-g)(1-pg)}\right),$$

$$p^{(\text{loss})} = \pi_{Q_1+1} \cdot \frac{q}{(1-g)(1-pg)}.$$

3.1. Time characteristics of the system

3.1.1. Time characteristics for serviced packets

$W^{(\text{serv})}(x)$ is the cumulative waiting time distribution function for the accepted into the system packet, $W_i^{(\text{serv})}(x)$ is the cumulative waiting time distribution function for the accepted into the system packet, if at the moment of its arrival there were i other packets in the system. Then:

$$W^{(\text{serv})}(x) = \frac{1}{p^{(\text{serv})}} \sum_{i=0}^{\infty} W_i^{(\text{serv})}(x) \cdot \pi_i,$$

$$\mathrm{w}_i^{(\text{serv})}(x) = \left(W_i^{(\text{serv})}(x)\right)'$$

— probability density function.

The auxiliary functions $W_{i,j}^{(\text{serv})}(x)$ and $\mathrm{w}_{i,j}^{(\text{serv})}(x) = \left(W_{i,j}^{(\text{serv})}(x)\right)'$ $(i, j \geqslant 0)$ are the distribution functions and the densities of distribution functions of the time spent by the served packet in the queue, if there were i other packets in the queue before the considered one and j others after it.

a) If $i = 0$, then the cumulative distribution functions $W_i^{(\text{serv})}(x) = 1, (x = 0)$. **b)** If $0 < i \leqslant Q_1$ — (the safe zone is not completely filled) then the received in the system packet will be in the safe zone (cannot be dropped). Then

$$\mathrm{w}_i^{(\text{serv})}(x) = \mu e^{-\mu x} \cdot \overline{A}(x) + \int_0^x e^{-\mu y} d(y) \cdot \mathrm{w}_{i,1}^{(\text{serv})}(x-y).$$

b.1) $0 < i + j \leqslant Q_1$, $j > 0$ (taking into account the packets that came after ours), the threshold value Q_1 has not been overcome in the queue, that is, the renovation mechanism has not turned on. Then

$$\mathrm{w}_{ij}^{(\mathrm{serv})}(x) = \frac{\mu^{j+1}x^j}{j!}e^{-\mu x} \cdot \overline{A}(x) + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}dA(y) \cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y).$$

b.2) $Q_1 < j+1$ $(j > 0)$ the renovation mechanism was activated, but our packet is in a safe zone. Then

$$\begin{aligned}\mathrm{w}_{ij}^{(\mathrm{serv})}(x) = {} & \frac{\mu^{j+1}x^j}{j!}p^{j-(Q_1-i)+1} \cdot \overline{A}(x) + \frac{\mu^{Q_1-i+1}x^{Q_1-i}}{(Q_1-i)!} \cdot qe^{-\mu x} \cdot \overline{A}(x) + \\ & + \sum_{k=1}^{1+(j-(Q_1-i)-1)} \tilde{\pi}_k(j-(Q_1-i)-k) \cdot \frac{\mu^{k+Q_1-i}x^{k+Q_1-i-1}}{(k+Q_1-i-1)!}e^{-\mu x} \cdot \overline{A}(x) + \\ & + \int\limits_0^x e^{-\mu y}dA(y) \cdot \mathrm{w}_{i,j+1}^{(\mathrm{serv})}(x-y) + \\ & + \int\limits_0^x \sum_{k=1}^{j-(Q_1-i)-1} \frac{(\mu y)^k}{k!}e^{-\mu y} \cdot p^k dA(y) \cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y) + \\ & + \int\limits_0^x \sum_{k=1-(Q_1-i)}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y} \cdot p^{i-Q_1-i} dA(y) \cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y),\end{aligned}$$

$$\begin{aligned}\mathrm{w}_{ij}^{(\mathrm{serv})}(x) = {} & \sum_{k=1}^{j-(Q_1-1)} \tilde{\pi}_k(j-(Q_1-i)-k) \cdot \frac{\mu^{k+Q_1-i}x^{k+Q_1-i-1}}{(k+Q_1-i-1)!}e^{-\mu x} \cdot \overline{A}(x) \\ & + \int\limits_0^x \sum_{k=1}^{j-(Q_1-i)-1} \frac{(\mu y)^k}{k!}e^{-\mu y} \cdot p^k dA(y) \cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y) + \\ & + \int\limits_0^x \sum_{k=1-(Q_1-i)}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y} \cdot p^{i-Q_1-i} dA(y) \cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y).\end{aligned}$$

c) $i \geqslant Q_1 + 1$ — at the time of receipt of our packet, the safe zone is filled and there are packets outside the safe zone — the renovation mechanism is enabled. Then

$$\mathrm{w}_{i,0}^{(\mathrm{serv})}(x) = \mu e^{-\mu x}p \cdot \overline{A}(x) + \int\limits_0^x e^{-\mu y}dA(y) \cdot \mathrm{w}_{i,1}^{(\mathrm{serv})}(x-y),$$

$$\mathrm{w}_{i,j}^{(\mathrm{serv})}(x) = \frac{\mu^{j+1}x^j}{j!}e^{-\mu x}p^{j+1}\overline{A}(x) + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y} \cdot p^k dA(y) \cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y).$$

