

# Magnetic Wormholes and Regular Black Holes with Trapped Ghosts

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We construct explicit examples of globally regular static, spherically symmetric solutions in general relativity with scalar and electromagnetic fields which describe traversable wormholes (with flat and AdS asymptotics) and regular black holes, in particular, black universes. A black universe is a nonsingular black hole where, beyond the horizon, there is an expanding, asymptotically isotropic universe. The scalar field in our solutions is minimally coupled to gravity, has a nonzero self-interaction potential, while its kinetic energy is negative in a restricted strong-field region of space–time and positive outside it. Thus in such configurations a “ghost” (as are called fields with negative kinetic energy) is trapped in a small part of space, and this may in principle explain why no ghosts are observed under usual conditions. The configurations obtained contain different numbers of Killing horizons, from zero to four.

**Key words and phrases:** black holes, wormholes, nonsingular cosmology, phantom matter, electromagnetic field.

## 1. Introduction

Modern cosmological observations (see, e.g., [1, 2]) favor, to a certain extent, the existence of phantom matter which violates all standard energy conditions ( $w = p/\rho$ , the pressure to energy density ratio, for phantom matter is smaller than  $-1$ ). Various kinds of phantom matter are discussed in cosmology as possible dark energy candidates. Meanwhile, macroscopic phantom matter has not yet been observed. There exist theoretical arguments both *pro et contra* phantom fields, and the latter seem somewhat stronger, see, e.g., a discussion in [3].

An opportunity of interest has been suggested in [4]: there can be a kind of matter which possesses phantom properties only in a restricted region of space, a strong-field region, whereas far away from it all standard energy conditions are observed. As an example of such matter, it was suggested [4] to use a minimally coupled scalar field with the Lagrangian<sup>1</sup>

$$L_s = \frac{1}{2}h(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi), \quad (1)$$

where  $h(\varphi)$  and  $V(\varphi)$  are arbitrary functions. If  $h(\varphi)$  has a variable sign, it cannot be absorbed by re-definition of  $\varphi$  in its whole range. Cases of interest are those where  $h > 0$  (that is, the scalar field is canonical, with positive kinetic energy) in a weak field region and  $h < 0$  (the scalar field is of phantom, or ghost nature) in some restricted region where, e.g., a wormhole throat can be expected. In this sense it can be said that the ghost is trapped. A possible transition between  $h > 0$  and  $h < 0$  in cosmology was considered in [5].

It is also well known that phantom matter, in addition to supporting a growing cosmological acceleration, can lead to numerous local effects. Among them is the

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<sup>1</sup>We choose the metric signature  $(+, -, -, -)$ , the units  $c = \hbar = 8\pi G = 1$ , and the sign of  $T_\mu^\nu$  such that  $T_0^0$  is the energy density.

existence of many kinds of regular space–time configurations which are otherwise impossible, at least in the framework of general relativity, such as static traversable wormholes and a particular type of regular black holes called “black universes” [6, 7]. Black universes are, in our view, of particular interest since they combine avoidance of singularities in both black holes and cosmology. These are regular black holes (spherically symmetric ones in the known examples) where a possible explorer, after crossing the event horizon, gets into an expanding universe instead of a singularity. Thus such hypothetical configurations combine the properties of a wormhole (absence of a center, a regular minimum of the area function) and a black hole (a Killing horizon separating  $R$  and  $T$  regions). Moreover, the Kantowski–Sachs cosmology in the  $T$  region of such space–time is asymptotically isotropic and approaches a de Sitter mode of expansion, which makes such models potentially viable as models of our accelerating Universe.

It turns out that both wormholes and black universes can be supported not only by purely phantom matter but also in the “trapped ghost” framework. Thus, examples of trapped-ghost wormholes have been obtained in [4], and a generalization of the solutions obtained there has led to a family of black-universe solutions [8]. In additions to explicit examples, some general features were revealed [4, 8] for static, spherically symmetric configurations of self-gravitating scalar fields with the Lagrangian (1).

In this paper, we would like to discuss new features of wormhole and black-universe configurations which appear if, in addition to the field (1), an electromagnetic field is invoked as a source of gravity. As in [4, 8], we deal with static, spherically symmetric space–times, therefore the only kinds of electromagnetic fields are a radial electric (Coulomb) field and a radial magnetic (monopole) field. It should be stressed that in the latter case it is not necessary to assume the existence of magnetic charges (monopoles): in both wormholes and black universes a monopole magnetic field can exist without sources due the space–time geometry. In the wormhole case it perfectly conforms to Wheeler’s idea of a “charge without charge” [9]: electric or magnetic lines of force simply thread the wormhole. In the case of a black universe, the picture is different on different sides: in the static region a possible observer sees a black hole with an electric or magnetic charge; in the cosmological region, this corresponds to a primordial electric or magnetic field. For definiteness, we will speak of magnetic fields.

