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THE NONLINEAR BENDING OF SIMPLY SUPPORTED ELASTIC PLATE

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In this article, assumptions in Classical Plate Theory (CPT) are explained; followed by concepts involved in Finite Element discretization for elastic Plate bending in CPT. Computer implementation aspects and Numerical Results of CPT elements are also included for analyzing nonlinear bending of simply supported elastic plates.

Key words: Classical Plate Theory (CPT), Elastic Plate bending in CPT, Kirchhoff model, Conforming and Non-Conforming Elements, nondimensionalized deflections and stresses, isotropic plate structures

Introduction

A plate is a three-dimensional structural element, which is characterized by two key properties. Firstly, its geometrically three-dimensional solid whose thickness is very small (thin when compared with other dimensions of the faces (length, width, diameter, etc.). Secondly, the static or dynamic loads carried by plates are predominantly perpendicular to the plate faces.

By "thin," it is meant that the plate's transverse dimension, or thickness h, is small compared to the length and width dimensions. A mathematical expression of "thin" in the aforementioned paragraph is: $t/h < 1$ thickness, and L represents a representative length or width.

Prominently a plate has two special geometric features; viz,

Thinness: One of the plate dimensions, called its thickness, is much smaller than the other two.

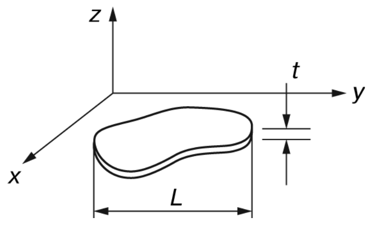


Figure 1. Plate and associated (x, y, z) coordinate system

Flatness: The mid surface of the plate, which is the locus of the points located halfway between the two plate surfaces, is a plane.

In this article attention is focused on the Kirchhoff model for bending of thin (but not too thin) plates. The term "thin" is to be interpreted in the engineering sense and not in the mathematical sense. For example, h/Lc is typically 1/5 to 1/100 for most plate structures.

Consider first a flat surface, called the plate reference surface or simply its mid surface or mid plane (see Figure 2, *b*). We place the axes x and y on that surface to locate its points. The third axis, z is taken normal to the reference surface forming a right-handed Cartesian system. Axis x and y are placed in the mid plane, forming a right-handed Rectangular Cartesian Coordinate (RCC) system.

If the plate is shown with a horizontal mid surface, as in Figure 2 below, we shall orient z *upwards*.

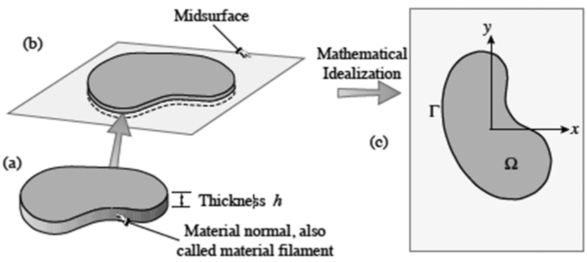


Figure 2. Idealization of plate as two-dimensional mathematical problem

Next, imagine material normals, also called material filaments, directed along the normal to the reference surface (that is, in the z direction) and extending $h/2$ above and $h/2$ below it. The magnitude h is the plate thickness. We will generally allow the thickness to be a function of x, y, that is $h = h(x, y)$, although most plates used in practice are of uniform thickness because of fabrication considerations.

The end points of these filaments describe two bounding surfaces, called the plate surfaces. The one in the $+z$ direction is by convention called the top surface whereas the one in the $-z$ direction is the bottom surface.

Assumptions in Classical Theory of Plates

The classical plate theory (CPT) is based on the Kirchhoff hypothesis. Three assumptions involved in this hypothesis are:

1. A cross-section perpendicular to the middle surface prior to deformation remains plane and perpendicular to the deformed middle surface (Fig. 3).