3.1.2. Time characteristics for dropped packets

Let $W^{(\text{loss})}(x)$ be the cumulative distribution functions of the time spent by the packet in the queue before dropping.

$$W^{(\text{loss})}(x) = \frac{1}{p^{(\text{loss})}} \cdot \sum_{i=0}^{\infty} W_i^{(\text{loss})}(x)\pi_i.$$

$W_i^{(\text{loss})}(x)$ is the conditional probability that in a time less than x the packet that has found exactly i of other packets in the system will be dropped from the queue. The auxiliary functions $W_{i,j}^{(\text{loss})}(x)$ and $\mathrm{w}_{i,j}^{(\text{loss})}(x) = \left(W_{i,j}^{(\text{loss})}(x)\right)'$ $(i, j \geqslant 0)$ are the distribution functions and the densities of distribution functions of the time spent by the dropped packet in the system, if there were i other packets in the queue before the considered one and j others after it.

a) $0 \leqslant i \leqslant Q_1$ (that is, the system was either empty, or at least there was one free space in the safe zone)

$$W_i^{(\text{loss})}(x) = 0.$$

b) $Q_1 < i$ $(i \geqslant Q_1 + 1)$

$$\mathrm{w}_{i,0}^{(\text{loss})}(x) = \mu e^{-\mu x} q \cdot \overline{A}(x) + \int\limits_0^x e^{-\mu y} dA(y) \cdot \mathrm{w}_{i,1}^{(\text{loss})}(x-y),$$

$$\mathrm{w}_{i,j}^{(\text{loss})}(x) \sum_{k=1}^{j+1} \frac{\mu^k x^{k-1}}{(k-1)!} e^{-\mu x} \cdot \tilde{\pi}_k(j+i-Q_1-k)\overline{A}(x) + \\ + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} \cdot \sum_{l=0}^{j-k} \pi_k(l) dA(y) \cdot \mathrm{w}_{i,j-k-l+1}^{(\text{loss})}(x).$$

4. The third model

Consider the $GI/M/1/\infty$ queuing system, shown in the figure 3.

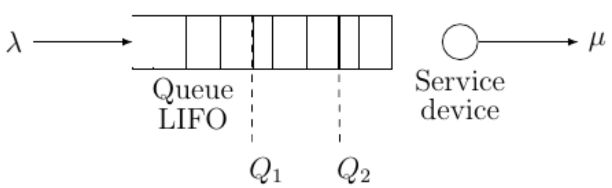

Figure 3. Queuing system model 3

In this section, a single-server queueing system with an infinite queue capacity and two threshold values is considered. Threshold values:

— Q_1 — the threshold value in the queue, when overcoming which by the queue length packets (from Q_1+1) will be dropped from the queue with a probability q.
— Q_2 — the threshold value in the queue to which packets are dropped (i.e. packets standing in the queue up to the Q_2 threshold are not dropped).

4.1. The service probability and loss probability of the received packet

Let's introduce the probability $p^{(\mathrm{serv})}$ that the packet, entering the system, will be served, auxiliary probabilities $p_i^{(\mathrm{serv})}$ $(i \geqslant 0)$ of incoming packet to be served if there were other i $(i \geqslant 0)$ packets in the system, and auxiliary probabilities $p_{i,j}^{(\mathrm{serv})}(x)$ that during the time x the packet, which found exactly i other packets in the system at the moment of arrival and behind which there are j more packets, will be served

$$p^{(\mathrm{serv})} = \sum_{i=0}^{\infty} p_i^{(\mathrm{serv})} \pi_i,$$

where π_i — the stationary probabilities [37], [38].

Let's consider several cases

a) The first one, when the system is empty: $p_0^{(\mathrm{serv})} = 1$.

b) The second case is when $1 \leqslant i \leqslant Q_2$, so $p_i^{(\mathrm{serv})} = 1$.

c) The third case $Q_2 < i \leqslant Q_1$ includes two subcases:

c.1) the first subcase, $Q_2 + 1 \leqslant i + 1 + j \leqslant Q_1 + 1$ — the Q_1 threshold in the queue has not been overcome (taking into account the packets after the considered one), that is, the renovation mechanism has not turned on

$$p_{i,j}^{(\mathrm{serv})}(x) = \overline{A}(x) \cdot \frac{(\mu x)^{j+1}}{(j+1)!} e^{-\mu x} + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} dA(y) \cdot p_{i,j-k+1}^{(\mathrm{serv})}(x-y).$$

c.2) the second subcase, $i + 1 + j > Q_1 + 1$ — the Q_1 threshold in the queue has been overcome, so the renovation mechanism has been activated

