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Towards the Analysis of the Queuing System Operating in the Random Environment with Resource Allocation

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The mathematical model of the system, that consists of a storage device and several homogeneous servers and operates in a random environment, and provides incoming applications not only services, but also access to resources of the system, is being constructed. The random environment is represented by two independent Markov processes. The first of Markov processes controls the incoming flow of applications to the system and the size of resources required by each application. The incoming flow is a Poisson one, the rate of the flow and the amount of resources required for the application are determined by the state of the external Markov process. The service time for applications on servers is exponential distributed. The service rate and the maximum amount of system resources are determined by the state of the second external Markov process. When the application leaves the system, its resources are returned to the system. In the system under consideration, there may be failures in accepting incoming applications due to a lack of resources, as well as loss of the applications already accepted in the system, when the state of the external Markov process controlling the service and provision of resources changes. A random process describing the functioning of this system is constructed. The system of equations for the stationary probability distribution of the constructed random process is presented in scalar form. The main tasks for further research are formulated.

Key words and phrases: queuing system, random environment, Markov modulated Poisson process, Markov modulated service process, resource allocation

1. Introduction

The mathematical model of the analysis of the functioning of modern telecommunication systems must take into account the influence of external factors, which may be realized within the framework of the queuing theory (the theory of teletraffic) [1–4] with the help of arrival and/or service processes controlled by some external random process. The application of the Markov modulated arrival process (MMAP), Markov modulated service process (MMSP)) [3, 5–9] allows us to construct not only the adequate mathematical model, but also to obtain good analytical results for different tasks [10–22].

The mathematical modeling of modern telecommunication systems when incoming applications in addition to services also require some fixed or variable volume of resources [23–29] is the actual problem.

We will try to apply Markov modulated Poisson process (MMPP) theory [5–9] to construct the mathematical model of the system, that consists of a storage device and several homogeneous servers and operates in a random environment, provides incoming applications not only services, but also access to resources of the system, is being constructed. The random environment is represented by two independent Markov processes. The first of Markov processes controls the incoming flow of applications to the system and the size of resources required by each application. The service rate and the maximum amount

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of system resources are determined by the states of the second external Markov process. The initial stages of this study were presented in [30]. The system of equations for the stationary probability distribution of the random process, describing the behavior of the system, is the main goal of this part of the research.

2. System description

We will consider the queueing system $MMPP_2|MMSP_2|n|r|R_1, R_2$ (according to Kendall–Basharin notation [1]), functioning in the random environment (Markov modulated Poisson arrival process and Markov modulated service process), with $1 \leqslant n < \infty$ homogeneous servers and the buffer of $r \leqslant \infty$ capacity.

The random environment is present by two-state Markov process (MP) $\eta_1(t)$, which control the incoming Poisson process. If the external Markov process $\eta_1(t)$ is in state 1 then the rate of incoming Poisson process is λ_1 and each arriving application requires the fixed k_1 amount of system resources. If the MP $\eta_1(t)$ is in state 2 the each application arrives according the Poisson law with the rate λ_2 and requires the fixed amount of system resources of size k_2.

The second external two-state Markov process $\eta_2(t)$ controls the service process on system servers and the maximum amount of system resources. If MP $\eta_2(t)$ is in the state 1, then the maximum value of system resources is $R_1 < \infty$, the service time of an application (on each of n homogeneous servers) is subject to the exponential distribution with the rate μ_1. If MP $\eta_2(t)$ is in the state 2, then the amount of system resources R_2 is unlimited, the service time of an application (on each of n homogeneous servers) is subject to the exponential distribution but with the rate mu_2.

The transitions of Markov processes η_1 and η_2 from one state to another are determined by the corresponding infinitesimal matrices $\Lambda = (\lambda_{ij})_{i,j=1,2}$ and $M = (\mu_{ij})_{i,j=1,2}$.

After the end of the service each application returns to the system the resources, occupied by this application.