2. The transverse normals do not experience elongation (i.e. they are inextensible).

3. The transverse normals rotate such that they remain perpendicular to the mid-surface after deformation.

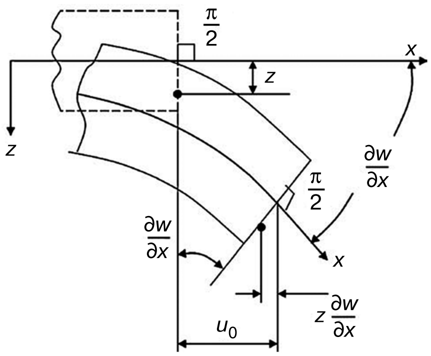


Figure 3. Deformation of the cross section in the *xz* plane according to the Kirchhoff assumptions

Finite Elements for elastic Plate bending in CPT

Historically the first model of thin plate bending was developed by Lagrange, Poisson and Kirchhoff. It is known as the Kirchhoff plate model. In the finite element literature Kirchhoff plate elements are often called C^1 plate elements because that is the continuity order nominally required for the transverse displacement shape functions.

The application of the finite element method (FEM) to the analysis of Kirchhoff plate bending demands the continuity in the first derivative of the expansion of the deflection w. The reader is referred to Zienckiewicz's excellent book for details.

Conforming vs Non-Conforming Elements. Displacement should be compatible between adjacent elements. There should not be any discontinuity or overlapping while deformed. The adjacent elements must deform without causing openings, overlaps or discontinuity between the elements.

On each element displacements and test functions are interpolated using shape functions and the corresponding nodal values.

$$u_3(x_1,\ x_2)=\sum_{k=1}^{N_p} N^k(x_1,\ x_2)u_3^k,\ v_3(x_1,\ x_2)=\sum_{k=1}^{N_p} N^k(x_1,\ x_2)v_3^k.$$

Where, N^k is a shape functions and u_3^k, v_3^k the Nodal values.

To obtain the FE equations the preceding interpolation equations are introduced into the weak form.

Similar to Euler-Bernoulli Beam the internal virtual work depends on the second order derivatives of the deflection, u_3 and virtual deflection, υ.

The problem domain is partitioned into a collection of pre-selected finite elements (either triangular or rectangular).

Conforming Elements. Elements that satisfy all the three convergence requirements and compatibility condition are called **Compatible** or **Conforming** elements.

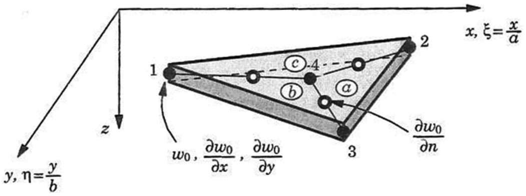


Figure 4. Conforming triangular element

Triangular element: A conforming triangular element due to Clough and Tocher is an assemblage of three triangles as shown in fig. 4. In each sub-triangle, the transverse deflection is represented by the polynomial ($i = 1, 2, 3$):

$$w_0{}^i(x, y) = a_i + b_i\xi + c_i\eta + d_i\xi\eta + e_i\xi^2 + f_i\eta^2 + g_i\xi^2 + h_i\xi^2\eta + k_i\xi\eta^2 + l_i\eta.$$

Where, (ξ, η) are the local coordinates, as shown in fig. 4 above.

Rectangular element: Conforming rectangular element with w_0, $\partial w_0/\partial x$, $\partial w_0/\partial y$ and $\partial^2 w_0/\partial x\partial y$ as the nodal variables (see fig. 5) was developed by Bogner. This element has fewer degrees of freedom, which are less accurate in theory, but in practice the analytical solution of the problems frequently are not sufficiently high to achieve the accuracy of the triangular elements. This is good choice of element for thin plate analysis.