One of the motivations for the present study was that modern observations testify to a possible existence of a global magnetic field up to  $10^{-15}$  Gauss, causing correlated orientations of sources remote from each other [10], and some authors point out the possible primordial nature of such a magnetic field.

The paper is organized as follows. In Section 2 we present the basic equations and make some general observations. In Section 3 we obtain explicit examples of trapped-ghost wormhole and black-universe solutions using the inverse-problem method, and Section 4 contains a discussion and a brief conclusion.

## 2. Basic Equations

We consider the total action

$$S = \frac{1}{2} \int \sqrt{-g} d^4x \left[ R + 2h(\varphi)(\partial\varphi)^2 - 2V(\varphi) - F_{\mu\nu}F^{\mu\nu} \right], \quad (2)$$

where  $R$  is the scalar curvature,  $g = \det(g_{\mu\nu})$ , and  $F_{\mu\nu}$  is the electromagnetic field tensor, in static, spherically symmetric space–times. The metric can be written in the form

$$ds^2 = A(u)dt^2 - \frac{du^2}{A(u)} - r^2(u)d\Omega^2, \quad (3)$$

where we are using the so-called quasiglobal gauge  $g_{00}g_{11} = -1$ ;  $A(u)$  is called the redshift function and  $r(u)$  the area function;  $d\Omega^2 = (d\vartheta^2 + \sin^2\vartheta d\varphi^2)$  is the linear element on a unit sphere. The metric is only formally static: it is really static if  $A > 0$ ,

but it describes a Kantowski–Sachs type cosmology if  $A < 0$ , and  $u$  is then a temporal coordinate. In cases where  $A$  changes its sign, regions where  $A > 0$  and  $A < 0$  are called  $R$ - and  $T$ -regions, respectively.

Let us specify which kinds of functions  $r(u)$  and  $A(u)$  are required for the metric (3) to describe a wormhole or a black universe.

1. The range of  $u$  should be  $u \in \mathbb{R}$ , where both  $A(u)$  and  $r(u)$  should be regular,  $r > 0$  everywhere, and  $r \rightarrow \infty$  at both ends.
2. A flat, de Sitter or AdS asymptotic behavior as  $u \rightarrow \pm\infty$ .
3. In the wormhole case, absence of horizons (zeros of  $A(u)$ ), and flat or AdS asymptotics at both ends.
4. In the black-universe case, a flat or AdS asymptotic at one end and a de Sitter asymptotic at the other.

The existence of two asymptotic regions with  $r \sim |u|$  (by item 2) requires at least one regular minimum of  $r(u)$  at some  $u = u_0$ , at which

$$r = r_0 > 0, \quad r' = 0, \quad r'' > 0, \quad (4)$$

where the prime stands for  $d/du$ . (In special cases where  $r'' = 0$  at the minimum, we inevitably have  $r'' > 0$  in its neighborhood.)

The necessity of violating the weak and null energy conditions at such minima follows from the Einstein equations. Indeed, one of them reads

$$2A r''/r = -(T_t^t - T_u^u), \quad (5)$$

where  $T_\mu^\nu$  are components of the total stress-energy tensor (SET).

In an  $R$ -region ( $A > 0$ ), the condition  $r'' > 0$  implies  $T_t^t - T_u^u < 0$ ; in the usual notations  $T_t^t = \rho$  (density) and  $-T_u^u = p_r$  (radial pressure) it is rewritten as  $\rho + p_r < 0$ , which manifests violation of the weak and null energy conditions. It is the simplest proof of this well-known violation near a throat of a static, spherically symmetric wormhole ([11]; see also [12]).

However, a minimum of  $r(u)$  can occur in a  $T$ -region, and it is then not a throat but a bounce in the evolution of one of the Kantowski–Sachs scale factors (the other scale factor is  $[-A(u)]^{1/2}$ ). Since in a  $T$ -region  $t$  is a spatial coordinate and  $u$  temporal, the meaning of the SET components is  $-T_t^t = p_t$  (pressure in the  $t$  direction) and  $T_u^u = \rho$ ; nevertheless, the condition  $r'' > 0$  applied to (5) again leads to  $\rho + p_t < 0$ , violating the energy conditions. In the intermediate case where a minimum of  $r(u)$  coincides with a horizon ( $A = 0$ ), the condition  $r'' > 0$  holds in its vicinity, along with all its consequences. Thus the energy conditions are violated near a minimum of  $r$  in all cases.

