METHOD OF LOGICAL NETWORK OPERATOR FOR URBAN TRAFFIC CONTROL SYNTHESIS PROBLEMS

G.H.A. Alnovani[1], A.I. Diveev[2], K.A. Pupkov[3], E.A. Sofronova[1]

[1]Peoples' Friendship University of Russia
Ordjonikidze str., 3, Moscow, Russia, 117923

[2]Institution of Russian Academy of Sciences
Dorodnicyn Computing Centre of RAS
Vavilova str., 40, Moscow, Russia, 119333

[3]Bauman Moscow State Technical University
2-Baumanskaya str., 5, Moscow, Russia, 105005

The problem of maximal capacity of traffic at the rush hours is considered. The control is performed by adjusting the traffic lights phases at intersections. The mathematical model of traffic flow is given. The model is described by the oriented graph with changeable configuration. The optimal control problem for traffic flows is formulated as a problem of discrete dynamic optimization. To solve the problem the genetic algorithm and the method of logical network operator are used.

Key words: logical network operator, synthesis of control, traffic simulation.

INTRODUCTION

Nowadays the traffic control problem is important for big cities. At rush hours a considerable number of cars fill roads between intersections. If sections cannot coupe with a demanded number of cars then an overflow occurs. The cars stack in the intersections and do not respond to the traffic light phases. It leads to traffic jams. To avoid the overflow it is necessary to take into consideration the number of cars and traffic lights phases at adjoining roads.

The traffic light phases control at intersections helps to increase the traffic capacity and minimize the probability of traffic jams. Known mathematical models of traffic flow control [1—5] do not consider the control by means of traffic lights. As a rule the models use a continuous flow moving in all admissible directions. The traffic flow control problem by means of traffic lights is to divide the flow and then to direct the flows in possible directions taking into account overflow of roads and traffic capacity in intersection.

If the network contains intersections on the main road then the «green wave» mode is applied. At this mode the traffic light phases are calculated to maximize the velocity of the main traffic. When the main traffic reaches the intersection, the green phase is switched on. In this paper we consider that all traffic flows are equal and the network configuration is free.To solve the problem it is necessary to develop the mathematical model of traffic control by adjusting the traffic light phases. The number of cars and traffic lights phases at adjoining roads must be taken into consideration.

THE MATHEMATICAL MODEL OF THE CONTROL OBJECT

To develop the mathematical model of traffic flow control we use an oriented graph with adjustable configuration. Let each road with the traffic flow between two neighboring intersections be the node in a graph. Let each maneuver between roads in the intersection be the edge in the graph. Then we obtain the directed graph for urban road network.

Let us consider the road network shown in Fig. 1. The network has two intersections and fourteen sections. The numbers of sections are given in the circles. Possible maneuvers at intersections are dot lined.

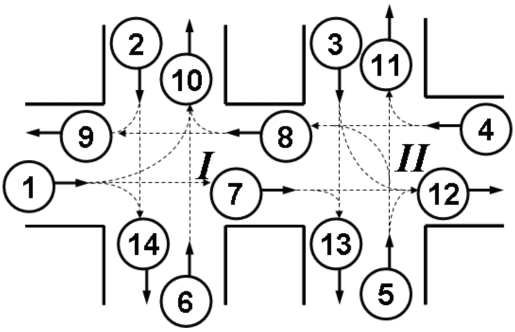

Fig. 1. The road network

The oriented graph for the given network is shown in Fig. 2.

The traffic is controlled by changes of traffic lights phases. Suppose that at intersection 1 in the network in Fig. 2 we have a three phase traffic light, and at intersection 2 a four phase traffic light.

For example in intersection 1 we have:

a) phase 1 allows a flow from 1 to 7 and 14, and from 8 to 9 and 10;
b) phase 2 allows a flow from 1 to 7, 10 and 14, and from 2 to 9;
c) phase 3 allows a flow from 2 to 9 and 14, and from 6 to 10.