$$\begin{aligned} p_{i,j}^{(\mathrm{serv})}(x) = \overline{A}(x) \cdot \frac{(\mu x)^{j+1}}{(j+1)!} e^{-\mu x} \cdot p^{i+j+1-(Q_1+1)} + \\ + \int\limits_0^x \sum_{k=0}^{i+j-Q_1} \frac{(\mu y)^k}{k!} e^{-\mu y} p^k dA(y) \cdot p_{i,j-k+1}^{(\mathrm{serv})}(x-y) + \\ + \int\limits_0^x \sum_{k=i+j-Q_1+1}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} p^{i+j-Q_1} dA(y) \cdot p_{i,j-k+1}^{(\mathrm{serv})}(x-y). \end{aligned}$$

d) the fourth case is when the Q_1 threshold in the queue has been overcome at the moment of the arrival of the considered packet, $(i > Q_1)$ so the renovation mechanism has been already activated

$$p_{i,j}^{(\text{serv})}(x) = \overline{A}(x) \cdot \frac{(\mu x)^{j+1}}{(j+1)!} e^{-\mu x} p^{j+1} + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} p^k dA(y) \cdot p_{i,j-k}^{(\text{serv})}(x-y),$$

$$p_i^{(\text{serv})} = \int\limits_0^\infty p_{i,0}^{(\text{serv})}(x) dx.$$

Loss probability of the received packet

$$p^{(\text{loss})} = \sum_{i=0}^{\infty} p_i^{(\text{loss})} \pi_i,$$

where $p_i^{(\text{loss})}$ — the probability that the incoming packet will be dropped if at the moment of its arrival there were i, $i \geqslant 0$ other packets in the system, and $p_{i,j}^{(\text{loss})}(x)$ is the probability that in time less than x the packet, before which there are i other packets in the queue and after which there are other j packets, will be dropped, $i, j \geqslant 0$.

a) $p_1^{(\text{loss})} = 0$, $i = \overline{0, Q_2}$;

b) $Q_2 < i \leqslant Q_1$ the threshold value of Q_1 has not been reached at the time of receipt;

b.1) $i + 1 + j \leqslant Q_1 + 1$ — (the threshold has not been crossed even taking into account the application that came later)

$$p_{i,j}^{(\text{loss})}(x) = \int\limits_0^y \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} dA(y) \cdot p_{i,j-k+1}^{(\text{loss})}(x-y).$$

b.2) $i+1+j > Q_1+1$ — (the Q_1 threshold was overcome due to applications after the incoming one)

$$p_{i,j}^{(\text{loss})}(x) = \overline{A}(x) \sum_{k=1}^{i+j+1-(Q_1+1)} \frac{(\mu x)^k}{k!} e^{-\mu x} p^{k-1} q +$$
$$+ \int\limits_0^x \sum_{k=0}^{i+j-Q_1} \frac{(\mu y)^k}{k!} e^{-\mu y} p^k dA(y) p_{i,j-k+1}^{(\text{loss})}(x-y) +$$
$$+ \int\limits_0^x \sum_{k=i+j-Q_1+1}^{j} \frac{(\mu y)^k}{k!} e^{-\mu y} p^{i+j-Q_1} dA(y) p_{i,j-k+1}^{(\text{loss})}(x-y).$$

c) $i > Q_1$

$$p_{i,j}^{(\text{loss})}(x) = \overline{A}(x) \sum_{k=1}^{j+1} \frac{(\mu x)^k}{k!} e^{-\mu x} p^{k-1} q + \\ + \int_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!} \cdot e^{-\mu y} p^k dA(y) p_{i,j-k+1}^{(\text{loss})}(x-y);$$

$$p_i^{(\text{loss})} = \int_0^\infty p_{i,0}^{(\text{loss})}(x) dx.$$

4.2. Time characteristics of the system

Let $W^{(\text{loss})}(x)$ and $W^{(\text{serv})}(x)$ be the cumulative distribution functions of the time spent in the system by the packet before being dropped or served. The auxiliary functions $W_{i,j}^{(\text{serv})}(x)$ and $\mathrm{w}_{i,j}^{(\text{serv})}(x) = \left(W_{i,j}^{(\text{serv})}(x)\right)'$, $W_{i,j}^{(\text{loss})}(x)$ and $\mathrm{w}_{i,j}^{(\text{loss})}(x) = \left(W_{i,j}^{(\text{serv})}(x)\right)'$ $(i, j \geqslant 0)$ are the distribution functions and the densities of distribution functions of the time spent by the served (lossed) packet in the queue, if there were i other packets in the queue before the considered one and j others after it. Then

$$W^{(\text{serv})}(x) = \frac{1}{p^{(\text{serv})}} \sum_{i=0}^{\infty} W_{i,j}^{(\text{serv})}(x) \cdot \pi_i,$$

$$W^{(\text{loss})}(x) = \frac{1}{p^{(\text{loss})}} \sum_{i=0}^{\infty} W_{i,j}^{(\text{loss})}(x) \cdot \pi_i.$$

a) If a packet enters the empty system ($i = 0$), it immediately starts to be served.