The functioning of the system may be defined by the multidimensional random process $\zeta(t) = \{\xi_1(t), \xi_2(t), R(t), \eta_1(t), \eta_2(t)\}$, where random process $\xi_1(t) = (\xi_{1s}(t), \xi_{1q}(t))$ describes the number of applications with demand on k_1 amount of resources (applications of the first type) on the servers ($\xi_{1s}(t)$) and in the buffer ($\xi_{1q}(t)$) at the time moment t. Respectively, the random process $\xi_2(t) = (\xi_{2s}(t), \xi_{2q}(t))$ — the number of application with demand on k_2 amount of resources (applications of the second type) on the servers ($\xi_{2s}(t)$) and in the buffer ($\xi_{2q}(t)$) at the time moment t. $R(t)$ — the available at time t amount of system resources. If the state of the Markov process η_2 is 1, then $R(t) = \max(0, R_1 - k_1\xi_1(t)\mathbf{1} - k_2\xi_2(t)\mathbf{1})$, if the state of the Markov process η_2 is 2 then $R(t) = R_2 = \infty$.

If the amount of the system resources $R(t)$ at the moment of the new application arrival is less then k_1 (for the first type application) or k_2 (for the second type application) amount of resources needed in addition to service (i.e. $R(t) < k_1$ or $R(t) < k_2$), then the incoming application is lost. Also the accepted to the system applications may be dropped from the buffer due to the transition Markov chain η_2 from state 2 with unlimited amount R_2 of system resources to the state 1 with limited amount of resources $R(t) = R_1 - k_1\xi_1(t)\mathbf{1} - k_2\xi_2(t)\mathbf{1}$.

In order to avoid downtime of servers it is supposed that the maximum value of system resources $R_1 < \infty$ is sufficient for all servers to be occupied, that is $R_1 \geqslant n \cdot \max(k_1, k_2)$.

The goal of this paper is to derive the system of equations for random process $\zeta(t)$ steady-state probability distribution. The main goals of the study as a whole are to obtain main time-probability characteristics of the system as for this general case (also for the case when the maximum values of system resources are finite, but different for all states of governing external Markov process), and for special cases of only one external governing Markov process.

3. The steady-state probability distribution. The system of equations (scalar form)

The set $\mathcal{X}$ of states of the random process $\zeta(t) = \{\xi_1(t), \xi_2(t), R(t), \eta_1(t), \eta_2(t)\}$ may be presented as $\mathcal{X} = \{(i_s; i_q), (j_s; j_q), R_1(i_s+i_q; j_s+j_q)|R_2, l, m\}$. Here, i_s and i_q ($0 \leqslant i_s \leqslant n$, $i_q \geqslant 0$) are numbers of the first type applications on servers (i_s) and in the buffer (i_q); j_s and j_q ($0 \leqslant j_s \leqslant n$, $j_q \geqslant 0$) are numbers of the second type applications on servers (j_s) and in the buffer (j_q). It should be noted that $0 \leqslant i_s + j_s \leqslant n$. The argument $l = 1, 2$ describes the state of the external Markov process η_1 as well as the $m = 1, 2$ — the state of the Markov process η_2. $R_1(i_s + i_q; j_s + j_q) = R_1 - (i_s + i_q)k_1 - (j_s + j_q)k_2$ — the current amount of the system resources in the state 1 of Markov process η_2.

In the case of the buffer of unlimited capacity, the entire set of states can be divided into 10 subsets corresponding to the following states:

