Discrete & Continuous Models
& Applied Computational Science
2019, 27 (2) 124–132
http://journals.rudn.ru/miph

Mathematical modeling

Research article

UDC 519.6;699.841
DOI: 10.22363/2658-4670-2019-27-2-124-132

Seismic stability of oscillating building on kinematic supports

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(received: February 4, 2019; accepted: November 14, 2019)

The design of kinematic supports is considered, which allows to damp the oscillation energy of seismic waves during earthquakes. The building rests on supports that have the geometry of straight cylinders. When horizontal ground oscillations occur, the supports are deflected at a small angle ψ. At the same time, their centre of gravity rises and tends to return to its original position under the action of two forces on each support: the weight of the building evenly distributed over all the supports, and the weight of the support itself. The first force is applied to the highest point of the support, the second one is applied to the centre of gravity of the support, so that the rotational moments of two forces act on the support.

It should be noted that under very strong vibrations of the ground, the projection of the centre of gravity could move beyond the base of the support. In this case, the supports will begin to tip over. We confine ourselves to considering such deviations that the rotational moments of the forces of gravity still tend to return the supports to their initial state of equilibrium.

Key words and phrases: ensuring seismic stability of buildings during earthquakes, the equation of motion of a physical pendulum, vibration damping.

1. Introduction

The amount of energy transferred to the building depends on the relation between the spectra of seismic effects and natural oscillations of the building. The closer the peaks of the spectra, the greater the energy transferred to the building under similar conditions. This energy is mainly absorbed by the inelastic deformations of the structure. Based on the above facts, two main tasks can be formulated aimed at ensuring the seismic resistance of a building:

— to separate the spectra and thereby reduce the amount of energy transmitted to the building and

— to provide the absorption of the remaining energy using special measures.

In the publications on deterministic analysis of the dynamics of constructions, describing the time-dependent motion of the system under the action of seismic load (strongly oscillating and irregular) [1–8], various types of the building supports are studied, aimed at efficient damping the energy of the spatial movement of the building caused by an earthquake. Among them the most successful solutions have been proposed by A. M. Kurzanov and Yu. P. Cherepinsky [6]. In this paper, we consider in detail the design and operation of Kurzanov's kinematic supports, which have been well-proven in experimental studies [1, 7, 8]. Our theoretical study of the functioning of kinematic supports is aimed at their mathematical modelling and the subsequent selection of parameters of the model of supports in order to solve both of the above problems of seismic stability of buildings. Note that in recent years the interest in the study of kinematic supports that provide the seismic resistance of buildings has noticeably increased [9–11]. See also [12–19].

2. Mathematical model of the functioning of kinematic supports

The design of kinematic supports is considered, which allows damping the oscillation energy of seismic waves during earthquakes. The building having the weight Mg rests on n supports, each having the weight mg. The supports have the geometry of straight cylinders of height h and base diameter a.

Under the action of an incident seismic wave, the whole construction “building + n supports” comes into a complex movement. Of all the seismic waves, let us consider horizontal waves. When horizontal ground oscillations occur, the supports are deflected by an angle ψ. At the same time, their centre of gravity rises and due to this fact the building, moving horizontally, rises and acquires additional potential energy. After that, the building tends to return to its original position and acquires additional kinetic energy, with the result that each of the supports acquires additional kinetic energy under the action of two forces on each support: the weight of the building evenly distributed on each support, and the weight of the support itself. The first force is applied to the highest point of the support (see Figure 1), the second one is applied to the centre of gravity of the support (see Figure 2), so that the rotational moments of two forces act on the support. Below we consider only a part of the system, namely, a support isolated from the seismic impact.

Each support has the shape of a cylinder with the height h and diameter $a = 2r$. It should be noted that under very strong vibrations of the ground, the projection of the centre of gravity can move horizontally beyond the base of the support, i.e., $\varphi > \alpha$, where α is the angle between the diagonal of the support and its height, so that $\alpha = \arctan\left(\frac{a}{h}\right)$. In this case, the supports will begin to tip over. We confine ourselves to considering such deviations, when the rotational moments of the forces of gravity still tend to return the supports to the initial state of equilibrium, that is, we will consider the case $\varphi < \alpha$. Then $\alpha = \varphi + \psi$, where ψ is the angle of deviation of the flat base of the support from the horizontal, while the supports perform non-linear oscillations. To compose the equation of motion of a support, we take into account that it is a physical pendulum.