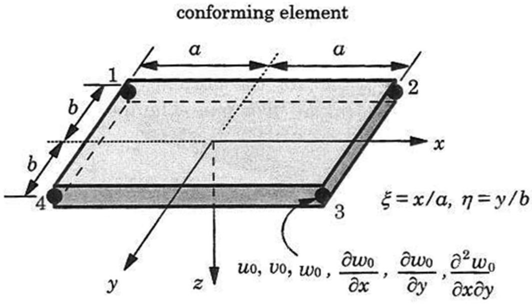


Figure 5. Conforming rectangular element

Non-Conforming Elements. Elements that violate continuity conditions are known as **Non-conforming** elements.

The Adini-Clough element is a nonconforming element. Despite this deficiency the element is known to give good results.

Triangular element: The first successful nonconforming triangular plate-bending element was the original BCIZ, (Nonconforming element of Bazeley, Cheung, Irons and Zienkeiwicz), and it consists of three degrees of freedom (DOF) (w_0, θ_x, θ_y) at the vertex nodes (see fig. 6). The element performs very well in bending as well as vibration problems (with a consistent mass matrix).

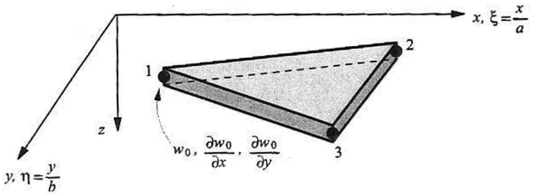


Figure 6. Non-conforming triangular element with 3 DOF (w_0, $\partial w_0/\partial x$, $\partial w_0/\partial y$) per node

Rectangular element: Non-conforming rectangular element has w_0, θ_x and θ_y as the nodal variables (see fig. 7).

Non-conformity: The element is C^0 continuous, since the functions are cubic along an edge and we have four degrees of freedom to specify the function. The normal derivatives are not continuous across inter element boundaries in general.

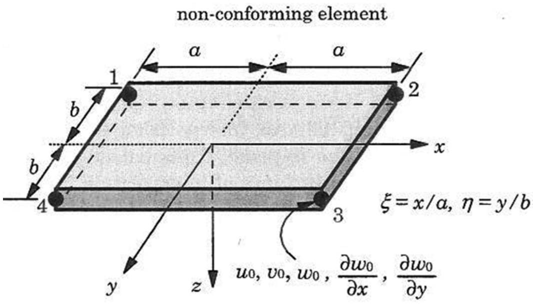


Figure 7. Non-conforming rectangular element

Computer implementation aspects and Numerical Results of CPT elements

The conforming and non-conforming rectangular finite elements discussed in this article are implemented into a computer program using bilinear interpolation of (u_0, υ_0) and Hermite cubic interpolation of w_0.

Results of Linear Analysis. We consider the bending of rectangular plates with various edge conditions to evaluate the elements developed herein. The foundation modulus k is set to zero in all examples. The linear stiffness coefficients are evaluated using 4 × 4 Gauss rule.

Example 1: Consider a simply supported (SS-1) rectangular plate under uniformly distributed load. The geometric boundary conditions of the computational domain (see the shaded quadrant in fig. 8) are:

$$u_0 = \partial w_0/\partial x = 0 \text{ at } x = 0;\ \upsilon_0 = \partial w_0/\partial y = 0 \text{ at } y = 0;\ \upsilon_0 = w_0 = \partial w_0/\partial y = 0 \text{ at } x = a/2;$$

$$u_0 = w_0 = \partial w_0/\partial x = 0 \text{ at } y = b/2;\ \partial^2 w_0/\partial x \partial y = 0 \text{ at } x = y = 0$$

(For conforming element only).