For intersection 2 we have:

a) phase 1 allows a flow from 4 to 8 and 11, and from 7 to 12 and 13;
b) phase 2 allows a flow from 3 to 12, and from 5 to 11 and 12;
c) phase 3 allows a flow from 3 to 12 and 13, and from 5 to 11 and 12;
d) phase 4 allows a flow from 5 to 8, 11 and 12.

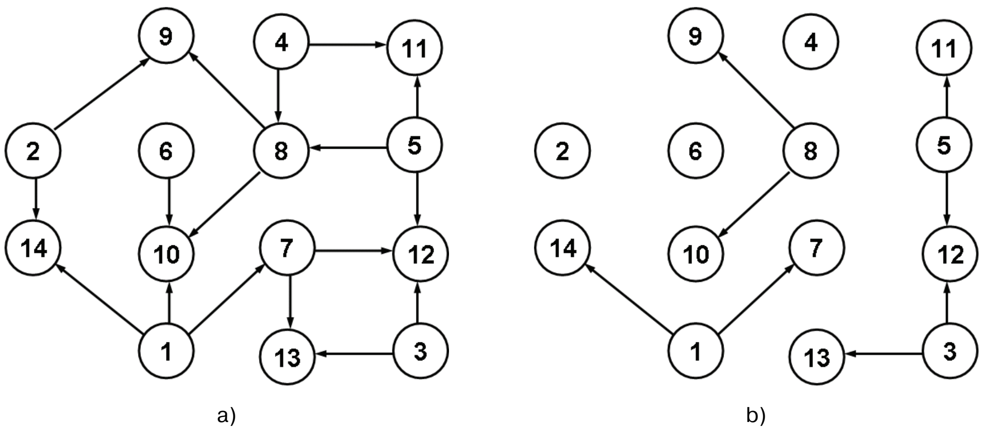

Fig. 2. The oriented graph with variable configuration

Any traffic light phase prohibits certain maneuvers between the roads. The prohibition of maneuvers is indicated in the graph as the absence of the edges.

For example, if the phase 1 is switched on in the intersection 1, and the phase 3 is switched on in the intersection 2, then we get the graph shown in Fig. 2b.

Graphs for urban road networks with all possible maneuvers in intersections are named the basic graphs. Graphs with allowable maneuvers at the traffic lights in intersections are called partial graphs or configurations of the basic graphs. The basic graph for the network in Fig 1 is represented in Fig 2a. The configuration of the basic graph at the phase 1 of traffic lights is represented in Fig 2b. There are 12 configurations for the basic graph in Fig 2a.

We control the traffic flows by means of traffic lights phases in intersections. To describe the choice of phases in intersections we use a control vector

$$\mathbf{u} = \left[u_1 \dots u_M\right]^T, \ \mathbf{u} \in \mathrm{U} = \mathrm{U}_1 \times \mathrm{U}_2 \times \dots \times \mathrm{U}_M, \qquad (1)$$

where M is a number of intersections in a road network, $u_i \in \mathrm{U}_i = \left\{1, \dots, u_i^+\right\}$, u_i^+ is a number of traffic light phases in the intersection i, $i = \overline{1, M}$.

A maximum number of configurations for the road networks is

$$\left|\mathrm{U}\right| = \prod_{i=1}^{M} u_i^+. \qquad (2)$$

To describe a basic graph we use a connectivity matrix

$$\mathbf{A} = \left[a_{ij}\right], \ a_{ij} \in \left\{0, 1\right\}, \ i, j = \overline{1, L}, \qquad (3)$$

where L is a number of roads in a network or nodes in a graph.

To describe relations between components of control vector and edges of the basic graph we use a control matrix

$$\mathbf{C} = \left[c_{ij}\right], \ c_{ij} \in \left\{0, 1, 2, ..., M\right\}, \ i, j = \overline{1, L}, \qquad (4)$$

where c_{ij} is a number of control vector component that relates to some edge between nodes i and j. If the edge is absent in the basic graph, then $a_{ij} = 0$, $i, j = \overline{1, L}$ and $c_{ij} = 0$.

