INTERACTION OF A PROTECTING WALL WITH A SOIL MASSIF IN PROCESS EXTRACTIONS OF FOUNDATION PIT

Abu Mahadi Mohamed Ibrahim, V.I. Elfimov

Department of Building Structures and Constructions
Engineering faculty
Peoples Friendship University of Russia
Mikluha-Maklay str., 6, Moscow, Russia, 117198

The computing procedure to determine the soil pressure on retaining structure and it's displacements during earth excavation from a pit is developed.

At the first stage the protecting construction (slurry wall, sheet piling, row of piles) is enclosed in a ground from the both sides, then the earth excavation from one side of the screen begins. The variations of soil pressure on the both sides of protecting screen are traced in the process of computing with proposed method. The critical depth of a pit at the moment of failure is determined. The retaining structure in the calculating scheme is simulated as an elastic beam construction. The soil is presented as a nonlinear model, in which the modulus of rigidity decreases when the soil stress is nearing to the ultimate limit value. The mathematical procedure consists of iteration of sequential separate calculations by Gauss's method.

Key words: soil pressure, retaining structure, nonlinear model.

Introduction. The determination of soil pressure on protecting structures is one of the most important engineering problem in designing of civil, industrial, transport and hydraulic engineering earthworks. In real constructions the large role is played by yielding (or pliability) of a protecting structure. The computing procedure evaluating soil pressure and displacement of a protecting construction should reflect mechanical interaction of retaining structure with soil.

Main principles and results of numerical experiments with the computing procedure reflecting interaction of a flexible protecting construction with soil during earth excavation from a pit is presented in this paper.

Model of soil and structure interaction. The diagram of flexible retaining wall with applied forces is presented in fig. 1. The soil pressure on a retaining structure (axis z) under the pit bottom (to left of z — in soil mass 1) is represented by a system of point forces R_i; the soil pressure behind the protection (to right of z — in soil mass 2) — by forces P_i.

The number of forces P_i is equal n ($n = 7$ in fig. 1); the number of forces R_i is equal $(n - L)$ ($L = 3$ in fig. 1). Number L is determined by the depth of a pit h. The distance between the points of application of these forces is $2d$, where $d = H/2n$.

When the number of applied forces n increases, the solution of this problem in discrete statement comes nearer to a solution in continuous functions.

The protecting construction is considered as flexible (yielding) beam or beam plate, which is conditionally divided into unit plots or elements a_1, a_2, ... a_n (fig. 1).