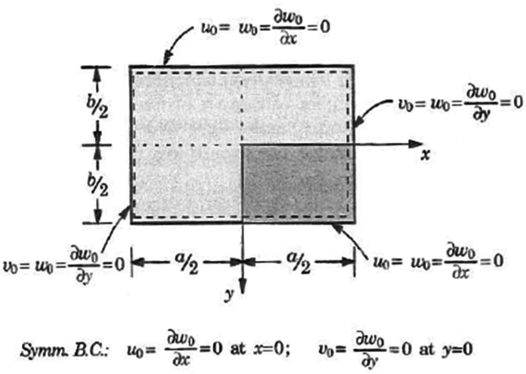


Figure 8. Boundary conditions for rectangular plates with biaxial symmetry

Table 1 shows a comparison of non-dimensionalized finite element solutions with the analytical solutions (see Reddy [3]) of isotropic and orthotropic square plates under uniformly distributed transverse load q_0. The stresses were evaluated at the center of the element. Hence, the locations of the maximum normal stresses are ($a/8$, $b/8$), ($a/16$, $b/16$), and ($a/32$, $b/32$) for uniform meshes 2 × 2, 4 × 4, and 8 × 8, respectively, while those of σ_{xy} are ($3a/8$, $3b/8$), ($7a/16$, $7b/16$), and ($15a/32$, $15b/32$) for the three meshes. The analytical solutions were evaluated using m, n = 1,3...19. The exact maximum deflection occurs at $x = y = 0$, maximum stresses σ_{xx} and σ_{yy} occur at (0,0,$h/2$), and the maximum shear stress σ_{xy} occurs at ($a/2$, $b/2$, $-h/2$).

Table 1

Maximum transverse deflections and stresses* of simply supported plates under a uniformly distributed load q_0 (linear analysis)

Variable	Non-conforming			Conforming			
	2 × 2	4 × 4	8 × 8	2 × 2	4 × 4	8 × 8	Analytical solution
Isotropic plate (ν = 0,25)							
$\underline{\omega} \times 10^2$	4.8571	4.6425	4.5883	4.7619	4.5952	4.5739	4.5698

End of Table 1

Variable	Non-conforming			Conforming			
	2 × 2	4 × 4	8 × 8	2 × 2	4 × 4	8 × 8	Analytical solution
$\underline{\sigma}_{xx}$	0.2405	0.2673	0.2740	0.2239	0.2637	0.2731	0.2762
$\underline{\sigma}_{xy}$	0.1713	0.1964	0.2050	0.1688	0.1935	0.2040	0.2085
Orthotropic plate ($E_1/E_2 = 25$, $G_{12} = G_{13} = 0{,}5E_2$, $v = 0{,}25$)							
$\underline{\omega} \times 10^2$	0.7082	0.6635	0.6531	0.7710	0.6651	0.6522	0.6497
$\underline{\sigma}_{xx}$	0.7148	0.7709	0.7828	0.5560	0.7388	0.7743	0.7866
$\underline{\sigma}_{yy}$	0.0296	0.0253	0.0246	0.0278	0.0249	0.0245	0.0244
$\underline{\sigma}_{xy}$	0.0337	0.0421	0.0444	0.0375	0.0416	0.0448	0.0463

$* \; \overline{\omega} = \dfrac{\omega_0 E_2 h^3}{q_0 a^4}, \; \overline{\sigma} = \dfrac{\sigma h^2}{q_0 a^2}.$

Example 2: Here we consider a clamped square plate under uniformly distributed load. The boundary conditions are taken to be:

$$u_0 = \partial w_0/\partial x = 0 \text{ at } x = 0; \upsilon_0 = \partial w_0/\partial y = 0 \text{ at } y = 0;$$

$$\text{at } x = a/2;\ u_0 = \upsilon_0 = w_0 = \partial w_0/\partial x = \partial w_0/\partial y = 0$$

$$\text{at } y = b/2;\ u_0 = \upsilon_0 = w_0 = \partial w_0/\partial x = \partial w_0/\partial y = 0$$

$$\partial^2 w_0/\partial x \partial y = 0 \text{ on clamped edges (For conforming element only)}$$