In what follows, we will assume that the space–time is asymptotically flat as  $u \rightarrow \infty$  and consider different behaviors of the metric as  $u \rightarrow -\infty$ .

The scalar field  $\varphi(u)$  with the Lagrangian (1) in a space–time with the metric (3) has the SET

$$T_\mu^\nu[s] = h(u)A(u)\varphi'(u)^2 \text{diag}(1, -1, 1, 1) + \delta_\mu^\nu V(u). \quad (6)$$

The kinetic energy density is positive if  $h(\varphi) > 0$  and negative if  $h(\varphi) < 0$ , so the solutions sought for must be obtained with  $h > 0$  at large values of the spherical radius  $r(u)$  and  $h < 0$  at smaller radii  $r$ . It has been shown [4] that this goal cannot be achieved for a massless field ( $V(\varphi) \equiv 0$ ). Thus we seek solutions with a nonzero potential  $V(\varphi)$ ,

The electromagnetic field compatible with the metric (3) can have the following nonzero components:

$$F_{01} = -F_{10} \text{ (electric), and } F_{23} = -F_{32} \text{ (magnetic),}$$

such that

$$F_{01}F^{01} = -q_e^2/r^4(u), \quad F_{23}F^{23} = q_m^2/r^4(u), \quad (7)$$

where the constants  $q_e$  and  $q_m$  have the meaning of electric and magnetic charges, respectively. The corresponding SET is

$$T_{\mu}^{\nu}[e] = \frac{q^2}{r^4(u)} \text{diag}(1, 1, -1, 1), \quad q^2 = q_e^2 + q_m^2. \quad (8)$$

Now, the set of equations to be solved can be written as follows:

$$2(Ar^2h\varphi')' - Ar^2h'\varphi' = r^2dV/d\varphi, \quad (9)$$

$$(A'r^2)' = -2r^2V + 2q^2/r^2; \quad (10)$$

$$r''/r = -h(\varphi)\varphi'^2; \quad (11)$$

$$A(r^2)'' - r^2A'' = 2 - 4q^2/r^2, \quad (12)$$

$$-1 + A'rr' + Ar'^2 = r^2(hA\varphi'^2 - V) - q^2/r^2, \quad (13)$$

Eq. (9) follows from (10)–(12), which, given the potential  $V(\varphi)$  and the kinetic function  $h(\varphi)$ , form a determined set of equations for the unknowns  $r(u)$ ,  $A(u)$ ,  $\varphi(u)$ . Eq. (13) (the  $\binom{1}{1}$  component of the Einstein equations), free from second-order derivatives, is a first integral of (9)–(12) and can be obtained from (10)–(12) by excluding second-order derivatives. Moreover, Eq. (12) can be integrated giving

$$r^4B'(u) \equiv r^4(A/r^2)' = 2u + 4q^2 \int \frac{du}{r^2(u)}. \quad (14)$$

where  $B(u) \equiv A/r^2$ .

### 3. Examples of Models with a Trapped Ghost

If one specifies the functions  $V(\varphi)$  and  $h(\varphi)$  in the Lagrangian (1), it is, in general, very hard to solve the above equations. Alternatively, to find examples of solutions possessing particular properties, one may employ the inverse problem method, choosing some of the functions  $r(u)$ ,  $A(u)$  or  $\varphi(u)$  and then reconstructing the form of  $V(\varphi)$  and/or  $h(\varphi)$ . We will do so, choosing a function  $r(u)$  that can provide wormhole and black-universe solutions. Given  $r(u)$  and the charge  $q$ , the function  $A(u)$  is found from (14) and  $V(u)$  from (10). The function  $\varphi(u)$  is found from (11) provided  $h(\varphi)$  is known; however, using the scalar field parametrization freedom, we can, vice versa, choose a monotonic function  $\varphi(u)$  (which will yield an unambiguous function  $V(\varphi)$ ) and find  $h(u)$  from Eq. (11).