To describe relations between the set of control vector component values and edges in the basic graph we use an allowable phase matrix

$$\mathbf{F} = \left[\mathrm{F}_{ij}\right], \ \mathrm{F}_{ij} \subseteq \mathrm{U}_{c_{ij}}, \ i, j = \overline{1, L}, \qquad (5)$$

where F_{ij} is a set of allowable values of control vector component $u_{c_{ij}}$ when the edge from node i to node j is not eliminate.

Matrices $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{F}$ can describe configurations of a basic graph according to the control vector $\mathbf{u} = \left[u_1 \ ... \ u_M\right]^T$. The structures of partial graphs are described by connectivity matrices of configurations.

$$\mathbf{A}(\mathbf{u}) = \left[a_{ij}(\mathbf{u})\right], \ i, j = \overline{1, L}, \qquad (6)$$

where

$$a_{ij}(\mathbf{u}) = \begin{cases} 1, \text{ if } u_{c_{ij}} \in \mathrm{F}_{ij}, \\ 0, \text{ if } u_{c_{ij}} \notin \mathrm{F}_{ij}, \end{cases} \quad i, j = \overline{1, L}. \tag{7}$$

The restrictions on maneuvers of traffic flows between roads are described by the capacity matrices

$$\mathbf{B} = \left[b_{ij} \right], \; b_{ij} \in \mathrm{R}^1, \; i, j = \overline{1, L}, \tag{8}$$

where b_{ij} is a restriction on maneuver from section i to section j.

A traffic flow distribution on roads is described by a distribution matrix

$$\mathbf{D} = \left[d_{ij} \right], \; d_{ij} \in \mathrm{R}^1, \; i, j = \overline{1, L}, \tag{9}$$

where d_{ij} is a part of traffic flow that comes from section i into section j.

Distribution matrix elements must fulfill the condition

$$\sum_{j=1}^{L} d_{ij} = 1, \text{ if } \sum_{j=1}^{L} a_{ij} \neq 0. \tag{10}$$

We have the following matrices $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{F}$ for the network in Fig 2.

$$\mathbf{A} = \begin{bmatrix}
0&0&0&0&0&0&1&0&0&1&0&0&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&1&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}, \quad
\mathbf{C} = \begin{bmatrix}
0&0&0&0&0&0&1&0&0&1&0&0&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&2&2&0\\
0&0&0&0&0&0&0&2&0&0&2&0&0&0\\
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0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix},$$

$$\mathbf{F} = \begin{bmatrix}
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{1,2\}&\varnothing&\varnothing&\{2\}&\varnothing&\varnothing&\varnothing&\{1,2\}\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{2\}&\varnothing&\varnothing&\varnothing&\varnothing&\{2,3\}\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{2,3\}&\{3\}&\varnothing\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{1\}&\varnothing&\varnothing&\{1\}&\varnothing&\varnothing&\varnothing\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{4\}&\varnothing&\varnothing&\{2,3,4\}&\{2,3,4\}&\varnothing&\varnothing\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{3\}&\varnothing&\varnothing&\varnothing&\varnothing\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{1\}&\{1\}&\varnothing\\
\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\varnothing&\{1\}&\{1\}&\varnothing&\varnothing&\varnothing&\varnothing\\
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\end{bmatrix}.$$

For the partial graph in Fig. 3 the configuration matrix $\mathbf{A}(\mathbf{u})$ with the control vector $\mathbf{u} = [1 \quad 3]^T$ has the following form

$$\mathbf{A}(\mathbf{u}) = \begin{bmatrix} 0&0&0&0&0&0&1&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}.$$

The configuration matrix $\mathbf{A}(\mathbf{u})$ manipulates the flow capacity matrix and the distribution matrix

$$\mathbf{B}(\mathbf{u}) = \mathbf{A}(\mathbf{u}) \odot \mathbf{B},\ \mathbf{D}(\mathbf{u}) = \mathbf{A}(\mathbf{u}) \odot \mathbf{D}, \tag{11}$$

where $\odot$ is a Hadamard product or element-wise production of matrices.