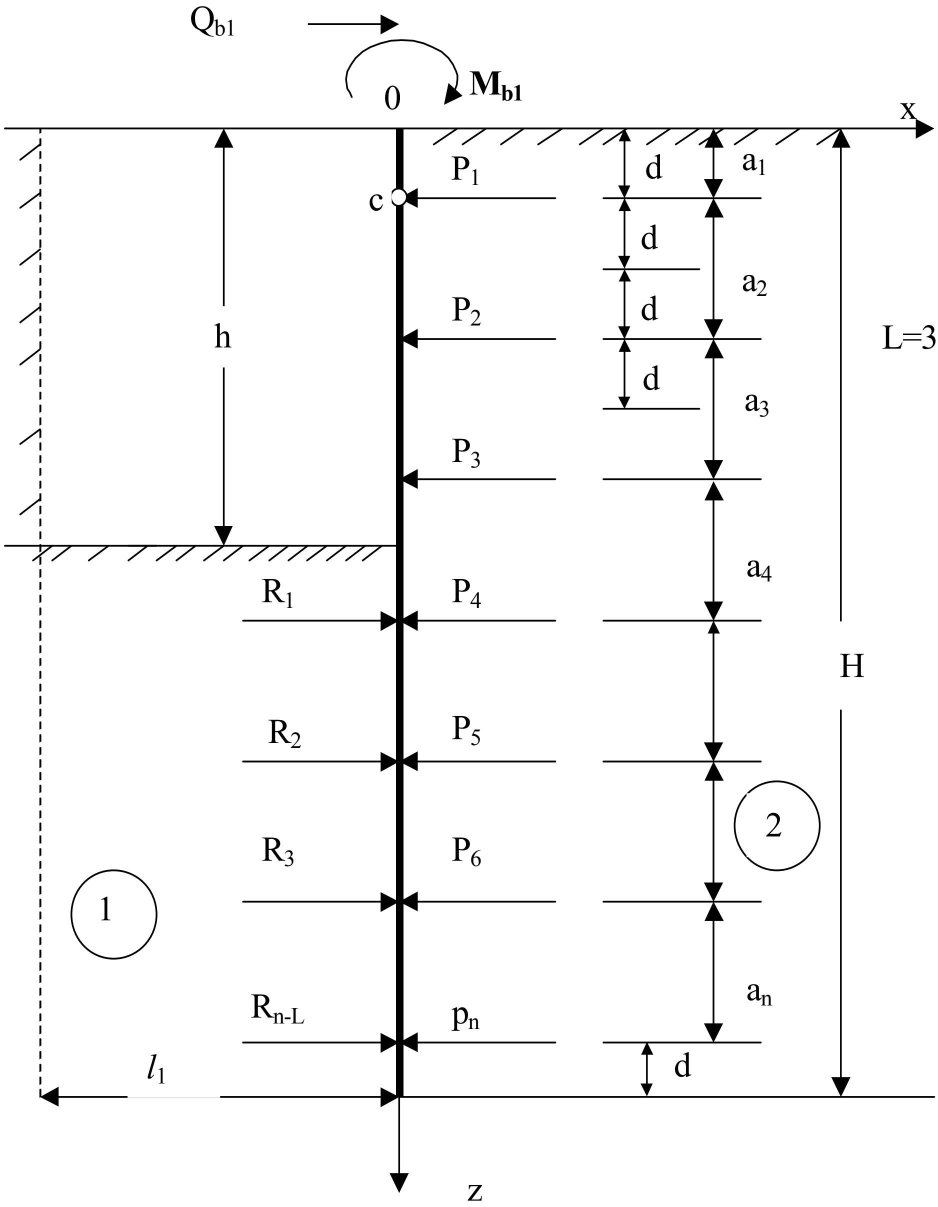

Fig. 1. Key diagram of flexible retaining wall

Each element of a beam is considered as a final element subjected to bending by the moments M_b, M_e and the transversal forces Q_b, Q_e applied at the beginning and extremity of an element (fig. 2). The relations between the beam sag S, deflection angle of it's cross-section φ and inner forces are expressed by the known formulas:

$$\varphi = -S', M = -GS'', Q = -GS''', \quad (1)$$

where $G = EJ$ — flexural rigidity of a beam; E — elasticity modulus of a beam material; J — moment of inertia of the beam cross-section; the primes mean derivation with respect to x; x — centerline of a beam element directed from the beginning of an element to its extremity.

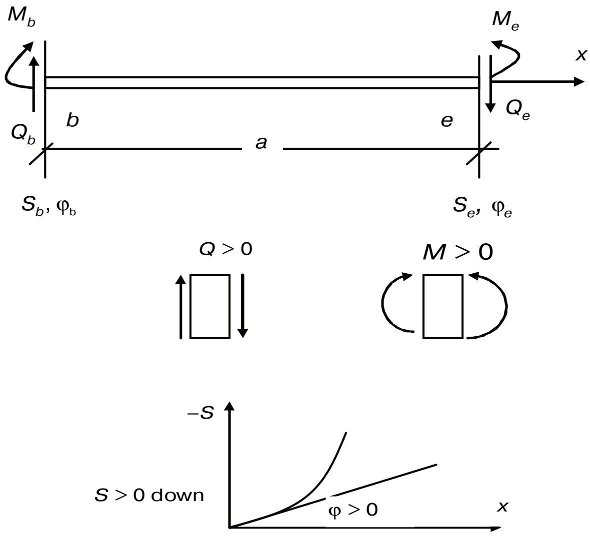

Fig. 2. Beam element with rule of signs

If to assume that there is no transversal distributed load between forces P_i and R_i, then from the equilibrium of an element follows:

$$Q_e = Q_b = Q, M_e = Q_b + kQ_b\,. \tag{2}$$

By means of an integration of the differential equation of the beam bent axis $GS^{1V} = 0$ within the limits of an element the following equalities can be obtained:

$$\varphi_e = \varphi_b + \frac{a}{G}\left(\frac{Q_b a}{2} + M_b\right), \tag{3}$$

$$s_e = s_b + \frac{a^2}{G}\left(-\frac{Q_b a}{6} - \frac{M_b}{2}\right) - a\varphi_b, \tag{4}$$

here S_e, S_b, φ_e, φ_b — the values of sag and turn angle of cross-sections at the extremity and the beginning of a beam element accordingly.