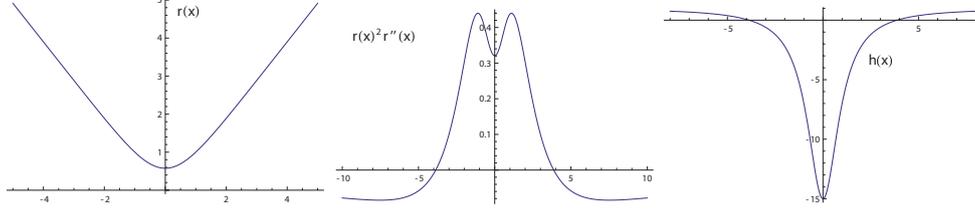
A simple example of the function  $r(u)$  compatible with the requirements 1–4 is

$$r(u) = a \frac{x^2 + 1}{\sqrt{x^2 + n}}, \quad n = \text{const} > 2. \quad (15)$$

where  $x = u/a$ , and  $a > 0$  is an arbitrary constant (the length scale). It differs from the function used in [4, 8] and leads to slightly simpler resulting expressions. Since

$$r''(x) = \frac{1}{a} \frac{x^2(2 - n) + n(2n - 1)}{(x^2 + n)^{5/2}},$$

we have  $r'' > 0$  at  $x^2 < n(2n - 1)/(n - 2)$  and  $r'' < 0$  at larger  $|x|$ , as required; it is also clear that  $r \approx a|x|$  at large  $|x|$ . It guarantees  $h < 0$  at small  $|x|$  and  $h > 0$  at large  $|x|$  (see Fig. 1, the left and middle panels).



**Figure 1.** Plots of  $r(x)$  (left),  $r^2 r''(x)$  (middle) and  $h(x)$  (right) for  $n = 3$  in Eq. (15)

In what follows, we will formally put  $a = 1$ , which will actually mean that the length scale is arbitrary but the quantities  $r$ ,  $q$ ,  $m$  (the Schwarzschild mass in our geometrized units) etc., with the dimension of length, are expressed in units of  $a$ , the quantities  $B$ ,  $V$  and others with the dimension  $(\text{length})^{-2}$  in units of  $a^{-2}$ , etc.; the quantities  $A$ ,  $\varphi$ ,  $h$  are dimensionless.

Now, the expression for  $B' = B'(x)$  can be written as

$$B'(x) = \frac{(x^2 + n)^2}{(x^2 + 1)^4} \left[ 6p - 2x + 2q^2 \left( \frac{(n-1)x}{1+x^2} + (n+1) \arctan x \right) \right], \quad (16)$$

where  $p$  is an integration constant. Further integration is also performed analytically but leads to rather cumbersome expressions for  $B(x)$  and other quantities, therefore we will restrict ourselves to the choice  $n = 3$ . An inspection shows that a particular choice of the parameter  $n > 2$  does not change the qualitative features of the solutions though certainly affects their numerical characteristics.

Integrating (16) for  $n = 3$ , we obtain

$$\begin{aligned} B = B_0 + & \frac{26 + 24x^2 + 6x^4 + 3px(69 + 100x^2 + 39x^4)}{6(1+x^2)^3} + \frac{39p}{2} \arctan x \\ & + \frac{q^2[107 + 383x^2 + 375x^4 + 117x^6 + 6x(69 + 169x^2 + 139x^4 + 39x^6) \arctan x]}{9(1+x^2)^4} \\ & + 13q^2 \arctan^2 x, \quad (17) \end{aligned}$$

where  $B_0$  is one more integration constant.

Now suppose that our system is asymptotically flat at  $x \rightarrow +\infty$ . Since  $B = A/r^2$  and  $A \rightarrow 1$  at infinity, we require  $B \rightarrow 0$  as  $x \rightarrow \infty$  and thus fix  $B_0$  as

$$B_0 = -\frac{13}{4}\pi(3p + \pi q^2). \quad (18)$$

Furthermore, comparing the asymptotic expression  $A = 1 - 2m/x + o(x)$  for  $A(x)$  with what is obtained from our expression for  $A = Br^2$ , we find a relation between the Schwarzschild mass  $m$  and our parameters  $p$  and  $q$ :

$$p = m - \frac{2}{3}\pi q^2. \quad (19)$$

Thus  $B$  is a function of  $x$  and two parameters, the mass  $m$  and the charge  $q$ .