Table 2

Maximum transverse deflections and stresses* of clamped (CCCC), isotropic and orthotropic, square plates ($a = b$) under a uniformly distributed load q_0 (linear analysis)

Variable	Non-conforming			Conforming		
	2 × 2	4 × 4	8 × 8	2 × 2	4 × 4	8 × 8
Isotropic plate ($v = 0{,}25$)						
$\underline{\omega} \times 10^2$	1.5731	1.4653	1.4342	1.4778	1.4370	1.4249
$\underline{\sigma}_{xx}$	0.0987	0.1238	0.1301	0.0861	0.1197	1.1288
$\underline{\sigma}_{xy}$	0.0487	0.0222	0.0067	0.0489	0.0224	0.0068
Orthotropic plate ($E_1/E_2 = 25$, $G_{12} = G_{13} = 0{,}5E_2$, $v = 0{,}25$)						
$\underline{\omega} \times 10^2$	0.1434	0.1332	0.1314	0.1402	0.1330	0.1311
$\underline{\sigma}_{xx}$	0.1962	0.2491	0.2598	0.1559	0.2358	0.2576
$\underline{\sigma}_{yy}$	0.0085	0.0046	0.0042	0.0066	0.0047	0.0043
$\underline{\sigma}_{xy}$	0.0076	0.0046	0.0019	0.0083	0.0048	0.0020

$* \; \overline{\omega} = \dfrac{\omega_0 E_2 h^3}{q_0 a^4}, \; \overline{\sigma} = \dfrac{\sigma h^2}{q_0 a^2}.$

Table 2 contains the nondimensionalized deflections and stresses. The locations of the normal stresses reported for the three meshes are:

2 × 2: $(a/8, b/8)$; 4 × 4: $(a/16, b/16)$; 8 × 8: $(a/32, b/32)$

and shear stresses reported for the three meshes are:

2 × 2: $(3a/8, 3b/8)$; 4 × 4: $(7a/16, 7b/16)$; 8 × 8: $(15a/32, 15b/32)$.

These stresses are not necessarily the maximum ones in the plate. For example for an 8 × 8 mesh, the maximum normal stress in the isotropic plate is found to be 0.2300 at (0.46875a, 0.03125b, -h/2) and the maximum shear stress u is 0.0226 at (0.28125a, 0.09375b, -h/2) for the non-conforming element. The conforming element yields slightly better solutions than the non-conforming element for deflections but not for the stresses, and both elements show good convergence.

Results of Nonlinear Analysis. Here we investigate geometrically nonlinear response of plates using the conforming and non-conforming plate finite elements. The nonlinear terms are evaluated using reduced integration. Full integration (F) means 4 × 4 Gauss rule and reduced integration (R) means 1 × 1 Gauss rule.

Example 3: Having established the credibility of the finite element for the linear analysis of CPT plates, we now employ the element in the nonlinear analyses. First, results are presented for single-layer isotropic square plates under uniform loading. The essential geometric boundary conditions used are (BC3 and BC5 in Table 1), simply supported (SS-3):

SS-3: $u = v = w = 0$ on all edges.		
Clamped (CC-l):	$u = v = w = 0$ on all edges,	Figures 9 and 10 show the nondimensionalized center deflection, $\bar{w} = w/h$, and nondimensionalized center stress, $\bar{\sigma} = \sigma a^2/Eh^2$, as a function of the load parameter, $P = q_0a^4/Eh^4$ for.
	$\psi_y = 0$ along edges parallel to x-axis,	
	$\psi_x = 0$ along edges parallel to y-axis.	

simply supported (SS-3) square plate, under uniformly distributed load. Figure 11 shows similar results for clamped (CC-l) square plate under uniformly distributed loadThe results are compared with the Ritz solution of Way [5], double Fourier-series solution of Levy [6], the finite-difference solution of Wang [7], the Galerkin solution of Yamaki [8], and the displacement finite-element solution of Kawai and Yoshimura [9]. Finite-element solutions were computed for the five degrees of freedom (NDF = 5), and for three degrees of freedom (NDF = 3); in the latter case, the in-plane displacements were suppressed. Since suppressing the in-plane displacements stiffens the plate, the deflections are smaller and stresses are larger than those obtained by including the in-plane displacements. Solutions of the other investigators were read from the graphs presented in their papers. The present solutions are in good agreement with the results of other investigators.