1) the system is empty — the states $\{(0; 0), (0; 0), R_1(0; 0), 1, 1\}$, $\{(0; 0), (0; 0), R_1(0; 0), 2, 1\}$, $\{(0; 0), (0; 0), R_2, 1, 2\}$, $\{(0; 0), (0; 0), R_2, 2, 2\}$;
2) there are only applications of the first type in the system, not all servers are occupied, the buffer is empty — $\{(i_s; 0), (0; 0), R_1(i_s; 0), 1, 1\}$, $\{(i_s; 0), (0; 0), R_1(i_s; 0), 2, 1\}$, $\{(i_s; 0), (0; 0), R_2, 1, 2\}$, $\{(i_s; 0), (0; 0), R_2, 2, 2\}$, $1 \leqslant i_s < n$;
3) there are only applications of the first type in the system, all servers are occupied, the buffer is empty — $\{(n; 0), (0; 0), R_1(n; 0), 1, 1\}$, $\{(n; 0), (0; 0), R_1(n; 0), 2, 1\}$, $\{(n; 0), (0; 0), R_2, 1, 2\}$, $\{(n; 0), (0; 0), R_2, 2, 2\}$;
4) there are only applications of the first type in the system, all servers are occupied, the buffer is not empty — $\{(n; i_q), (0; 0), R_1(n + i_q; 0), 1, 1\}$, $\{(n; i_q), (0; 0), R_1(n + i_q; 0), 2, 1\}$, $\{(n; i_q), (0; 0), R_2, 1, 2\}$, $\{(n; i_q), (0; 0), R_2, 2, 2\}$, $i_q \geqslant 1$;
5) there are only applications of the second type in the system, not all servers are occupied, the buffer is empty — $\{(0; 0), (j_s; 0), R_1(0; j_s), 1, 1\}$, $\{(0; 0), (j_s; 0), R_1(0; j_s), 2, 1\}$, $\{(0; 0), (j_s; 0), R_2, 1, 2\}$, $\{(0; 0), (j_s; 0), R_2, 2, 2\}$, $1 \leqslant j_s < n$;
6) there are only applications of the second type in the system, all servers are occupied, the buffer is empty — $\{(0; 0), (n; 0), R_1(0; n), 1, 1\}$, $\{(0; 0), (n; 0), R_1(0; n), 2, 1\}$, $\{(0; 0), (n; 0), R_2, 1, 2\}$, $\{(0; 0), (n; 0), R_2, 2, 2\}$;
7) there are only applications of the second type in the system, all servers are occupied, the buffer is not empty — $\{(0; 0), (n; j_q), R_1(0; n + j_q), 1, 1\}$, $\{(0; 0), (n; j_q), R_1(; n + j_q), 2, 1\}$, $\{(0; 0), (n; j_q), R_2, 1, 2\}$, $\{(0; 0), (n; j_n), R_2, 2, 2\}$, $j_q \geqslant 1$;
8) there are applications of both types in the system, not all servers are occupied, the buffer is empty — $\{(i_s; 0), (j_s; 0), R_1(i_s; j_s), 1, 1\}$, $\{(i_s; 0), (j_s; 0), R_1(i_s; j_s), 2, 1\}$, $\{(i_s; 0), (j_s; 0), R_2, 1, 2\}$, $\{(i_s; 0), (j_s; 0), R_2, 2, 2\}$, $1 \leqslant i_s \leqslant n - 2$, $1 \leqslant j_s \leqslant n - 1 - i_s$;
9) there are applications of both types in the system, all servers are occupied, the buffer is empty — $\{(i_s; 0), (n - i_s; 0), R_1(i_s; n - i_s), 1, 1\}$, $\{(i_s; 0), (n - i_s; 0), R_1(i_s; n - i_s), 2, 1\}$, $\{(i_s; 0), n - i_s; 0), R_2, 1, 2\}$, $\{(i_s; 0), n - i_s; 0), R_2, 2, 2\}$, $1 \leqslant i_s \leqslant n - 1$;
10) there are applications of both types in the system, all servers are occupied, the buffer is not empty — $\{(i_s; i_q), (n - i_s; j_q), R_1(i_s + i_q; n - i_s + j_q), 1, 1\}$, $\{(i_s; i_q), (n - i_s; j_q), R_1(i_s + i_q; n - i_s + j_q), 2, 1\}$, $\{(i_s; i_q), (n - i_s; j_q), R_2, 1, 2\}$, $\{(i_s; i_q), (n - i_s; j_q), R_2, 2, 2\}$, $1 \leqslant i_s \leqslant n - 1$, $i_q + j_q \geqslant 1$.

For the system with the buffer of finite size, three more groups of states will be introduced (the system is fully occupied by applications of only one type, the system is fully occupied by both type applications).

Since the states in which the amount of resources requested by applications exceeds the amount of resources of the entire system are impossible (due to our assumptions), then conditional indicator function — the Kronecker symbol — is introduced:

$$\delta\left(R_1(i_s+i_q, j_s+j_q)\right)=\begin{cases}1, & R_1-(i_s+i_q)k_1-(j_s+j_q)k_2 \geqslant 0,\\ 0, & R_1-(i_s+i_q)k_1-(j_s+j_q)k_2<0.\end{cases} \tag{1}$$

This indicator function will be used for the equations of transitions between the states of the groups (4), (6), (10) and for the transition from the states (3), (6) and (9) to the overlying states and for transitions from the overlying states to states of these groups.