At the junctions of beam elements it is necessary to meet the conditions of sag continuity (beam is not torn), continuities of deflection angles (the beam does not break), equality of bending moments (element equilibrium condition if no external concentrated moments at the junctions). For transversal forces in the point of elements junction there is a difference T between forces P and R: $T = P - R$:

$$T_i = Q_i - Q_{i+1} \tag{5}$$

From the equilibrium condition at the points of application of forces T_i follows: $M_{ei} = M_{b(i+1)}$

Taking into account (5) it is possible to define:

$$M_{b(i+1)} = M_{b1} + \sum_{j=1}^{i} Q_j a_j \;\; (i = 1, 2, ..., n-1), \tag{6}$$

where the values of Q_{b1} and M_{b1} are considered known.

In this problem of soil interaction with a protecting wall there are:

$(n-1)$ unknown values of $Q_2, Q_3, \ldots, Q_n$;

$(n-1)$ — of $M_{b1}, M_{b2}, \ldots, M_{b(n-1)}$;

n — of $\varphi_1, \varphi_2, \ldots, \varphi_n$;

n — of $S_1, S_2, \ldots, S_n$,

and also the sag S_{b1} and turn angle φ_{b1} of the upper point of a wall over the axis Z.

It is assumed that in soil mass 1 and 2 (under the pit bottom and behind a wall) the shear stress $\tau_{xy} = 0$. Therefore, the stresses σ_x and σ_z are considered main. The stress σ_z is calculated as a sum of soil weight and pressure p on the surface behind a wall

$$\sigma_z = -\gamma z - p, \tag{7}$$

where γ is soil density.

From the equations of equilibrium when $\tau_{xy} = 0$ in a plane strain problem follows:

$$\frac{\partial \sigma_x}{\partial x} = 0; \ \frac{\partial \sigma_z}{\partial z} = -\gamma. \tag{8}$$

The equations (7) and (8) show that σ_x and σ_z are functions of z.

In an elastic body the relation between strain ε_x and stresses σ_x and σ_z under the conditions of plane strain is of following type:

$$\frac{E}{1+\nu}\varepsilon_x = (1-\nu)\sigma_x - \nu\sigma_z, \tag{9}$$

here E — modulus of elasticity; ν — Poisson's ratio.

It is assumed, that the array 1 is limited by the distance l_1 from the wall. Outside l_1 horizontal displacements of soil are equal zero, i.e. $u(-l_1) = 0$. The wall displacement s to the left is positive (opposite to axis x), i.e. $s = -u(0)$.

As σ_x, σ_z and ε_x according to (9) are functions of z, it is possible to define:

$$\frac{1+\nu}{E}\left[(1-\nu)\sigma_x - \nu\sigma_z\right] = F(z).$$

Then

$$\varepsilon_x = \frac{\partial u}{\partial x} = F(z) \text{ and } u = Fx + C_1(z) \tag{10}$$

In the soil mass 1:

$$u(-l_1) = 0;$$
$$u(0) = -S.$$

For these conditions with the formula (10) is received

$$\frac{ES}{l_1(1+\nu)} = -\left[(1-\nu)\sigma_x - \nu\sigma_z\right]. \tag{11}$$

In the soil mass 2 the similar vertical plane fixed in horizontal direction is located on a distance l_2 from the wall, where u $(-l_2) = 0$; $u(0) = -Su$.

For these conditions with the formula (10) is received

$$\frac{ES}{l_2(1+\nu)} = (1-\nu)\sigma_z - \nu\sigma_z. \qquad (12)$$

As $p = -(\sigma_x)_1 2d$, and $p = -(\sigma_x)_2 2d$, from the formulas (11) and (12) follows the equations, connecting P, R and S in different points of the wall

$$P_m u + S_m h = g(2m-1); \ (m = 1, 2, ..., n) \qquad (13)$$

$$R_j u - S_{j+L}.h = g(2j-1); \ (j = 1, 2, ..., n-L) \qquad (14)$$

Here $g = \nu\gamma d$; $h = \dfrac{E}{l(1+\nu)}$; $u = \dfrac{1-\nu}{2d}$.