To describe the traffic values we use a flow vector

$$\mathbf{x} = [x_1 \ldots x_L]^T,\ x_i \in \mathbb{R}^1,\ i = \overline{1, L}, \tag{12}$$

where x_i is a value of flow on the section *i*.

We suppose that the traffic light phases are exactly divided on the given value called a control time. At each control time $k = \overline{0, N}$ we set the control vector $\mathbf{u}(k) = [u_1(k) \ldots u_M(k)]^T$ to change the graph configuration. Traffic flow vector $\mathbf{x}(k)$ depends on the chosen configuration, the properties of the network and the flow vector in the previous moment $\mathbf{x}(k-1)$. To build a mathematical model of traffic flow we use several assumptions.

Assumption 1. Cars do one maneuver at a time.

Assumption 2. Values of flow vectors are computed in two steps at one configuration. On the first step we compute a decrease of traffic flows by number of cars performing maneuvers. On the second step we compute an increase of traffic flows by the same number of car performing maneuvers.

On the first step flow values are reduced by values of the distribution matrix elements

$$\Delta\mathbf{x}'(k-1/2) = \left(\left(\mathbf{x}(k-1)\mathbf{1}_L^T\right) \odot \mathbf{D} \odot \mathbf{A}(\mathbf{u}(k))\right)\mathbf{1}_L, \tag{13}$$

$$\Delta\mathbf{x}''(k-1/2) = \left(\mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{B}\right)\mathbf{1}_L, \tag{14}$$

where $\mathbf{1}_L^T = \overbrace{[1\ldots1]}^{L}$.

Values of flow vector are determined as a difference between the value of flow vector at previous moment and the minimum among increments (13) and (14)

$$\mathbf{x}(k-1/2) = \mathbf{x}(k-1) - \min\left\{\Delta\mathbf{x}'(k-1/2), \Delta\mathbf{x}''(k-1/2)\right\}. \tag{15}$$

In (15) a minimum is computed by each component

$$x_i(k-1/2) = x_i(k-1) - \min\left\{\Delta x_i'(k-1/2), \Delta x_i''(k-1/2)\right\},\ i = \overline{1,L},$$

where $\mathbf{x}(k-1/2) = [x_1(k-1/2)\ldots x_L(k-1/2)]^T$, $\Delta\mathbf{x}'(k-1/2) = [\Delta x_1'(k-1/2)\ldots\Delta x_L'(k-1/2)]^T$, $\Delta\mathbf{x}''(k-1/2) = [\Delta x_1''(k-1/2)\ \ldots\ \Delta x_L''(k-1/2)]^T$.

Write (15) in the form $\mathbf{x}(k-1/2) = \mathbf{x}(k-1) - \left(\Delta\mathbf{x}'(k-1/2) - \left(\Delta\mathbf{x}'(k-1/2) - \right.\right.$ $\left.\left.\dot{-}\,\Delta\mathbf{x}''(k-1/2)\right)\right)$,

where $a \dot{-} b = \begin{cases} a-b, & \text{if } a > b, \\ 0 & \text{— otherwise.} \end{cases}$

In the second step we obtain

$$\Delta\mathbf{x}'(k) = \left(\left(\mathbf{x}(k-1)\mathbf{1}_L^T\right) \odot \mathbf{D} \odot \mathbf{A}\left(\mathbf{u}(k)\right)\right)^T \mathbf{1}_L,\ \Delta\mathbf{x}''(k) = \left(\mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{B}\right)^T \mathbf{1}_L,$$

where $\Delta\mathbf{x}'(k) = [\Delta x_1' \ldots \Delta x_L']^T$, $\Delta\mathbf{x}''(k) = [\Delta x_1'' \ldots \Delta x_L'']^T$.