$$\mathrm{w}_{0,0}^{(\text{serv})}(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geqslant 0, \end{cases}$$

$$\omega_{0,0}^{(\text{serv})}(s) = \int_0^\infty e^{-sx} \mathrm{w}_{0,0}^{(\text{serv})}(x) d(x) = 1,$$

$$\mathrm{w}_{0,0}^{(\text{loss})}(x) = 0.$$

b) If the total number of packets in the system has not overcome the threshold Q_2 $(0 < i \leqslant Q_1,\ i + j + 1 \leqslant Q_1)$, then the considered packet will be in the safe area and the renovation mechanism is not enabled.

$$\mathrm{w}_{i,0}^{(\text{serv})}(x) = \overline{A}(x) \cdot \mu e^{-\mu x} + \int_0^x e^{-\mu y} dA(y) \cdot \mathrm{w}_{i,1}^{(\text{serv})}(x-y).$$

$$\mathrm{w}_{i,j}^{(\mathrm{serv})}(x) = \overline{A}(x)\frac{\mu^{j+1}x^j}{j!}e^{-\mu x} + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y),$$

$$\omega_{i,j}^{(\mathrm{serv})}(s) = \frac{(-1)^j\mu^{j+1}}{j!}\overline{\alpha}^{(j)}(s+\mu) + \sum_{k=0}^{j}\frac{(-\mu)^k}{k!}\times\alpha^{(k)}(s+\mu)\cdot\omega_{i,j-k+1}^{(\mathrm{serv})}(s),$$

$$\mathrm{w}_{i,j}^{(\mathrm{loss})}(x) = 0.$$

c) The case, when at the moment of arrival of the considered packet there were $0 < i < Q_2$ other packets in the system (our packet was in the safe area), but currently the total number of packets in the system is equal to $i+j+1 > Q_1$ (so the renovation mechanism is enabled)

$$\begin{aligned}\mathrm{w}_{i,j}^{(\mathrm{serv})}(x) = {} & \frac{\mu^{i+j}x^j}{j!}e^{-\mu x}p^{i+j+1-Q_1}\overline{A}(x) + \\ & + \overline{A}(x)\int\limits_0^x \sum_{k=1}^{i+j+1-Q_1}\frac{(\mu y)^k}{k!}e^{-\mu y}p^{k-1}q\mu dy\frac{(\mu(x-y))^{Q_2-i-1}}{(Q_2-i-1)}e^{-\mu(x-y)} + \\ & + \int\limits_0^x \sum_{k=0}^{i+j+1-Q_1}\frac{(\mu y)^k}{k!}e^{-\mu x}p^{k-1}qdA(y)\mathrm{w}_{i,Q_2-i-1+1}^{(\mathrm{serv})}(x-y) + \\ & + \int\limits_0^x \sum_{k=i+j+1-Q_1-1}^{j}\frac{(\mu y)^k}{k!}e^{\mu y}p^{i+j+1-Q_1}dA(y)\mathrm{w}_{i,j-k}^{(\mathrm{serv})}(x),\end{aligned}$$

$$\mathrm{w}_{i,j}^{(\mathrm{loss})}(x) = 0.$$

d) The case, when at the moment of arrival of the considered packet there were $Q_2 < i < Q_1$ other packets in the system (our packet was out of the safe area), includes several subcases.

d.1) The first subcase — currently the total number of packets in the system is $Q_2 < i+j+1 \leqslant Q_1$ (the renovation mechanism is not enabled)

$$\mathrm{w}_{i,j}^{(\mathrm{serv})}(x) = \overline{A}(x)\frac{\mu^{j+1}x^j}{j!}e^{-\mu x} + \int\limits_0^x \sum_{k=0}^{j}\frac{(\mu y)^k}{k!}\times e^{-\mu y}dA(y)\cdot\mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y),$$

$$\mathrm{w}_{i,j}^{(\mathrm{loss})}(x) = \int\limits_0^x \sum_{k=0}^{i+j+1-Q_2}\frac{\mu y}{k!}e^{-\mu y}dAy\cdot\mathrm{w}_{i,j-k+1}^{(\mathrm{loss})}(x-y).$$

d.2) The second subcase, when currently the total number of packets in the system has overcome the threshold Q_1 $(i+j+1 > Q_1)$, so the renovation mechanism is activated

$$\mathrm{w}_{i,j}^{(\mathrm{serv})}(x) = \overline{A}(x)\frac{\mu^{j+i}x^j}{j!}e^{-\mu y}\cdot p^{i+j+1-Q_1} + \\ + \int\limits_0^x \sum_{k=0}^{i+j+1-Q_1} \frac{(\mu y)^k}{k!}e^{-\mu y}p^k dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y) + \\ + \int\limits_0^x \sum_{k=i+j+1-Q_1+1}^{j} \frac{(\mu y)^k}{k!}\cdot p^{i+j+1-Q_1}e^{-\mu y}dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y),$$