Now we know the metric completely, while the remaining quantities  $\varphi(x)$  and  $V(\varphi(x))$  are easily found from Eqs. (11) and (10), respectively. To construct  $V$  as an

unambiguous function of  $\varphi$  and to find  $h(\varphi)$ , it makes sense to choose a monotonic function  $\varphi(u)$ . It is convenient to assume

$$\varphi(x) = \frac{1}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}}, \quad (20)$$

so that  $\varphi$  has a finite range:  $\varphi \in (-\varphi_0, \varphi_0)$ ,  $\varphi_0 = \pi/(2\sqrt{3})$ , which is common to kink configurations. Thus we have  $x = u/a = \sqrt{3} \tan(\sqrt{3}\varphi)$ , whose substitution into the expression for  $V(u)$ , found from (10), gives  $V(\varphi)$  defined in this finite range. The function  $V(\varphi)$  can be extended to the whole real axis,  $\varphi \in \mathbb{R}$ , by supposing  $V(\varphi) \equiv 0$  at  $\varphi \geq \varphi_0$  and  $V(\varphi) = V(-\varphi_0) > 0$  at  $\varphi < -\varphi_0$ .

The expression for the kinetic coupling function  $h(\varphi)$  is then found from (11) as follows:

$$h(\varphi) = \frac{x^2 - 15}{x^2 + 1} = \frac{3 \tan^2(\sqrt{3}\varphi) - 15}{3 \tan^2(\sqrt{3}\varphi) + 1}. \quad (21)$$

The function  $h(\varphi)$  given by Eq. (21) is also defined in the interval  $(-\varphi_0, \varphi_0)$  and can be extended to  $\mathbb{R}$  by supposing  $h(\varphi) \equiv 1$  at  $|\varphi| \geq \varphi_0$ . The function  $h(x)$  is plotted in Fig. 1. Evidently, the null energy condition is violated only where  $h(\varphi) < 0$ .

Substituting the expressions (15) and (17) into (10), with  $A(u) = B/r^2$ , we obtain the potential  $V$  as a function of  $u$  or  $x = u/a$ . This expression is rather bulky and will not be presented here. Instead, we will present some plots for selected values of the parameters.

It is easy to see that asymptotic values of the function  $B(x)$  at  $x \rightarrow -\infty$  are directly related to those of the potential  $V$  which in this case plays the role of an effective cosmological constant:

$$V(-\infty) = -3B(-\infty), \quad (22)$$

so that negative  $B(-\infty)$  correspond to a de Sitter (dS) asymptotic, with  $B(-\infty) = 0$  it is flat and with  $B(-\infty) > 0$  it is anti-de Sitter (AdS). The solutions obtained may be classified by this asymptotic behavior and by the number and nature of horizons appearing there. The latter correspond to regular zeros of the function  $B(x)$ . It turns out that inclusion of the electromagnetic field makes the solutions much more diverse than it was found previously for purely scalar-vacuum configurations [4, 6].

To begin with, from (17) it follows that  $B(x)$  is an even function if and only if  $p = 0$ , hence  $m = (2/3)\pi q^2$ . Then  $V(x)$  is also an even function. Such symmetric configurations are asymptotically flat at both ends,  $x \rightarrow \pm\infty$  and can be classified as follows (see the corresponding curves in Fig. 2):

**A1:** Twice asymptotically flat (M-M) wormholes.

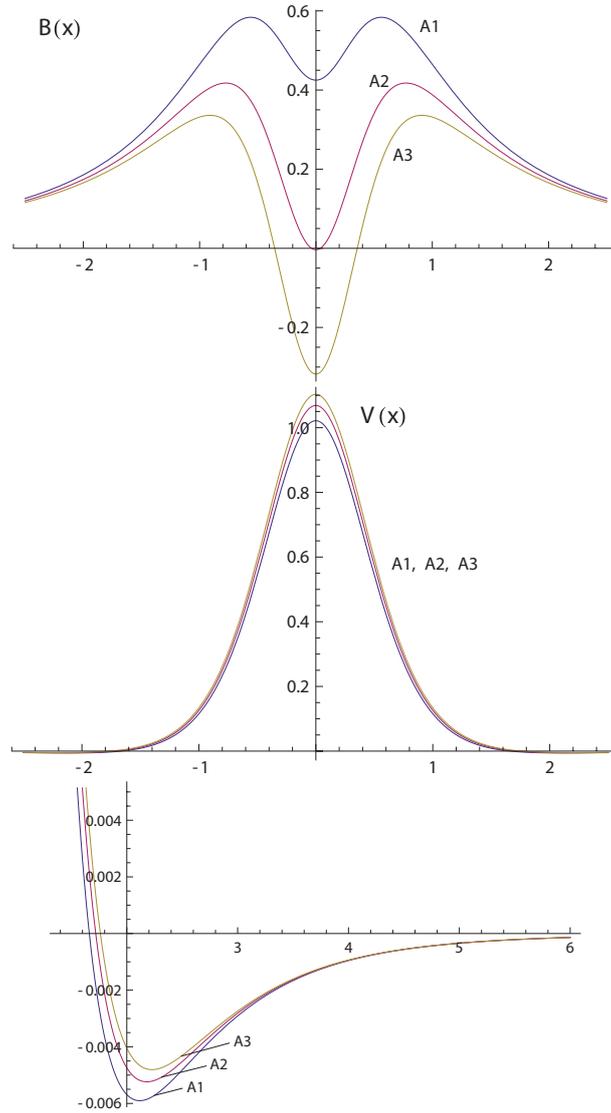
**A2:** Extremal regular black holes (M-M), with a double horizon (curve A2).