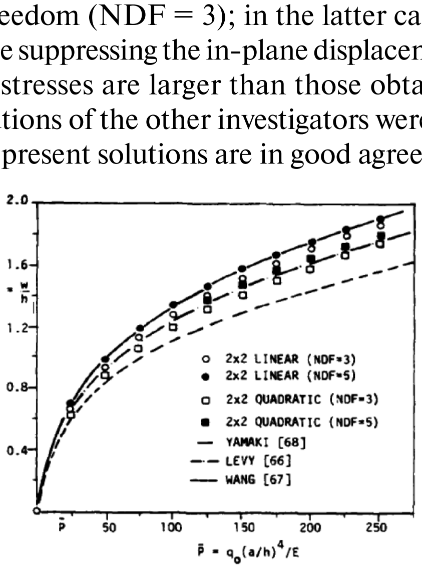


Figure 9. Load-deflection curves by various investigators for simply supported (SS-3) square plate under uniformly distributed load

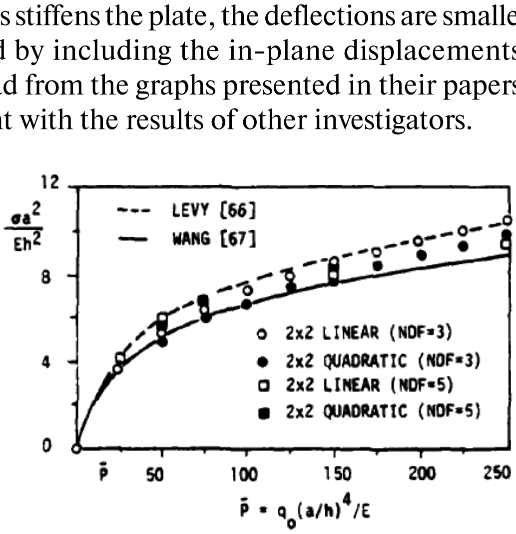
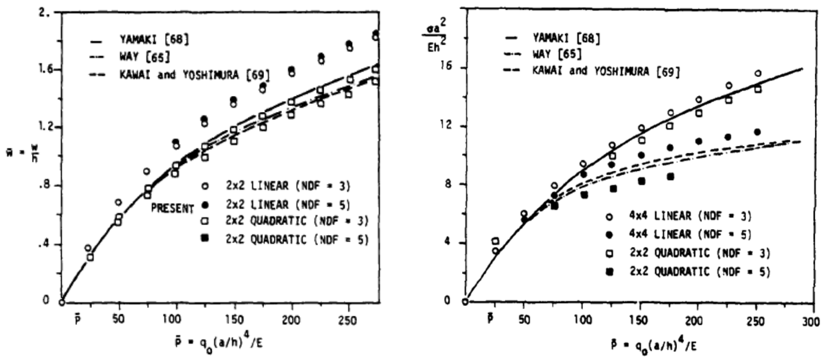


Figure 10. Nondimensionalized center stress versus the load parameter for simply supported (SS-3) square plate under uniformly distributed load



(a) Load-deflection curves (b) Center stress versus load parameter

Figure 11. Nondimensionalized center deflection and stress versus the load parameter for clamped (CC-I) square plate under uniformly distributed load

Table 2 shows nondimensionalized center deflection $\overline{w} = w/h$, center stress $\partial_x^A = \sigma_x{}^0 a^2/Eh^2$, and edge stress $= \partial_x^B = \sigma_x(a/2, 0)a^2/Eh^2$, of aclamped (CC-2) square plate under uniformly distributed load, qo. The boundary conditions are of type BC6, CC-2: $u = v = w = \psi_x = \psi_y = 0$ along all edges.