The first four equations consider the transition of the system from the zero state:

$$\begin{aligned}(\lambda_1+\mu_{1,2}+\lambda_{1,2})\,P\left((0;0),(0;0),R_1(0;0),1,1\right)=\mu_1 P\left((1;0),(0;0),R_1(1;0),1,1\right)+\\ +\mu_1 P\left((0;0),(1;0),R_1(0;1),1,1\right)+\lambda_{2,1} P\left((0;0),(0;0),R_1(0;0),2,1\right)+\\ +\mu_{2,1} P\left((0;0),(0;0),R_2,1,2\right),\end{aligned} \tag{2}$$

$$\begin{aligned}(\lambda_2+\mu_{1,2}+\lambda_{2,1})\,P\left((0;0),(0;0),R_1(0;0),2,1\right)=\mu_1 P\left((1;0),(0;0),R_1(1;0),2,1\right)+\\ +\mu_1 P\left((0;0),(1;0),R_1(0;1),2,1\right)+\lambda_{1,2} P\left((0;0),(0;0),R_1(0;0),1,1\right)+\\ +\mu_{2,1} P\left((0;0),(0;0),R_2,2,2\right),\end{aligned} \tag{3}$$

$$\begin{aligned}(\lambda_1+\mu_{2,1}+\lambda_{1,2})\,P\left((0;0),(0;0),R_2,1,2\right)=\mu_2 P\left((1;0),(0;0),R_2,1,2\right)+\\ +\mu_2 P\left((0;0),(1;0),R_2,1,2\right)+\lambda_{2,1} P\left((0;0),(0;0),R_2,2,2\right)+\\ +\mu_{1,2} P\left((0;0),(0;0),R_1(0;0),1,1\right),\end{aligned} \tag{4}$$

$$\begin{aligned}(\lambda_2+\mu_{2,1}+\lambda_{2,1})\,P\left((0;0),(0;0)R_2,2,2\right)=\mu_2 P\left((1;0),(0;0),R_2,2,2\right)+\\ +\mu_2 P\left((0;0),(1;0),R_2,2,2\right)+\lambda_{1,2} P\left((0;0),(0;0),R_2,1,2\right)+\\ +\mu_{1,2} P\left((0;0),(0;0),R_1(0;0),2,1\right).\end{aligned} \tag{5}$$

Now consider the case where only the first type of application is present in the system and not all servers are occupied:

$$\begin{aligned}(\lambda_1+\lambda_{1,2}+\mu_{1,2}+i_s\mu_1)\,P\left((i_s;0),(0;0),R_1(i_s;0),1,1\right)=\\ =\lambda_1 P\left((i_s-1;0),(0;0),R_1(i_s-1;0),1,1\right)+\lambda_{2,1} P\left((i_s;0,)(0;0),R_1(i_s;0),2,1\right)+\\ +\mu_{2,1} P\left((i_s;0),(0;0),R_2,1,2\right)+\mu_1(i_s+1) P\left((i_s+1;0),(0;0),R_1(i_s+1;0),1,1\right)+\\ +\mu_1 P\left((i_s;0),(1;0),R_1(i_s;1),1,1\right),\quad 1\leqslant i_s\leqslant n-1,\end{aligned} \tag{6}$$

$$\begin{aligned}(\lambda_2+\lambda_{2,1}+\mu_{1,2}+i_s\mu_1)\,P\left((i_s;0),(0;0),R_1(i_s;0),2,1\right)=\\ =\mu_1(i_s+1) P\left((i_s+1;0),(0;0),R_1(i_s+1;0),2,1\right)+\\ +\mu_1 P\left((i_s;0),(1;0),R_1(i_s;1),2,1\right)+\lambda_{1,2} P\left((i_s;0),(0;0),R_1(i_s;0),1,1\right)+\\ +\mu_{2,1} P\left((i_s;0),(0;0),R_2,2,2\right),\quad 1\leqslant i_s\leqslant n-1,\end{aligned} \tag{7}$$