$$\mathrm{w}_{i,j}^{(\mathrm{loss})}(x) = \overline{A}(x)\sum_{k=1}^{i+j+1-Q_1}\frac{\mu^k x^{k-1}}{(k-1)!}p^{k-1}qe^{-\mu x} + \\ + \int\limits_0^x \sum_{k=0}^{i+j+1-Q_1} \frac{(\mu u)^k}{k!}p^k e^{-\mu y} dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{loss})}(x-y) + \\ + \int\limits_0^x \sum_{k=i+j+1-Q_1+1}^{j} \frac{(\mu y)^k}{k!}\cdot p^{i+j+1-Q_1}e^{-\mu y}dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{loss})}(x-y).$$

e) The last case, when the threshold Q_1 was overcome ($i > Q_1$) at the moment of our packet arrival

$$\mathrm{w}_{i,j}^{(\mathrm{serv})}(x) = \overline{A}(x)\frac{\mu^{j+1}x^j}{j!}e^{-\mu x}p^{j+1} + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}p^k dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{serv})}(x-y),$$

$$\mathrm{w}_{i,j}^{(\mathrm{loss})}(x) = \overline{A}(x)\sum_{k=1}^{j+1}\frac{\mu^k x^{k-1}}{(k-1)!}e^{-\mu x}p^{k-1}q + \\ + \int\limits_0^x \sum_{k=0}^{j} \frac{(\mu y)^k}{k!}e^{-\mu y}p^k dA(y)\cdot \mathrm{w}_{i,j-k+1}^{(\mathrm{loss})}(x-y).$$

5. GPSS simulation results

Below (see table 1) is presented a table with GPSS simulation results that was performed with the following initial parameters: threshold value $Q_1 = 30$, arrival rate — 14 task per 1 unit of time, service rate — 16 task per 1 unit of time, and the simulation time is 100000 units of time) for different drop probabilities.

The table 2 shows the results of GPSS simulation that was performed with the following initial parameters: arrival rate — 14 task per 1 unit of time, service rate — 16 task per 1 unit of time, $q = 0.01$, and the simulation time is 100000 units of time) for different threshold values. For the third model the threshold value $Q_2 = 10$.

Table 1

Simulation results for different drop probabilities

q **propability**		**0.0025**	**0.005**	**0.01**	**0.025**	**0.05**	**0.1**	**0.15**
Generated tasks	sys.1	1401525	1401566	1401134	1400127	1400915	1399127	1398795
	sys.2	1400992	1401374	1401547	1400816	1401421	1400971	1401135
	sys.3	1401647	1401379	1400564	1400333	1400889	1400251	1399581
Serviced tasks	sys.1	1400084	1398863	1396791	1394210	1393457	1389597	1389540
	sys.2	1400752	1400843	1400879	1399692	1399428	1399166	1399030
	sys.3	1400537	1399411	1397201	1395975	1395643	1393555	1393104
Serviced tasks without calling the renv. mech.	sys.1	1379233	1381969	1385859	1388162	1388647	1386899	1387651
	sys.2	1378347	1381669	1385318	1388493	1387780	1391338	1391897
	sys.3	1379887	1382616	1385828	1389605	1390628	1390814	1391166
Dropped tasks	sys.1	1436	2698	4332	5917	7456	9530	9249
	sys.2	240	527	663	1117	1984	1803	2104
	sys.3	1091	1967	3357	4357	5240	6696	6472
Service Probability	sys.1	0.9990	0.9981	0.9969	0.9958	0.9947	0.9932	0.9934
	sys.2	0.9998	0.9996	0.9995	0.9992	0.9986	0.9987	0.9985
	sys.3	0.9992	0.9986	0.9976	0.9969	0.9963	0.9952	0.9954
Drop Probability	sys.1	0.0010	0.0019	0.0031	0.0042	0.0053	0.0068	0.0066
	sys.2	0.0002	0.0004	0.0005	0.0008	0.0014	0.0013	0.0015
	sys.3	0.0008	0.0014	0.0024	0.0031	0.0037	0.0048	0.0046
Average queue length	sys.1	6.0930	5.9230	5.7090	5.5240	5.4820	5.3080	5.2360
	sys.2	6.1800	6.0780	6.0220	5.8580	5.9530	5.7980	5.8550
	sys.3	6.1230	5.9360	5.7330	5.5720	5.5560	5.4120	5.3290
Maximum queue length	sys.1	92	71	63	67	54	46	43
	sys.2	92	64	61	65	60	51	49
	sys.3	92	71	71	67	54	46	43
Average waiting time	sys.1	0.497	0.483	0.467	0.453	0.449	0.437	0.431
	sys.2	0.503	0.495	0.491	0.478	0.485	0.473	0.478
	sys.3	0.499	0.484	0.469	0.456	0.454	0.444	0.438