The present solution is obtained using a 2 by 2 uniform mesh (in quarter plate) of nine-node isoparametric elements (2Q9) with (R) and without (F) reduced integration. The present solution for center deflection and stresses agree very closely with the finite element solution of Pica et al. [10], and the analytical solution of Levy [11]. The edge stress, for some reason, does not agree with the other two results.

Table 1

Types of boundary conditions used in the present study

Notation (Type)	Description of essential boundary conditions	
	Side 1	Side 2
BC1 (SS-I)	$v = w = \psi_y = 0$	$u = w = \psi_x = 0$
BC2 (SS-2)	$u = w = \psi_y = 0$	$v = w = \psi_x = 0$
BC3 (SS-3)	$u = v = w = 0$	$u = v = w = 0$
BC4 (SS-4)	$u = v = w = \psi_y = 0$	$u = v = w = \psi_x = 0$
BC5 (CC-i)	$u = v = w = \psi_x = 0$	$u = v = w = \psi_y = 0$
BC6 (CC-2)	$u = v = w = \psi_x = \psi_y = 0$	$u = v = w = \psi_x = \psi_y = 0$
BC7 (CC-3)	$u = w = \psi_x = 0$	$v = w = \psi_y = 0$
BC8 (CC-4)	$w = \psi_x = 0$	$w = \psi_y = 0$

Table 2

Nondimensionalized center deflection ($\overline{w}$), center stress (∂_x^A), and edge stress (∂_x^B) for clamped (CC-2) square plate under uniformly distributed load, (q_0) ¶

$p = q_0a^4/Eh^4$	$\overline{\omega} = \omega_0/h$				$\overline{\sigma}_x^A = \sigma_x^A a^2/Eh^2$				$\overline{\sigma}_x^B = \sigma_x^B a^2/Eh^2$			
	Pica				Pica				Pica			
	present F	(2Q9) R	et al.[1] [70]	Levy [71]	present F	(2Q9)[2] R	et al. [70]	Levy [71]	present F	(2Q9) R	et al. [70]	Levy [71]
17,79	0,1904	0,2455	0,2368	0,237	2,239	2,459	2,6319	2,6	0,8904	0,558	5,3163	5,58
38,3	0,3881	0,4784	0,3699	0,471	4,839	5,129	5,4816	5,2	1,692	1,394	11,216	11,52
63,4	0,5897	0,7045	0,6915	0,695	7,767	7,834	8,3258	8,0	2,373	2,672	17,726	18,03
95,0	0,7909	0,9147	0,9029	0,912	10,97	10,46	11,103	11,1	2,915	4,389	24,967	25,32
134,9	0,9862	1,1189	1,1063	1,121	14,38	13,09	13,857	13,3	3,316	6,606	33,045	33,5

End of Table 2

$p = q_0a^4/Eh^4$	$\bar{\omega} = \omega_0/h$				$\bar{\sigma}_x^A = \sigma_x^A a^2/Eh^2$				$\bar{\sigma}_x^B = \sigma_x^B a^2/Eh^2$			
	Pica				Pica				Pica			
	present F	(2Q9) R	et al.[1] [70]	Levy [71]	present F	(2Q9)[2] R	et al. [70]	Levy [71]	present F	(2Q9) R	et al. [70]	Levy [71]
189,0	1,1791	1,3189	1,3009	1,323	17,98	15,75	16,497	15,9	3,568	9,325	41,885	42,4
245,0	1,3781	1,5155	1,4928	1,521	21,89	18,48	19,225	19,2	3,665	12,59	51,719	52,8
318,0	1,5672	1,7020	1,6786	1,714	25,89	21,21	21,994	21,9	3,627	16,29	62,325	63,9
402,0	1,7469	1,8760	1,8555	1,902	29,91	23,89	24,780	25,1	3,464	20,30	73,407	75,8

[1] 4Q9 (4 by 4 mesh of 9-node elements).
[2] computed at the nearest Gauss points.