$$\begin{aligned}(\lambda_1+\lambda_{1,2}+\mu_{2,1}+i_s\mu_2)\,P\left((i_s;0),(0;0),R_2,1,2\right)=\\ =\lambda_1 P\left((i_s-1;0),(0;0),R_2,1,2\right)+\lambda_{2,1} P\left((i_s;0),(0;0),R_2,2,2\right)+\\ +\mu_{1,2} P\left((i_s;0),(0;0),R_1(i_s;0),1,1\right)+\mu_2(i_s+1) P\left((i_s+1;0),(0;0),R_2,1,2\right)+\end{aligned}$$

$$+\mu_2 P\left((i_s;0),(1;0),R_2,1,2\right), \quad 1 \leqslant i_s \leqslant n-1, \quad (8)$$

$$(\lambda_2+\lambda_{2,1}+\mu_{2,1}+i_s\mu_2)\,P\left((i_s;0),(0;0),R_2,2,2\right)=\lambda_{1,2}P\left((i_s;0),(0;0),R_2,1,2\right)+ \\ +\mu_{1,2}P\left((i_s;0),(0;0),R_1(i_s;0),2,1\right)+\mu_2(i_s+1)P\left((i_s+1;0),(0;0),R_2,2,2\right)+ \\ +\mu_2 P\left((i_s;0),(1;0),R_2,2,2\right), \quad 1 \leqslant i_s \leqslant n-1. \quad (9)$$

The system contains only applications of the first type, all servers are occupied, but the buffer is empty. According to the assumptions, the maximum amount of system resources is sufficient for all servers to be occupied, but it is not sufficient for arriving applications to occupy the buffer. Therefore, it is necessary to use the indicator function — verification of the existence of overlying states:

$$(n\mu_1+\mu_{1,2}+\lambda_{1,2}+\lambda_1\delta\left(R_1(n+1,0)\right))\,P\left((n,0),(0,0),R_1(n;0),1,1\right)= \\ =\lambda_1 P\left((n-1,0),(0,0),R_1(n-1;0),1,1\right)+\lambda_{2,1}P\left((n,0),(0,0),R_1(n;0),2,1\right)+ \\ +\mu_{2,1}P\left((n,0),(0,0),R_2,1,2\right)+\mu_1\delta\left(R_1(n;1)\right)P\left((n-1,1),(1,0),R_1(n;1),1,1\right)+ \\ +\mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(n+i_1;j_1)\right)\right)P\left((n,i),(0,j),R_2,1,2\right)+ \\ +n\mu_1\delta\left(R_1(n+1,0)\right)P\left((n,1),(0,0),R_1(n+1;0),1,1\right), \quad (10)$$

$$(n\mu_1+\mu_{1,2}+\lambda_{2,1}+\lambda_2\delta\left(R_1(n,1)\right))\,P\left((n,0),(0,0),R_1(n;0),2,1\right)= \\ =\lambda_{1,2}P\left((n,0)(0,0),R_1(n;0),1,1\right)+\mu_1\delta\left(R_1(n,1)\right)P\left((n-1,1),(1,0),R_1(n;1),2,1\right)+ \\ +\mu_{2,1}P\left((n,0),(0,0),R_2,2,2\right)+n\mu_1\delta\left(R_1(n+1,0)\right)P\left((n,1),(0,0),R_1(n+1;0),2,1\right)+ \\ +\mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(n+i_1;j_1)\right)\right)P\left((n,i),(0,j),R_2,2,2\right), \quad (11)$$

$$(n\mu_2+\mu_{2,1}+\lambda_{1,2}+\lambda_1)\,P\left((n,0),(0,0),R_2,1,2\right)=\lambda_1 P\left((n-1,0),(0,0),R_2,1,2\right)+ \\ \lambda_{2,1}P\left((n,0),(0,0),R_2,2,2\right)+\mu_{1,2}P\left((n,0),(0,0),R_1(n;0),1,1\right)+ \\ +\mu_2 P\left((n-1,1),(1,0),R_2,1,2\right)+n\mu_2 P\left((n,1),(0,0),R_2,1,2\right), \quad (12)$$

$$(n\mu_2+\mu_{2,1}+\lambda_{2,1}+\lambda_2)\,P\left((n,0),(0,0),R_2,2,2\right)=\lambda_{1,2}P\left((n,0),(0,0),R_2,1,2\right)+ \\ +\mu_{1,2}P\left((n,0),(0,0),R_1(n;0),2,1\right)+\mu_2 P\left((n-1,1),(1,0),R_2,2,2\right)+ \\ +n\mu_2 P\left((n,1),(0,0),R_2,2,2\right). \quad (13)$$