Thus the full set of equations of the problem of soil interaction with a protecting wall in a ground will include:

a) the equations of equilibrium: the sum of projections of forces on horizontal axes and sum of moments related to the point c (see fig. 1) are equal to zero:

$$\sum_{j=1}^{n-L} R_j - \sum_{m=1}^{n} P_m = -Q_1 \qquad (15)$$

$$\sum_{j=1}^{n-L} R_j (j+L-1) - \sum_{m=2}^{n} P_m (m-1) = \frac{Q_1}{2} + \frac{M_{н1}}{2d} \qquad (16)$$

b) the equations (13) and (14);

c) the equations derived from (3)—(6)

$$\left. Q_j + \sum_{k=1}^{j-1} P_k = Q_1 \ \ (j = 2, 3, ..., L+1) \right\} \qquad (17)$$

$$\left. Q_j + \sum_{k=1}^{j-1} P_k - \sum_{m=1}^{J-L-1} R_m = Q_1 \ \ (j = L+2, ..., n) \right\} \qquad (18)$$

$$M_{b,i+1} - \sum_{j=1}^{i} Q_j a_j = M_{b1} + a_1 Q_{b1} \ \ (i = 1, 2, \ldots, n-1) \qquad (19)$$

$$\varphi_1 - \varphi_{b1} = ccc \qquad (20)$$

$$-S_1 + S_{b1} - a_1\varphi_{b1} = ddd \qquad (21)$$

$$-\varphi_i + \varphi_{i-1} + \rho_i Q_i + \chi_i M_{bi} = 0 \ \ (i = 2, 3, ..., n) \qquad (22)$$

$$S_{i-1} - S_i - \delta_i Q_i - \rho_i M_{bi} - a_i \varphi_{i-1} = 0 \ \ (i = 2, 3, ..., n) \qquad (23)$$

where: $\rho_i = \dfrac{a_i^2}{2G_i}$, $\delta_i = \dfrac{a_i^3}{6G_i}$, $\chi_i = \dfrac{a_i}{G_i}$, $ccc = \dfrac{a_1}{G_1}\left(\dfrac{a_1 Q_1}{2} + M_{b1}\right)$, $ddd = \dfrac{a_1^2}{2G_1}\left(\dfrac{a_1 Q_1}{3} + M_{b1}\right)$.

In total we have ($6n - L$) equations with unknowns P_i, S_i, φ_i ($3n$ unknowns), Q_i, M_{bi} ($2(n-1)$ unknowns), R_i ($(n-L)$ unknowns), S_{b1} and φ_{b1}. All these equations are linear and can be solved by Gauss's method.

This method to determine the stress-strain values in soil has one disadvantage — the model, accepted in (9) is linear-deforming body, and do not quite correspond to real soil behavior. When stress-strain state of soil masses 1 and 2 come nearer to the extreme soil strength, the linear dependence (9) should be replaced by nonlinear.

With this purpose the authors have developed a nonlinear model of soil deformation with nonlinear relations between wall displacements and side pressure of soil. The extreme state of soil was taken into account as Mohr-Coulomb strength condition:

$$\sigma_3 = m\sigma_1 + b, \tag{25}$$

where σ_1 and σ_3 — main stresses, $\sigma_1 \le \sigma_3$; $m = \frac{1-\sin\varphi}{1+\sin\varphi}$; $b = \frac{2c\cdot\cos\varphi}{1+\sin\varphi}$; φ — angle of internal friction of soil; c — cohesion.