Table 2

Simulation results for different threshold values

Threshold value Q_1		**10**	**20**	**25**	**30**	**40**	**50**	**75**
Generated tasks	sys.1	1399202	1401573	1401188	1401134	1399645	1400335	1400451
	sys.2	1399603	1400523	1399393	1401547	1402003	1400032	1399596
	sys.3	1399603	1400753	1400647	1400564	1399680	1400321	1400448
Serviced tasks	sys.1	1368353	1389618	1393927	1396791	1398462	1399917	1400367
	sys.2	1387180	1397457	1397721	1400879	1401813	1399986	1399562
	sys.3	1387180	1393344	1395743	1397201	1398764	1399969	1400374
Serviced tasks without calling the renv. mech.	sys.1	1166280	1343186	1370099	1385859	1394747	1398969	1400319
	sys.2	1145456	1336931	1365038	1385318	1396545	1398819	1399341
	sys.3	1145456	1346681	1372422	1385828	1395050	1399021	1400326
Dropped tasks	sys.1	30833	11955	7261	4332	1176	407	83
	sys.2	12423	3065	1672	663	190	42	33
	sys.3	12423	7409	4902	3357	916	337	73
Service Probability	sys.1	0.9780	0.9915	0.9948	0.9969	0.9992	0.9997	0.9999
	sys.2	0.9911	0.9978	0.9988	0.9995	0.9999	1.0000	1.0000
	sys.3	0.9911	0.9947	0.9965	0.9976	0.9993	0.9997	0.9999
Drop Probability	sys.1	0.0220	0.0085	0.0052	0.0031	0.0008	0.0003	0.0001
	sys.2	0.0089	0.0022	0.0012	0.0005	0.0001	0.0000	0.0000
	sys.3	0.0089	0.0053	0.0035	0.0024	0.0007	0.0002	0.0001
Average queue length	sys.1	4.564	5.273	5.5330	5.7090	5.9110	5.934	6.158
	sys.2	5.069	5.7	5.8540	6.0220	6.0780	6.014	6.089
	sys.3	5.069	5.37	5.5630	5.7330	5.9210	5.933	6.158
Maximum queue length	sys.1	67	64	71	63	80	76	89
	sys.2	67	75	62	61	64	76	102
	sys.3	67	75	59	71	80	76	89
Average waiting time	sys.1	0.381	0.433	0.454	0.467	0.484	0.485	0.502
	sys.2	0.418	0.466	0.479	0.491	0.496	0.491	0.497
	sys.3	0.418	0.441	0.456	0.469	0.485	0.485	0.502

6. Conclusion

Based on the simulation results 1, the following conclusions can be drawn. The largest number of dropped packets, as expected, is observed in the first model, the smallest — in the second model (due to the safe zone). The third model shows an average result compared to the first and the second models. The largest number of serviced packets is in the second model, then — in the third model. The smallest number of serviced packets is in the first model.

The probability of a packet to be dropped is about five times greater for the first model than for the second model, and 20–30 percent more than for the third model.

The average waiting time for the second model is about 5–10 percent greater than the same characteristic for the first and third models.

As the value of the renovation probability q increases, the drop probability increases for all three models, and the service probability decreases accordingly. Also, with an increase of the renovation probability q, both the average and maximum queue lengths decrease, and the average waiting time also decreases.

Based on the simulation results 2, the following conclusions can be drawn. With an increase of the threshold value Q_1 responsible for switching on the renovation mechanism, the number of dropped packets decreases for all three models (the second model is characterized by the smallest number of dropped packets), the service probability increases to unity (the second model), and the drop probability decreases almost to zero. The average and maximum queue lengths increase, and the values for the first and third models become approximately the same. The average waiting time also increases, and again for the first and third models, the values become approximately the same.

The third model, which generalizes the first and the second models, shows average results compared to the above models, and is more preferable for use as a queue length management model.

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References

[1] F. Baker and G. Fairhurst. "IETF Recommendations Regarding Active Queue Management. RFC 7567". (Jul. 2015), [Online]. Available: `https://tools.ietf.org/html/rfc7567`.

[2] K. Nichols and V. Jacobson, "Controlling queue delay", *Communications of the ACM*, vol. 55, no. 7, pp. 42–50, May 2012. DOI: `10.1145/2209249.2209264`.

[3] T. Hoeiland-Joergensen *et al.* "The flow queue codel packet scheduler and active queue management algorithm. RFC 8290". (2018), [Online]. Available: `https://www.rfc-editor.org/info/rfc8290`.

[4] S. Jung, J. Kim, and J.-H. Kim, "Intelligent active queue management for stabilized QoS guarantees in 5G mobile networks", *IEEE Systems Journal*, vol. 15, pp. 4293–4302, 2021. DOI: `10.1109/JSYST.2020.3014231`.

[5] W.-c. Feng, D. Kandlurz, D. Sahaz, and K. Shin, “BLUE: a new class of active queue management algorithms”, University of Michigan, Tech. Rep., Sep. 2000.