In the system there are only applications of the first type, all servers are occupied, there are also applications in the buffer. The indicator function is used to check the possibility of transition to (from) overlying states:

$$(n\mu_1+\mu_{1,2}+\lambda_{1,2}+\lambda_1\delta\left(R_1(n+i_q+1,0)\right))\,P\left((n,i_q),(0,0),R_1(n+i_q;0),1,1\right)= \\ =\lambda_1 P\left((n,i_q-1),(0,0),R_1(n+i_q-1;0),1,1\right)+ \\ +\lambda_{2,1}P\left((n,i_q),(0,0),R_1(n+i_q;0),2,1\right)+\mu_{2,1}P\left((n,i_q),(0,0),R_2,1,2\right)+ \\ +\mu_1\delta\left(R_1(n+i_q;1)\right)P\left((n-1,i_q+1)(1,0),R_1(n+i_q;1),1,1\right)+$$

$$+\mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(n+i_q+i_1;j_1)\right)\right)P\left((n,i_q+i),(0,j),R_2,1,2\right)+$$
$$+n\mu_1\delta\left(R_1(n+i_q+1,0)\right)P\left((n,i_q+1),(0,0),R_1(n+i_q+1;0),1,1\right),\quad i_q\geqslant 1,\quad (14)$$

$$(n\mu_1+\mu_{1,2}+\lambda_{2,1}+\lambda_2\delta\left(R_1(n+i_q+1,0)\right))P\left((n,i_q)(0,0),R_1(n+i_q;0),2,1\right)=$$
$$=\lambda_{1,2}P\left((n,i_q),(0,0),R_1(n+i_q;0),1,1\right)+\mu_{2,1}P\left((n,i_q),(0,0),R_2,2,2\right)+$$
$$+n\mu_1\delta\left(R_1(n+i_q+1,0)\right)P\left((n,i_q+1),(0,0),R_1(n+i_q+1;0),2,1\right)+$$
$$+\mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(n+i_q+i_1;j_1)\right)\right)P\left((n,i_q+i),(0,j),R_2,2,2\right)+$$
$$+\mu_1\delta\left(R_1(n+i_q;1)\right)P\left((n-1;i_q+1),(1,0),R_1(n+i_q;1),2,1\right),\quad i_q\geqslant 1,\quad (15)$$

$$(n\mu_2+\mu_{2,1}+\lambda_{1,2}+\lambda_1)P\left((n,i_q),(0,0),R_2,1,2\right)=$$
$$=\lambda_1P\left((n,i_q-1),(0,0),R_2,1,2\right)+\lambda_{1,2}P\left((n,i_q),(0,0),R_2,1,2\right)+$$
$$+\mu_{1,2}\delta\left(R_1(n+i_q;0)\right)P\left((n,i_q),(0,0),(R_1(n+i_q;0),1,1\right)+$$
$$+\mu_2P\left((n-1,i_q+1),(1,0),R_2,1,2\right)+$$
$$+n\mu_2P\left((n,i_q+1),(0,0),R_2,1,2\right),\quad i_q\geqslant 1,\quad (16)$$

$$(n\mu_2+\mu_{2,1}+\lambda_{2,1}+\lambda_2)P\left((n,i_q),(0,0),R_2,2,2\right)=\lambda_{2,1}P\left((n,i_q),(0,0),R_2,2,2\right)+$$
$$+\mu_{1,2}\delta\left(R_1(n+i_q;0)\right)P\left((n,i_q),(0,0),(R_1(n+i_q;0),2,1\right)+$$
$$+\mu_2P\left((n-1,i_q+1),(1,0),R_2,2,2\right)+n\mu_2P\left((n,i_q+1),(0,0),R_2,2,2\right),\ i_q\geqslant 1.\quad (17)$$