**A3:** Non-extremal regular black holes (M-M), with two simple horizons (curve A3).

The abbreviation (M-M) stands here for two flat (Minkowski) asymptotic regions; we will also use similar notations for de Sitter (dS) and anti-de Sitter (AdS) asymptotic behaviors.

The symmetric models form a one-parameter family, depending on  $q$ ; clearly, at  $q$  smaller than those appearing in Fig. 2 we also obtain wormholes (the simplest of them is with  $q = m = 0$  and  $V \equiv 0$ , it is the Ellis massless wormhole [13, 14]), while at larger  $q$  there are non-extremal black holes. The critical value of  $q$  that separates them is  $q \approx 0.4635$ , at which there emerges a double horizon corresponding to a double root of  $B(x)$ .

It is of interest that in the narrow range of  $q$  in which the behavior of  $B(x)$  drastically changes, the potential  $V(x)$  changes very little. We also notice that at large  $|x|$  the potential takes small negative values (see the lower right panel in Fig. 2)



**Figure 2.** Plots of  $B(x)$  (top) and  $V(x)$  (bottom) for symmetric configurations. Curves A1, A2, A3 correspond to  $q = 0.44, 0.4635, 0.48$ , respectively. The lower right panel shows the behavior of the potential at large  $x$

while in the strong field region it is large and positive. It is not by chance since in the general case  $V(x)$  behaves at large  $x$  as follows:

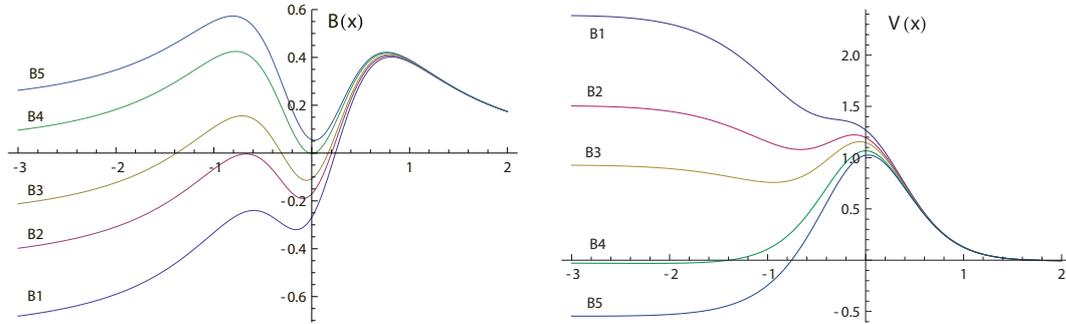
$$V(x) = -\frac{4m}{5x^5} + \frac{2(q^2 - 12)}{3x^6} + O(x^{-7}). \quad (23)$$

Thus in all examples considered below, since we everywhere take  $m > 0$ , the potential at large  $x$  behaves approximately as shown in Fig. 2, lower right panel.