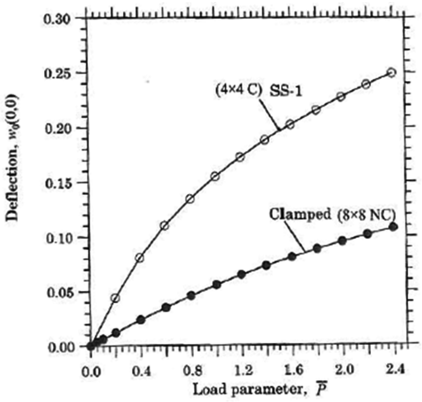


Figure 12. Load-deflection curves for simply supported and clamped orthotropic square plates under uniform load

Example 4: This example is concerned with the bending of simply supported (SS-I) orthotropic square plate under a uniformly distributed transverse load q_0. The geometry and material properties used are given below.

$$a = b = 12 \text{ in.}, h = 0.138 \text{ in.}, E1 = 3 \times 10^6 \text{ psi}, E2 = 1.28 \times 10^6 \text{ psi},$$

$$G_{12} = G_{23} = G_{13} = 0.37 \times 10^6 \text{ psi}, \upsilon_{12} = \upsilon_{23} = \upsilon_{13} = 0.32.$$

A load increment of $\Delta q_0 \approx 0.2$ psi and a uniform mesh of 4 × 4 in a quarter plate was used Figure 12 shows plots of the center deflection versus the intensity of the distributed load. The finite element results are in close agreement with the experimental results of Zaghloul and Kennedy [12].

Example 5: In this last example of this section we consider the bending of a clamped square plate under uniformly distributed load, q_0. The geometry and material parameters used are the same as those in Example 4. The boundary conditions of a clamped edge are taken to be:

$$u_0 = \upsilon_0 = w_0 = \frac{\partial w_0}{\partial z} = \frac{\partial w_0}{\partial y} = 0.$$

Of course, for conforming element, one may also impose ($\partial^2 w_0/\partial x \partial y = 0$). A uniform mesh of 8 × 8 non-conforming elements in a quarter plate is used, and a load increments of:

$$\{\Delta q_0\} = \{0.05, 0.05, 0.1, 0.2, 0.2 \ldots 0.2\}$$

psi was used. The linear solution at $q_0 \approx 0.05$ is found to be $w_0(0,0) = 0.00302$ in. A plot of the center deflection versus the intensity of the distributed load for the clamped orthotropic plate is included in Figure 12 above.

Conclusion

A finite element formulation based on Kirchhoff's plate model assumptions has been developed for the analysis of nonlinear bending of simply supported elastic plates.

Different continuity conditions, according to displacement gradients, can be introduced into the formulation of geometrically non-linear non-conforming elastic plate elements to ensure convergence.

We can also conclude with a note that the plate bending elements of the CPT discussed here are adequate for most engineering applications, which involve thin, and isotropic plate structures, which can be used to analyze both thin and thick plates.

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НЕЛИНЕЙНЫЙ ИЗГИБ ОПЕРТОЙ УПРУГОЙ ПЛАСТИНЫ

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В статье изложены допущения в классической теории пластин (КПП); с последующей формулировкой, применяемой в конечно-элементной дискретизации для упругой пластины в КПП. Аспекты компьютерных расчетов также включены для анализа нелинейного изгиба опертой упругой пластины.

Ключевые слова: классическая теория пластин (КПП), упругий изгиб пластины, модель Кирхгофа, соответствующие и несоответствующие элементы, безразмерные прогибы и напряжения, изотропные пластины

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