There are only application of the second type in the system, not all servers are occupied, the buffer is empty:

$$(\lambda_1+\lambda_{1,2}+\mu_{1,2}+j_s\mu_1)P\left((0;0),(j_s;0),R_1(0;j_s),1,1\right)=$$
$$=\mu_1(j_s+1)P\left((0;0),(j_s+1;0),R_1(0;j_s+1),1,1\right)+$$
$$+\mu_1P\left((1;0),(j_s;0),R_1(1;j_s),1,1\right)+\lambda_{2,1}P\left((0;0)(j_s;0),R_1(0;j_s),2,1\right)+$$
$$+\mu_{2,1}P\left((0;0),(j_s;0),R_2,1,2\right),\quad 1\leqslant j_s\leqslant n-1,\quad (18)$$

$$(\lambda_2+\lambda_{2,1}+\mu_{1,2}+j_s\mu_1)P\left((0;0),(j_s;0),R_1(0;j_s),2,1\right)=$$
$$=\lambda_2P\left((0;0),(j_s-1;0),R_1(0;j_s-1),2,1\right)+\mu_1P\left((1;0),(j_s;0),R_1(1;j_s),2,1\right)+$$
$$+\mu_1(j_s+1)P\left((0;0),(j_s+1;0),R_1(0;j_s+1),2,1\right)+$$
$$+\lambda_{1,2}P\left((0;0),(j_s;0),R_1(0;j_s),1,1\right)+$$
$$+\mu_{2,1}P\left((0;0),(j_s;0),R_2,2,2\right),\quad 1\leqslant j_s\leqslant n-1,\quad (19)$$

$$(\lambda_1+\lambda_{1,2}+\mu_{2,1}+j_s\mu_2)P\left((0;0),(j_s;0),R_2,1,2\right)=$$
$$=\lambda_{2,1}P\left((0;0),(j_s;0),R_2,2,2\right)+\mu_{1,2}P\left((0;0),(j_s;0),R_1(0;j_s),1,1\right)+$$
$$+\mu_2(j_s+1)P\left((0;0),(i_s+1;0),R_2,1,2\right)+$$

$$+ \mu_2 P\left((1;0),(i_s;0),R_2,1,2\right), \quad 1 \leqslant j_s \leqslant n-1, \quad (20)$$

$$\begin{gathered}(\lambda_2+\lambda_{2,1}+\mu_{2,1}+j_s\mu_2)\,P\left((0;0),(j_s;0),R_2,2,2\right) = \\ = \lambda_2 P\left((0;0),(j_s-1;0),R_2,2,2\right) + \lambda_{1,2} P\left((0;0),(j_s;0),R_2,1,2\right) + \\ + \mu_{1,2} P\left((0;0),(j_s;0),R_1(0;j_s),2,1\right) + \mu_2(j_s+1) P\left((0;0),(j_s+1;0),R_2,2,2\right) + \\ + \mu_2 P\left((1;0),(j_s;0),R_2,2,2\right), \quad 1 \leqslant i_s \leqslant n-1. \quad (21)\end{gathered}$$

In the system there are only applications of the second type, all servers are occupied, but the buffer is empty. The indicator function is used to check the possibility of transition to (from) overlying states:

$$\begin{gathered}(n\mu_1+\mu_{1,2}+\lambda_{1,2}+\lambda_1\delta\left(R_1(1,n)\right))\,P\left((0,0),(n,0),R_1(0;n),1,1\right) = \\ = \lambda_{2,1} P\left((0,0),(n,0),R_1(0;n),2,1\right) + \\ + \mu_{2,1} P\left((0,0),(n,0),R_2,1,2\right) + \mu_1\delta\left(R_1(1;n)\right) P\left(1,0\right),\left((n-1,1),R_1(1;n),1,1\right) + \\ + \mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(i_1;n+j_1)\right)\right) P\left((0,i),(n,j),R_2,1,2\right) + \\ + n\mu_1\delta\left(R_1(0,n+1)\right) P\left((0;0),(n;1),R_1(0;n+1),1,1\right), \quad (22)\end{gathered}$$