Concerning asymmetric configurations, it is natural to expect a critical behavior, i.e., transitions between different types of models, at values of  $m$  and  $q$  close to those appearing in Fig. 2 (but certainly with  $p \neq 0$ ). This idea is confirmed by a direct inspection, and Fig. 3 (left) shows the corresponding five modes of the behavior of  $B(x)$ :

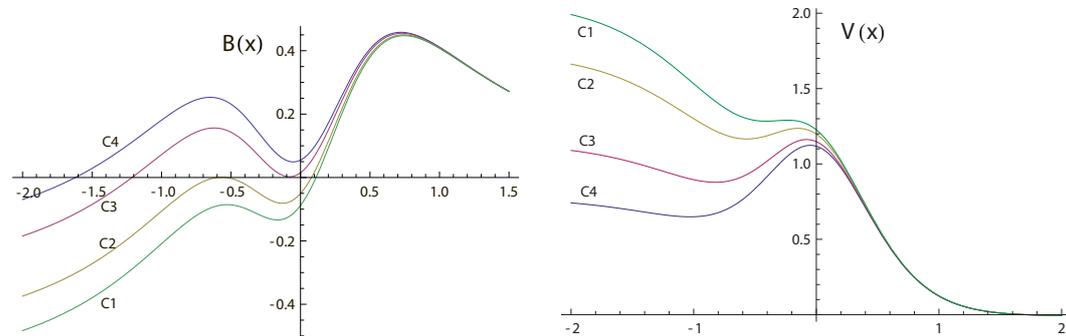
**B1:** A black universe (M-dS) with a single simple horizon.

- B2:** A black universe (M-dS) with two horizons (simple and double).
- B3:** A black universe (M-dS) with three simple horizons.
- B4:** A regular black hole with a double horizon, asymptotically AdS at the far end ( $x \rightarrow -\infty$ ).
- B5:** A wormhole, asymptotically AdS at the far end ( $x \rightarrow -\infty$ ).



**Figure 3.** Plots of  $B(x)$  (left) and  $V(x)$  (right) for asymmetric configurations at parameter values close to critical ones. The parameters are:  $m = 0.45$  and  $q = 0.457, 0.45942, 0.461, 0.4636, 0.465$  for curves **B1–B5**, respectively

The shape of the potential  $V(x)$  (Fig. 4, right) corresponds to Eq. (22): it is certainly zero at the flat asymptotic and is of the opposite sign to that of  $B$  at the other end.



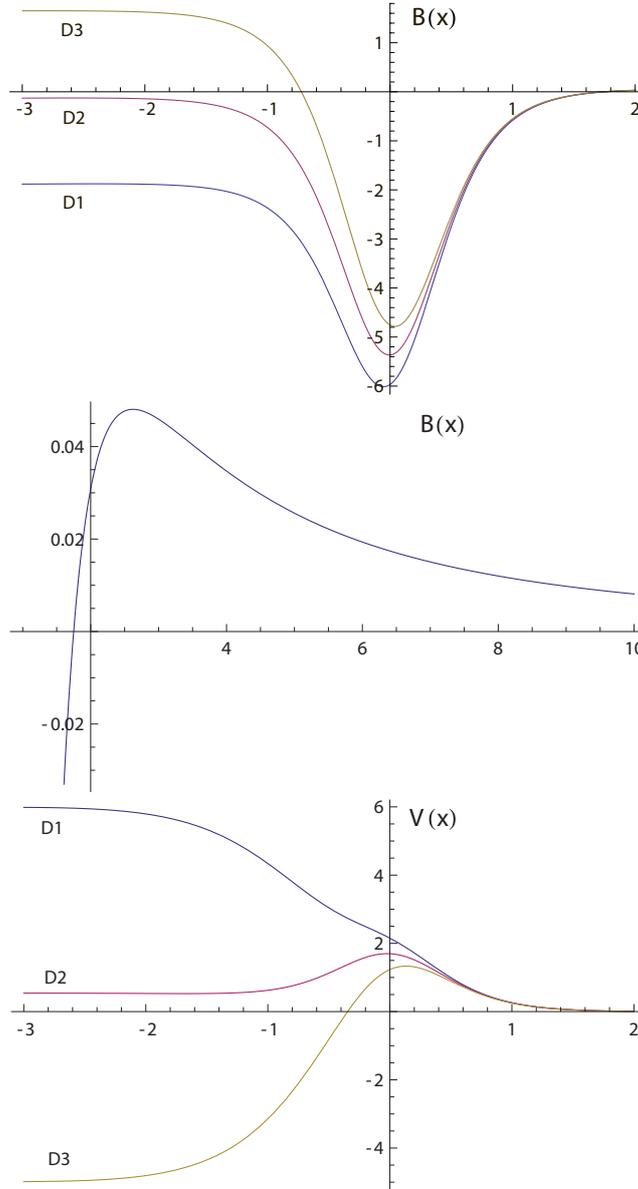
**Figure 4.** Plots of  $B(x)$  (left) and  $V(x)$  (right) for asymmetric configurations with the parameter values  $m = 0.43$  and  $q = 0.45, 0.45095, 0.4526, 0.4536$  for curves **C1–C4**, respectively

A somewhat different picture is observed if we slightly move down the mass and charge values, see Fig. 4. A qualitatively new feature as compared to Fig. 4 is that the function  $B(x)$  corresponding to a double horizon between two R regions (curve C3) has a negative limit as  $x \rightarrow -\infty$ . As a result, it is a black universe model instead of an M-AdS wormhole.