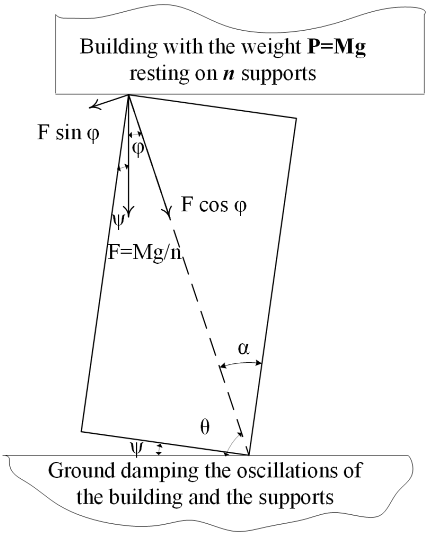

Figure 1. Rotational force generated by the pressure of the building on the supports

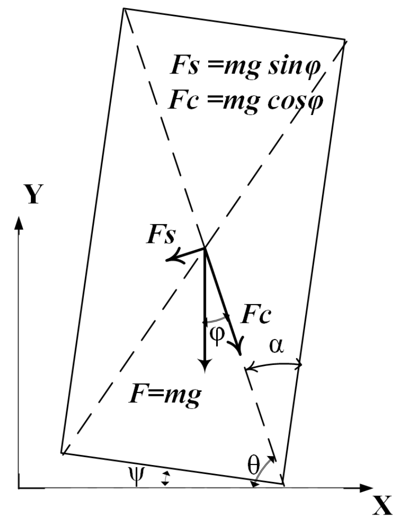

Figure 2. Rotational force generated by the weight of the support

Before the appearance of seismic effects, the "building + n supports" system is in a state of stable equilibrium, i.e., the supports stand on the ground, the building rests on the supports, with the centre of gravity in the lowest position and the lowest potential energy. After the deviation of the "building + n supports" system from the equilibrium position of the supports (along with the building), they rise and fall, rotating around one of their edges during the first half-period of the oscillatory motion. Then the supports touch their bases to the ground with a blow, followed by the loss of a part ε of the energy due to the inelastic impact of the ground. After that, due to the remaining $(1-\varepsilon)$ kinetic energy of the support (along with the building), they rise and fall, rotating around its other edge during the second half-period of the oscillatory motion. Then the next blow occurs and the oscillatory motion continues with damping.

During each of the half-periods, the movement occurs under the action of gravity forces and their rotational moments.

The returning force of a uniformly distributed building weight acting on a support is $F_M = -\frac{Mg}{n}\sin\varphi$. The distance from the point of application of the force F_M to the axis of rotation is equal to $L_M = 2l = \sqrt{a^2+h^2}$. The torque of the force is expressed as $F_M \bullet L_M = -\frac{Mg}{n}\sqrt{a^2+h^2}\sin\varphi$.

The returning force generated by the weight of the support itself is calculated as $F_m = mg\sin\varphi$. The distance from the point of application of the force F_M to the axis of rotation is l. The torque of the force is calculated using the formula $F_m * l$.

The moment of inertia of the support is equal to $J = \frac{m}{2}r^2 + \frac{m}{6}h^2$. The equation of motion of a support, assuming that it is a physical pendulum, has the form

$$F_m + F_M = -J\ddot{\varphi}. \qquad (1)$$

Taking into account the explicit form of the restoring forces, we obtain the relation

$$mgl\sin\varphi + \frac{2Mg}{n} l\sin\varphi = -\frac{12l^2+h^2}{24}\ddot{\varphi}, \tag{2}$$

which is reduced to the Lagrange differential equation describing the dynamics of the motion of the supports after a seismic shock, bringing the entire system out of equilibrium:

$$-\ddot{\varphi} = -\left(\frac{nm+2M}{12l^2+h^2}\right)\frac{24gl}{n}\sin\varphi. \tag{3}$$

In the case of limited oscillations, i.e., when $\psi < \alpha$, we get

$$\ddot{\psi} - \left(\frac{nm+2M}{12l^2+h^2}\right)\frac{24gl}{n}\sin(\alpha-\psi) = 0. \tag{4}$$

The obtained Eq. (4) is the equation of free oscillations (not disturbed by the continuing seismic effect).