[6] W.-c. Feng, D. Kandlur, and D. Saha, “The BLUE active queue management algorithms”, *Networking, IEEE/ACM Transactions on*, vol. 10, pp. 513–528, Sep. 2002. DOI: 10.1109/TNET.2002.801399.

[7] C. Zhang, J. Yin, and Z. Cai, “RSFB: a resilient stochastic fair blue algorithm against spoofing DDoS attacks”, in *9th International Symposium on Communications and Information Technology*, 2009, pp. 1566–1567. DOI: 10.1109/ISCIT.2009.5341033.

[8] T. Hoiland-Jorgensen, D. Taht, and J. Morton, “Piece of CAKE: a comprehensive queue management solution for home gateways”, in *IEEE International Symposium on Local and Metropolitan Area Networks (LANMAN)*, Jun. 2018, pp. 37–42. DOI: 10.1109/LANMAN.2018.8475045.

[9] J. Palmei, S. Gupta, P. Imputato, J. Morton, M. Tahiliani, S. Avallone, and D. Taht, “Design and evaluation of COBALT queue discipline”, in *IEEE International Symposium on Local and Metropolitan Area Networks (LANMAN)*, Jul. 2019, pp. 1–6. DOI: 10.1109/LANMAN.2019.8847054.

[10] A. Roy, J. L. Pachuau, and A. K. Saha, “An overview of queuing delay and various delay based algorithms in networks”, *Computing*, vol. 103, pp. 2361–2399, 2021. DOI: 10.1007/s00607-021-00973-3.

[11] W. de Morais, C. E. M. Santos, and C. M. Pedroso, “Application of active queue management for real-time adaptive video streaming”, *Telecommun Syst*, vol. 79, pp. 261–270, 2022. DOI: 10.1007/s11235-021-00848-0.

[12] J. George and R. Santhosh, “Congestion control mechanism for unresponsive flows in Internet through active queue management system (AQM)”, *Lecture Notes on Data Engineering and Communications Technologies*, vol. 68, pp. 765–777, 2022. DOI: 10.1007/978-981-16-1866-6_58.

[13] S. Singha, B. Jana, N. K. Mandal, S. Jana, S. Bandyopadhyay, and S. Midya, “Application of dynamic weight with distance to reduce packet loss in RED based algorithm”, *Lecture Notes in Networks and Systems*, vol. 292, pp. 530–543, 2022. DOI: 10.1007/978-981-16-4435-1_52.

[14] R. Adams, “Active queue management: a survey”, *Communications Surveys & Tutorials, IEEE*, vol. 15, pp. 1425–1476, Jan. 2013. DOI: 10.1109/SURV.2012.082212.00018.

[15] M. Menth and S. Veith, “Active queue management based on congestion policing (CP-AQM)”, in Jan. 2018, pp. 173–187. DOI: 10.1007/978-3-319-74947-1_12.

[16] A. Chydzinski and L. Chrost, “Analysis of AQM queues with queue size based packet dropping”, *Applied Mathematics and Computer Science*, vol. 21, pp. 567–577, Sep. 2011. DOI: 10.2478/v10006-011-0045-7.

[17] A. Chydzinski and P. Mrozowski, “Queues with dropping functions and general arrival processes”, *PloS one*, vol. 11, e0150702, Mar. 2016. DOI: 10.1371/journal.pone.0150702.

[18] M. Konovalov and R. Razumchik, “Numerical analysis of improved access restriction algorithms in a GI/G/1/N system”, *Journal of Communications Technology and Electronics*, vol. 63, pp. 616–625, Jun. 2018. DOI: 10.1134/S1064226918060141.

[19] M. Konovalov and R. Razumchik, “Comparison of two active queue management schemes through the M/D/1/N queue”, *Informatika i ee Primeneniya*, vol. 12, no. 4, pp. 9–15, 2018, in Russian. DOI: 10.14357/19922264180402.

[20] C. SSo-In, R. Jain, and J. Jiang, “Enhanced forward explicit congestion notification (E-FECN) scheme for datacenter Ethernet networks”, in *International Symposium on Performance Evaluation of Computer and Telecommunication Systems*, 2008, pp. 542–546.

[21] C. Gomez, X. Wang, and A. Shami, “Intelligent active queue management using explicit congestion notification”, in *IEEE Global Communications Conference (GLOBECOM)*, Sep. 2019, pp. 1–6. DOI: 10.20944/preprints201909.0077.v1.

[22] S. Shahzad, E.-S. Jung, J. F. Chung M., and R. Kettimuthu, “Enhanced explicit congestion notification (EECN) in TCP with P4 programming”, in *International Conference on Green and Human Information Technology (ICGHIT)*, Feb. 2020. DOI: 10.1109/ICGHIT49656.2020.00015.

[23] S. Wang, J. Zhang, T. Huang, T. Pan, J. Liu, and Y. Liu, “A-ECN minimizing queue length for datacenter networks”, *IEEE Access*, vol. 8, pp. 49 100–49 111, 2020. DOI: 10.1109/ACCESS.2020.2979216.