These relations are approximated by the following formulas:

In soil mass 1 (to left of the wall, see fig. 1)

$$\psi_1 = \frac{\sigma_{x1}}{\sigma_{z1}} = \begin{cases} m + a\cdot\exp(\beta s) & if \;\; s \le 0 \\ \xi + b - b\cdot\exp(-\alpha s) & if \;\; s \ge 0 \end{cases} \tag{26}$$

In soil mass 2 (to right of the wall)

$$\psi_2 = \frac{\sigma_{x2}}{\sigma_{z2}} = \begin{cases} \xi + b - b\cdot\exp(\alpha s) & if \;\; s \le 0 \\ m + a\cdot\exp(-\beta s) & if \;\; s \ge 0 \end{cases} \tag{27}$$

Here ξ — initial factor of side pressure accepted in the linear model;

$$\xi = \nu/(1-\nu);\;\; a = \xi - m;\;\; b = \frac{1}{m} - \xi;\;\; \beta = -\frac{A}{a\sigma_z};\;\; \alpha = -\frac{A}{b\sigma_z};\;\; A = \frac{E}{l(1-\nu^2)};$$

Here E, ν — modulus of elasticity and Poisson's ratio accepted in the linear model of soil; S — wall displacement ($S > 0$ to the left); L — breadth of soil regions 1 and 2.

The relations (26) and (27) correspond to the case of loose soil, when $c = 0$. For soils with cohesion, the similar formulas can be easily written.

The functions Ψ_1, Ψ_2 calculated with the formulas (26) and (27) for $\nu = 0{,}3$; $E =$ $= 20$ MH/m^2; g = 20 kH/m^3; $l = 10$ m; are shown in fig. 3.

The similar diagrams for linear model of soil would be straight lines with the same inclination angle as the tangents for a curve Ψ_1 and Ψ_2 in a point where $S = 0$, $\psi = \xi$.

The calculations are conducted with iterative method.

At the beginning (first stage) the linear model is used. At the second stage the equations (13) and (14) of the full set of equations of this problem (i.e. equations expressing relation between wall displacement and soil pressure on the wall in soil masses 1 and 2) are corrected.

The new equations look like:

$$R = P_\xi + kS, \tag{28}$$

here $P_\xi = e_p.\xi$;

$$e_p = -2d\sigma_z; \tag{29}$$

$$k = \frac{R(S) - P_\xi}{S}; \tag{30}$$

where $R(S) = e_p\Psi_1$, where Ψ_1 is determined by (26), depending on the sign of S.

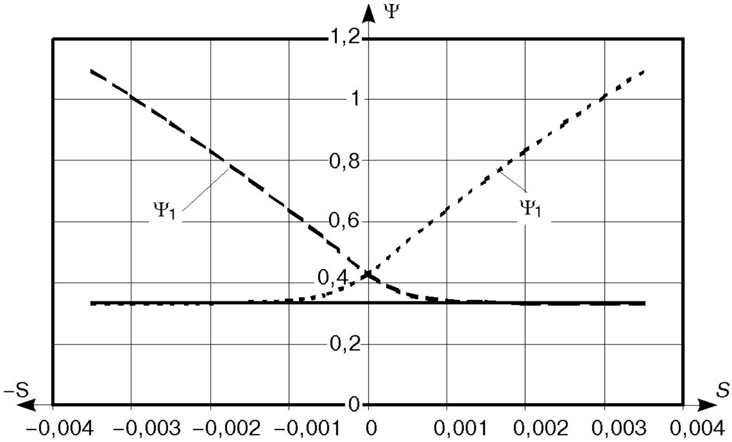

Fig. 3. Function Ψ

For soil mass 2:

$$P = P_\xi + kS \tag{31}$$

where k is determined by (30), but now $R(S) = e_p\Psi_2$.

As the equations (28) and (31) are linear, at the second step of iteration the system of $(6n - L)$ linear algebraic equations is solved again. The results of this step are used for determination of the values k in the formula (30), and the next step of iteration is done.

The examples of calculations with this method are shown in figures 4—6. With the above given mechanical properties of soil and retaining wall, the wall fails at the pit depth of 2.625 m, as it is shown in fig. 5.