Thus we get

$$\mathbf{x}(k) = \mathbf{x}(k-1/2) + \min\left\{\Delta\mathbf{x}'(k), \Delta\mathbf{x}''(k)\right\} \text{ or}$$
$$\mathbf{x}(k) = \mathbf{x}(k-1/2) + \Delta\mathbf{x}'(k) - \left(\Delta\mathbf{x}'(k) \dot{-} \Delta\mathbf{x}''(k)\right).$$

As a result we obtain the mathematical model of traffic flow control

$$\begin{aligned}\mathbf{x}(k) = \mathbf{x}(k-1) - &\left(\left(\mathbf{x}(k-1)\mathbf{1}_L^T\right) \odot \mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{D} - \left(\left(\mathbf{x}(k-1)\mathbf{1}_L^T\right) \odot\right.\right. \\ &\left.\left.\odot\, \mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{D} \dot{-} \mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{B}\right)\right)\mathbf{1}_L + \left(\left(\mathbf{x}(k-1)\mathbf{1}_L^T\right) \odot\right. \\ &\left.\odot\, \mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{D} - \left(\left(\mathbf{x}(k-1)\mathbf{1}_L^T\right) \odot \mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{D} \dot{-} \mathbf{A}\left(\mathbf{u}(k)\right) \odot \mathbf{B}\right)\right)^T \mathbf{1}_L.\end{aligned} \tag{16}$$

THE PROBLEM STATEMENT

Suppose we have an urban network with L roads and M intersections. Traffic light has u_i^+ phases in intersection i. We have a connectivity matrix (3) for a basic graph, a control matrix (4), an allowable phase matrix (5), a capacity matrix (8) and a distribution matrix (9). Restrictions on the flow values for each road are given x_i^+, $i=\overline{1,L}$. The mathematical model (16) and the initial values of flows $x_i(0)$, $i=\overline{1,L}$ for traffic flow control are also given. It's necessary to find the control in the form

$$\mathbf{u}=\mathbf{h}(\mathbf{x}). \tag{17}$$

where $\mathbf{h}(\mathbf{x})$: $\mathbb{R}^L \to \mathrm{U}_1\times\ldots\times\mathrm{U}_M$, $\mathrm{U}_i=\{0,1,\ldots,u_i^+\}$, $i=\overline{1,M}$.

The control has to minimize the object function

$$J_1=\sum_{i\in I_1}x_i(N)-\sum_{j\in I_2}x_j(N)\to\text{minimum}, \tag{18}$$

where I_1 is the set of input roads' numbers, I_2 is the set of output roads' numbers.

All flow values have to satisfy the restrictions

$$x_i(k)\le x_i^+,\ i=\overline{1,L},\ k=\overline{1,N}. \tag{19}$$

We include restrictions (19) in the object function (18)

$$J_1=s\sum_{k=1}^{N}\sum_{i=1}^{L}\left(\left(\frac{x_i(k)}{x_i^+}-1\right)+\left|\frac{x_i(k)}{x_i^+}-1\right|\right)x_i^+ +$$

$$+\sum_{i\in I_1}x_i(N)-\sum_{j\in I_2}x_j(N)\to\text{minimum}, \tag{20}$$

where s is a penalty coefficient.

Assume that the network includes the roads without restrictions. These can be output roads $i\in I_2$. Then in (19) we can substitute expressions $x_i^+=\infty$ by $x_i^+=-1$.

THE NETWORK OPERATOR METHOD

To solve our problem we use the network operator method [6—9]. The network operator allows to describe mathematical expression in a form of directed graph. The edges of the graph relate to unary operations. The nodes of the graph relate to binary operations. The source nodes relate to the arguments of the mathematical expressions. For the traffic control problem we have integer unary and binary operations

$$O_1=(\varphi_1(z),\ldots,\varphi_8(z)), \tag{21}$$

$$O_2=(\omega_0(z',z''),\ldots,\omega_7(z',z'')), \tag{22}$$

where $\omega_0(z',z'')=\max\{z'(\bmod z^+),z''(\bmod z^+)\}$, $\omega_1(z',z'')=\min\{z'(\bmod z^+),z''(\bmod z^+)\}$,