Less diverse is the solution behavior at larger values of the parameters, as exemplified in Fig. 5:

- D1, D2:** Black universes (M-dS) with a single simple horizon,
- D3:** Regular black holes (M-AdS) with two simple horizons.

The behavior of  $B(x)$  at large  $x$  is not evident from the upper left panel in Fig. 5 and is shown in the upper right panel. It naturally corresponds, as in all other cases considered here, to a Schwarzschild asymptotic with a positive mass.



**Figure 5. Plots of  $B(x)$  (top) and  $V(x)$  (bottom) for asymmetric configurations with the parameter values  $m = 1$  and  $q = 0.68$  (curve D1),  $q = 0.69$  (curve D2) and  $q = 0.7$  (curve D3). The upper right panel shows the behavior of  $B(x)$  at large  $x$  for all these parameter values**

## 4. Conclusion

It has been shown [4] that a minimally coupled scalar field may change its nature from canonical to ghost in a smooth way without creating any space–time singularities. This feature, in particular, allows for construction of wormhole [4] and black-universe models [8] (trapped-ghost wormholes and black universes) where the ghost is present in some restricted region around the throat (of arbitrary size) whereas in the weak-field region far from it the scalar has usual canonical properties.

In the present paper, we have obtained similar models with an electromagnetic field and found that its inclusion leads to a greater diversity of qualitatively different

configurations. More specifically, we have found as many as 10 types of models, classified by the types of asymptotic behavior and the number and nature of horizons. They have been represented above by the curves A1–A3, B1–B5, C3 and D3. At zero charge  $q$  we return to the situation discussed in [8], where only three configuration types were revealed: M–M wormholes (represented here by the curve A1), M–AdS wormholes (curve B5) and black universes with a single simple horizon (curves B1, C1, C4, D1, D2). The reason is that in a pure scalar–vacuum system the field equations forbid the function  $B(u)$  to have a regular minimum.

We have been assuming that the space–time is asymptotically flat as  $x \rightarrow \infty$ . It is clear that if we abandon this assumption, then the number of possible qualitatively different configurations in the scalar–electrovacuum system under consideration will be still larger. To see how they can look, let us note that in Eq. (17) the constant  $B_0$  is additive. Therefore, changing  $B_0$ , we simply move up or down the plot of  $B(x)$ , thus changing the asymptotic behavior and number and nature of horizons in our model. For instance, if we slightly move down the curve A3 in Fig. 2, we will obtain a configuration with two de Sitter asymptotics (dS–dS), separated by four simple horizons.

We conclude that the present field system creates quite a number of diverse models which can be of interest both as descriptions of local objects (black holes, wormholes) and as a basis for building singularity-free cosmological scenarios.

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## Магнитные кротовые норы и регулярные черные дыры с захваченными духами

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Построены явные примеры глобально регулярных статических сферически-симметричных решений ОТО со скалярным и электромагнитным полями, описывающих кротовые норы (с плоскими и АдС асимптотиками) и регулярные черные дыры, в частности, чёрные вселенные. Чёрная вселенная — это несингулярная чёрная дыра, в которой за горизонтом находится расширяющаяся асимптотически изотропная вселенная. В наших решениях скалярное поле минимально связано с гравитацией и имеет ненулевой потенциал самодействия, а его кинетическая энергия отрицательна в ограниченной области пространства–времени с сильными полями, а вне этой области положительна. Таким образом, в полученных конфигурациях «дух» (этим названием обозначаются поля с отрицательной кинетической энергией) захвачен в малой области пространства, и это в принципе может объяснить отсутствие наблюдаемых «духов» в обычных условиях. Полученные конфигурации содержат разное число горизонтов Киллинга, от нуля до четырёх.

**Ключевые слова:** чёрные дыры, кротовые норы, несингулярная космология, фантомная материя, электромагнитное поле.