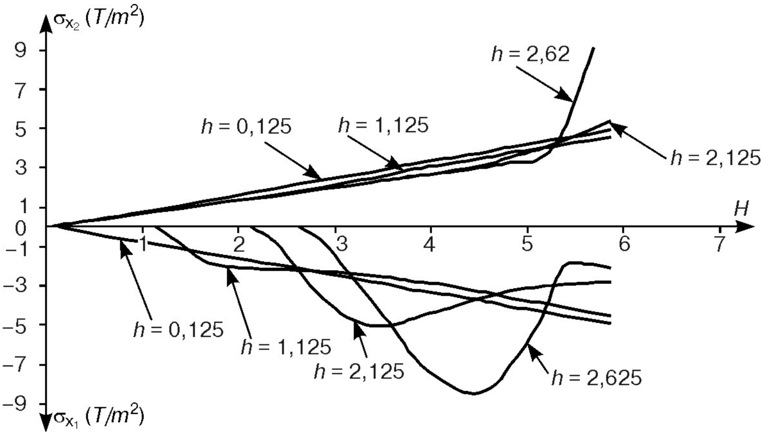

Fig. 4. Side pressure on a wall for various depth of a pit

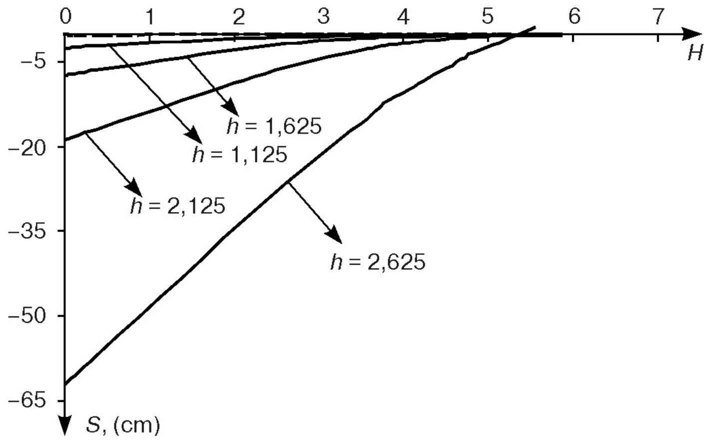

Fig. 5. Wall displacement for various depths

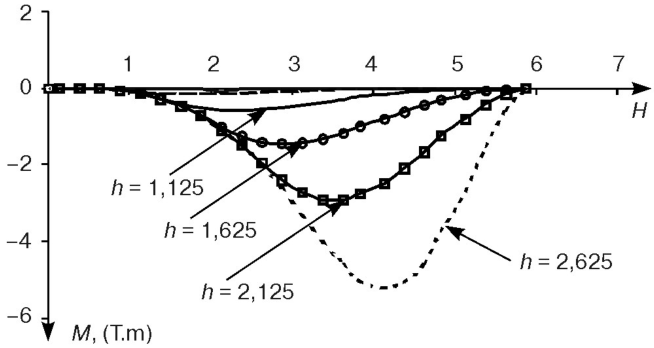

Fig. 6. Moment diagram for various depths

Conclusion. The developed method of determination the stress-strain parameters of retaining structure during earth excavation from a pit takes into account interaction of flexing retaining wall with soil and also introduces nonlinearity of soil behavior when inner forces in soil are greater than elastic limit and Hook's law does not hold.

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ВЗАИМОДЕЙСТВИЕ ШПУНТОВОЙ СТЕНКИ С ГРУНТОВЫМ МАССИВОМ В ПРОЦЕССЕ ВЫЕМКИ КОТЛОВАНА

Абу Махади Мохаммед Ибрагим, В.И. Елфимов

Кафедра строительных конструкции и сооружений
Инженерный факультет
Российский университет дружбы народов
ул. Миклухо-Маклая, 6, Москва, Россия, 117198

Разработана расчетная схема определения давления грунта на ограждающую конструкцию и ее смещения в процессе выемки котлована. В исходном положении ограждающая конструкция (шпунт, стена в грунта, свайный ряд) окружена грунтом с обеих сторон. Затем начинается выемка грунта с левой стороны от ограждения. В ходе расчета следят за изменением давления грунта с обеих сторон ограждения. Определяется критическая глубина выемки котлована, при которой происходит обрушение стенки. Ограждающая конструкция моделируется в расчетной схеме упругой балочной плитой. Грунт описывается нелинейной моделью, в которой модуль сдвига уменьшается по мере приближения напряженного состояния грунта к предельному. Математическая процедура сводится к итерациям последовательных отдельных расчетов по методу Гаусса.

Ключевые слова: механика, механика грунтов, основание, фундамент, подпорная стена, шпунт.