[24] A. Bashir, E. Machnev, and E. Mokrov, “Queueing model of hysteretic congestion control for cloud wireless sensor networks”, in *13th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, 2021, pp. 104–108. DOI: 10.1109/ICUMT54235.2021.9631576.

[25] S. Li, Q. Xu, J. Gaber, Z. Dou, and J. Chen, “Congestion control mechanism based on dual threshold DI-RED for WSNs”, *Wireless Personal Communications*, vol. 115, pp. 2171–2195, 2020. DOI: 10.1007/s11277-020-07676-6.

[26] S. Singha, B. Jana, S. Jana, and N. K. Mandal, “An innovative active queue management model through threshold adjustment using queue size”, *Advances in Intelligent Systems and Computing*, vol. 1406, pp. 257–273, 2022. DOI: 10.1007/978-981-16-5207-3_23.

[27] A. Kreinin, “Queueing systems with renovation”, *Journal of Applied Mathematics and Stochastic Analysis*, vol. 10, pp. 431–443, Jan. 1997. DOI: 10.1155/S1048953397000464.

[28] M. Konovalov and R. Razumchik, *Queueing systems with renovation vs. queues with red. supplementary material*, 2017. arXiv: 1709.01477.

[29] A. V. Gorbunova and A. V. Lebedev, “Queueing system with two input flows, preemptive priority, and stochastic dropping”, *Automation and Remote Control*, vol. 81, no. 12, pp. 2230–2243, 2020. DOI: 10.1134/S0005117920120073.

[30] S. Floyd and V. Jacobson, "Random early detection gateways for congestion avoidance", *IEEE/ACM Transactions on Networking*, vol. 1, pp. 397–413, Sep. 1993. DOI: 10.1109/90.251892.

[31] K. Ramakrishnan, S. Floyd, and D. Black. "RFC3168: The Addition of Explicit Congestion Notification (ECN) to IP". (2001), [Online]. Available: https://tools.ietf.org/html/rfc3168.

[32] S. Floyd, R. Gummadi, and S. Shenker, *Adaptive RED: an algorithm for increasing the robustness of RED's active queue management*, Sep. 2001.

[33] A. V. Korolkova, D. S. Kulyabov, and A. I. Chernoivanov, "On the classification of RED algorithms", *Bulletin of Peoples' Friendship University of Russia*, no. 3, pp. 34–46, 2009, in Russian.

[34] W.-C. Feng, "Improving Internet congestion control and queue management algorithms", The University of Michigan, Tech. Rep., 1999.

[35] H. C. C. Viana, I. Zaryadov, V. Tsurlukov, T. Milovanova, E. Bogdanova, A. Korolkova, and D. Kulyabov, "The general renovation as the active queue management mechanism. Some aspects and results", *Communications in Computer and Information Science*, vol. 1141, pp. 488–502, 2019. DOI: 10.1007/978-3-030-36625-4_39.

[36] H. C. C. Viana, I. S. Zaryadov, and T. A. Milovanova, "Queueing systems with different types of renovation mechanism and thresholds as the mathematical models of active queue management mechanism", *Discrete and Continuous Models and Applied Computational Science*, vol. 28, no. 4, pp. 305–318, 2020. DOI: 10.22363/2658-4670-2020-28-4-305-318.

[37] H. C. C. Viana, I. S. Zaryadov, and T. A. Milovanova, "Two types of single-server queueing systems with threshold-based renovation mechanism", *Lecture Notes in Computer Science*, vol. 13144, pp. 196–210, 2021. DOI: 10.1007/978-3-030-92507-9_17.

[38] H. C. C. Viana and I. S. Zaryadov, "Single-server queuing systems with exponential service times and threshold-based renovation", in *13th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, 2021, pp. 91–97. DOI: 10.1109/ICUMT54235.2021.9631585.

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Анализ систем массового обслуживания с пороговым механизмом обновления и инверсионной дисциплиной обслуживания

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Аннотация. В работе представлено исследование трёх систем массового обслуживания с пороговым механизмом обновления и инверсионной дисциплиной обслуживания. В модели первого типа пороговое значение отвечает только за активацию механизма обновления — механизма вероятностного сброса заявок. Во второй модели пороговое значение не только включает механизм обновления, но и определяет в накопителе границы области, из которой поступившие в систему заявки не могут быть сброшены. В модели третьего типа, обобщающей предыдущие две модели, используются два пороговых значения: одно для активации механизма сброса заявок, второе — для задания безопасной зоны в накопителе. На основе полученных ранее результатов представлены основные вероятностно-временные характеристики рассмотренных моделей. С помощью имитационного моделирования проведён анализ и сравнение поведения изученных моделей.

Ключевые слова: система массового обслуживания, активное управление очередью, механизм обновления, пороговое значение, временные характеристики, GPSS