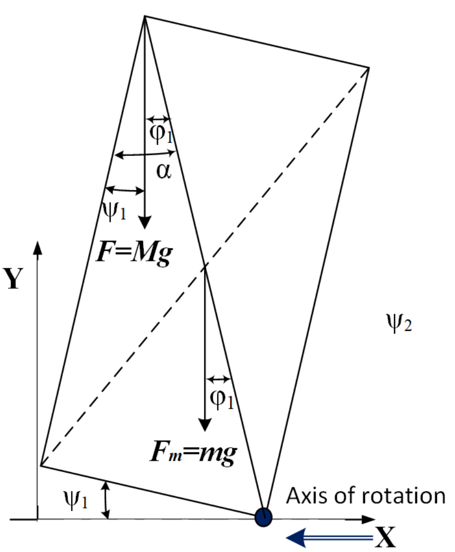

Figure 3. Position of the support at the moment of maximum lifting of the centre of gravity ($t = 0$)

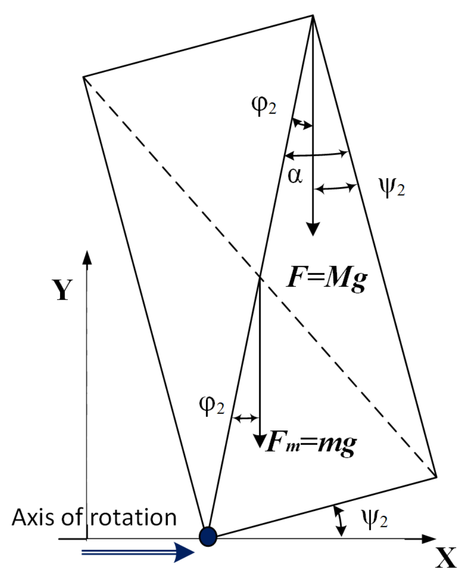

Figure 4. The support of the building. Maximum deviation after changing the axis of rotation

For a complete description of the evolutionary process of support oscillations, we supplement Eq. (4) with the initial conditions, taking the maximum deflection of the support after the seismic shock shown in Figure 3 for the initial position at zero time. Then at the moment of time corresponding to the highest ascent of the centre of gravity of the support (the centre of rotation of the support is located at its lower right point, see Figure 3) the movement of an individual support is described by Eq. (4) with the initial conditions (we assume the deflection angle positive)

$$\psi_1(0) = \psi_{const}, \quad \dot{\psi}_1(0) : \psi_{const} > 0. \tag{5}$$

Under the influence of the gravity of the support and the entire building, the centres of gravity of the supports will tend to return to their original position. In this case, the angle $\psi_1 > 0$, $\psi_1 \to 0$, in accordance with the solution of the Cauchy problem (4), will tend to zero. At that moment, when the magnitude of the angle equals zero, the inertial forces will force the building and the supports to move further in the same horizontal direction, which will lead (after the impact of the support on the base surface) to a change of the rotation axis of the supports, see Figure 4. The motion of the support system and the building itself is still described by exactly the same equations, but with the variable ψ_1 changed for ψ_2:

$$\ddot{\psi}_2 - \left(\frac{nm + 2M}{12l^2 + nm}\right) \frac{24gl}{n} \sin(\alpha - \psi_2) = 0 \tag{6}$$

and with other initial conditions (again we consider the deflection angle ψ_2 to be positive)

$$\begin{aligned} \psi_2(0) &= 0, \\ \dot{\psi}_2(0) &= \dot{\psi}_1(0). \end{aligned} \tag{7}$$

It is important to note that at the time of the collision of the support with the ground surface, the system consisting of the building and the supports loses some fraction of the kinetic energy. Setting the restitution coefficient to be equal to $C_r = (1 - \varepsilon) < 1$ and considering the kinetic energy losses, we arrive at the relation

$$\frac{J}{2}\dot{\varphi}^2_{after} = C_r \frac{J}{2}\dot{\varphi}^2_{befor} \Rightarrow \dot{\varphi}_{after} = \sqrt{C_r}\dot{\varphi}_{befor}, \tag{8}$$

which, in turn, makes it possible to determine the new velocities of the supports after their collisions with the surface and recalculate the initial conditions of the problems (5) and (7) when going through the zero value of the rotation angle using the formulas

$$\begin{aligned} \psi_{k+1}(t + 0) &= \psi_k(t - 0), \\ \dot{\psi}_{k+1}(t + 0) &= -\sqrt{C_r}\dot{\psi}_k(t - 0), \end{aligned} \tag{9}$$

for the generalized equation of motion of the supports and the building:

$$\ddot{\psi} = sign(\psi) \left(\frac{nm + 2M}{12l^2 + nm}\right) \frac{24gl}{n} \sin(\alpha - sign(\psi)\psi). \tag{10}$$

The formulated system can be solved by means of any stable numerical method, for example, the 4^{th}-order Runge–Kutta method with automatic step selection. We get the following plots for the solutions (Figures 5–7).