$\omega_2(z',z'')=(z'+z'')(\bmod z^+)$, $\omega_3(z',z'')=(z'\cdot z'')(\bmod z^+)$, $\omega_4(z',z'')=\begin{cases}z''(\bmod z^+),\text{if } z'=z'',\\ 0\text{ — otherwise},\end{cases}$

$\omega_5(z',z'')=\begin{cases}(z''+1)(\bmod z^+),\text{ if } z'>z'',\\ (z''-z')(\bmod z^+)\text{ — otherwise},\end{cases}$ $\omega_6(z',z'')=\begin{cases}z'(\bmod z^+),\text{if } z'>z'',\\ (z''-z')(\bmod z^+)\text{ — otherwise},\end{cases}$

$$\omega_7(z',z'')=\begin{cases} z^+-1, \text{ if } z'>z'',\\ (z''-z')(\bmod z^+) \text{— otherwise,}\end{cases} \quad \varphi_1(z)=z(\bmod z^+),\ \ \varphi_2(z)=(z+1)(\bmod z^+),$$

$$\varphi_3(z)=\begin{cases}(z-1)(\bmod z^+), \text{ if } z>0,\\ 0 \text{ — otherwise,}\end{cases} \quad \varphi_4(z)=z^+-1-z(\bmod z^+),$$

$$\varphi_5(z)=\begin{cases}2z, \text{ if } 2z<z^+,\\ z^+-1-(2z)(\bmod z^+) \text{ — otherwise,}\end{cases} \quad \varphi_6(z)=\begin{cases}3z, \text{ if } 3z<z^+,\\ z^+-1-(3z)(\bmod z^+) \text{ — otherwise,}\end{cases}$$

$$\varphi_7(z)=\frac{z}{2},\ \ \varphi_8(z)=\frac{z}{3},\ \ z^+=\max\{u_i^+, i=\overline{1,M}\}.$$

We use an input integer variable $\mathbf{y}$ for control (17)

$$\mathbf{u}=\mathbf{h}(\mathbf{y}), \tag{23}$$

where $\mathbf{y}=[y_1\ldots y_L]^T$, $y_i(k)=\left\lfloor \frac{x_i(k)}{y_i^+\Delta y_i}\right\rfloor$, $i=\overline{1,L}$, $i\notin I_0$, $y_i^+=\begin{cases}0, \text{if } x_i^+=\infty, x_i(0)=0,\\ x_i(0), \text{if } x_i^+=\infty, x_i(0)\neq 0,\\ x_i^+ \text{ — otherwise,}\end{cases}$

$i=\overline{1,L}$, $i\notin I_0$, $\Delta y_i=\frac{1}{u^+}$, $u^+=\max\{u_i^+, i=\overline{1,M}\}$.

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МЕТОД ЛОГИЧЕСКОГО СЕТЕВОГО ОПЕРАТОРА ДЛЯ СИНТЕЗА УПРАВЛЕНИЯ ПОТОКАМИ ТРАНСПОРТА В СЕТИ ГОРОДСКИХ ДОРОГ

Г.Х.А. Алновайни[1], А.И. Дивеев[2], К.А. Пупков[3], Е.А. Софронова[1]

[1]Кафедра кибернетики и мехатроники
Российский университет дружбы народов
ул. Орджоникидзе, 3, Москва, Россия, 117923

[2]Вычислительный центр им. А.А. Дородницына РАН
ул. Вавилова, 40, Москва, Россия, 119333

[3]Кафедра системы автоматического управления
Московский государственный технический университет им. Н.Э. Баумана
ул. 2-я Бауманская, 5, Москва, Россия, 105005

Рассмотрена задача максимальной пропускной способности дорог в часы пик. Управление осуществляется за счет согласования фаз светофоров на перекрестках. Приведена математическая модель управления транспортного потока. Она описывается ориентированным графом с изменяемой конфигурацией. Задача оптимального управления транспортными потоками сформулирована как задача дискретной динамической оптимизации. Для решения задачи используется генетический алгоритм и метод логического сетевого оператора.

Ключевые слова: логический сетевой оператор, синтез управления, моделирование транспортных потоков.