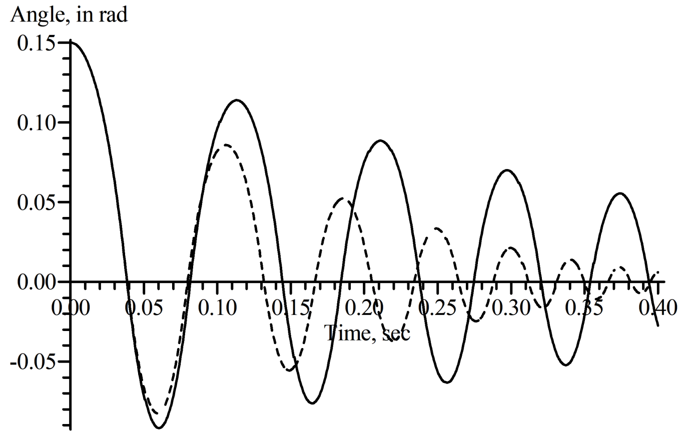

Figure 5. The angle of deviation from the vertical. The solid line corresponds to the recovery coefficient $C_r = 0.9$, the dashed line — to $C_r = 0.8$

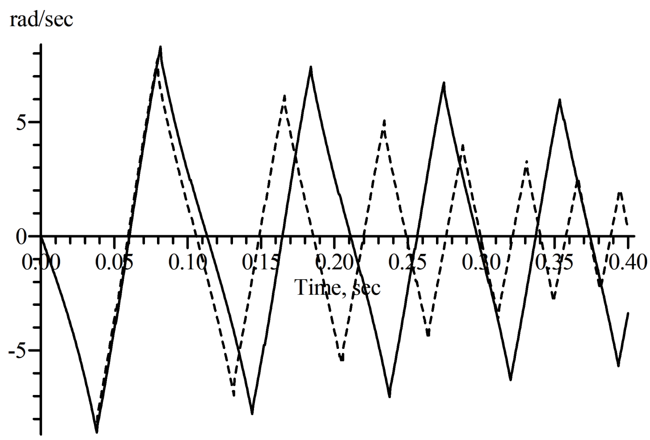

Figure 6. The rate of change of the deviation angle from the vertical. The solid line corresponds to the recovery coefficient $C_r = 0.9$, the dashed line — to $C_r = 0.8$

3. Conclusion

The model (4) is a nonlinear conservative dynamical system. The motion of the supports and the building body during the non-linear oscillations can be considered as oscillations of the coupled physical pendulums. At the same time, the coupling of pendulums is not conservative, but contains a factor proportional to the rolling friction of the supports on the base of the building body. Moreover, the rolling radius depends on the angle: the larger the angle ψ, the smaller the radius, which means greater friction force between the n supports and the base of the building.

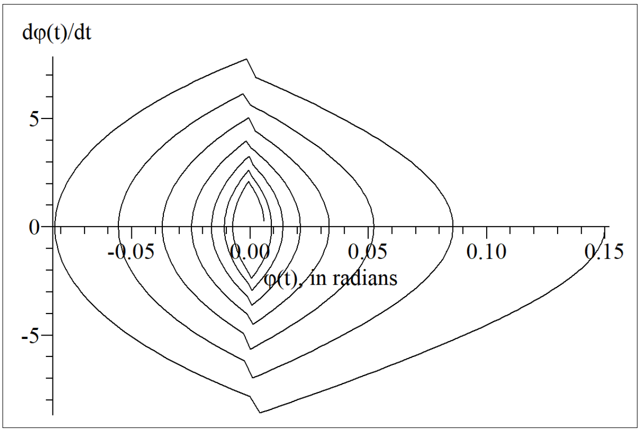

Figure 7. Phase portrait of the "oscillatory system" ($C_r = 0.8$).

In addition, of course, this entire unified system, including the reaction of the soil, moves under the action of a seismic strongly oscillating and irregular disturbance generated by an earthquake. Only as a result of the joint consideration of all these factors and we can count on an adequate description of the deterministic dynamics of the building under the influence of seismic perturbations from an earthquake.

However, even the analysis of equation (4) allows detuning of the natural frequencies of free vibrations of a building system on supports by varying the weight of the building and the number of supports.

The equation obtained by us is valid for angles $\psi < \alpha$, but it can be easily generalized to the case $\psi > \alpha$, when the movement of the supports will cease to be oscillatory, and the whole structure will lose stability. However, this case, interesting while considering the driving forces of horizontal seismic vibrations, will be analysed elsewhere.

Acknowledgments

The publication has been prepared under support from the "RUDN University Program 5-100" and the Russian Foundation for Basic Research, grants No. 18-07-00567 and No. 18-51-18005.

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For citation:

S. P. Karnilovich, K. P. Lovetskiy, L. A. Sevastianov, E. L. Shchesnyak, Seismic stability of oscillating building on kinematic supports, Discrete and Continuous Models and Applied Computational Science 27 (2) (2019) 124–132. DOI: 10.22363/2658-4670-2019-27-2-124-132.

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