$$\begin{gathered}(n\mu_1+\mu_{1,2}+\lambda_{2,1}+\lambda_2\delta\left(R_1(0,n+1)\right))\,P\left((0,0),(n,0),R_1(0;n),2,1\right) = \\ = \lambda_2 P\left((0,0),(n-1,0),R_1(0;n-1),1,1\right) + \lambda_{1,2} P\left((0,0)(n,0),R_1(0;n),1,1\right) + \\ + \mu_1\delta\left(R_1(1,n)\right) P\left(1,0\right),\left((n-1,1),R_1(1;n),2,1\right) + \mu_{2,1} P\left((0,0),(n,0),R_2,2,2\right) + \\ + n\mu_1\delta\left(R_1(0,n+1)\right) P\left((0,0),(n,1),R_1(0;n+1),2,1\right) + \\ + \mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(i_1;n+j_1)\right)\right) P\left((0,i),(n,j),R_2,2,2\right), \quad (23)\end{gathered}$$

$$\begin{gathered}(n\mu_2+\mu_{2,1}+\lambda_{1,2}+\lambda_1)\,P\left((0,0),(n,0),R_2,1,2\right) = \lambda_{2,1} P\left((0,0),(n,0),R_2,2,2\right) + \\ + \mu_{1,2} P\left((0,0),(n,0),R_1(0;n),1,1\right) + \mu_2 P\left((1,0),(n-1,1),R_2,1,2\right) + \\ + n\mu_2 P\left((0,0),(n,1),R_2,1,2\right), \quad (24)\end{gathered}$$

$$\begin{gathered}(n\mu_2+\mu_{2,1}+\lambda_{2,1}+\lambda_2)\,P\left((0,0),(n,0),R_2,2,2\right) = \lambda_2 P\left((0,0),(n-1,0),R_2,1,2\right) + \\ + \lambda_{1,2} P\left((0,0),(n,0),R_2,1,2\right) + \mu_{1,2} P\left((0,0),(n,0),R_1(0;n),2,1\right) + \\ + \mu_2 P\left((1,0),(n-1,1),R_2,2,2\right) + n\mu_2 P\left((0,0),(n,1),R_2,2,2\right). \quad (25)\end{gathered}$$

In the system there are only applications of the second type, all servers are occupied, the buffer is not empty. The indicator function is used to check the possibility of transition to (from) overlying states:

$$\begin{gathered}(n\mu_1+\mu_{1,2}+\lambda_{1,2}+\lambda_1\delta\left(R_1(1,n+j_q)\right))\,P\left((0,0),(n,j_q),R_1(0;n+j_q),1,1\right) = \\ = \lambda_{2,1} P\left((0,0),(n,j_q),R_1(0;n+j_q),2,1\right) + \mu_{2,1} P\left((0,0,(n,j_q),R_2,1,2\right) + \\ + \mu_1\delta\left(R_1(1;n+J_q)\right) P\left((1,0)(n-1,j_q+1),R_1(1;n+j_q),1,1\right) +\end{gathered}$$

$$+\mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(i_1;n+j_q+j_1)\right)\right)P\left((0,i),(n,j_q+j),R_2,1,2\right)+$$
$$+n\mu_1\delta\left(R_1(0,n+j_q+1)\right)P\left((0,0),(n,j_q+1),R_1(0;n+j_q+1),1,1\right),\quad j_q\geqslant 1,\quad (26)$$

$$\left(n\mu_1+\mu_{1,2}+\lambda_{2,1}+\lambda_2\delta\left(R_1(0;n+j_q+1)\right)\right)P\left((0,0)(n,j_q),R_1(0;n+j_q),2,1\right)=$$
$$=\lambda_2 P\left((0,0),(n,j_q-1),R_1(0;n+j_q-1),1,1\right)+$$
$$+\lambda_{1,2}P\left((0,0),(n,j_q),R_1(0;n+j_q),1,1\right)+\mu_{2,1}P\left((0,0),(n,j_q),R_2,2,2\right)+$$
$$+n\mu_1\delta\left(R_1(0,n+j_q+1)\right)P\left((0,0),(n,j_q+1),R_1(0;n+j_q+1),2,1\right)+$$
$$+\mu_{2,1}\sum_{i+j=1}^{\infty}\prod_{i_1+j_1=1)}^{j+j}\left(1-\delta\left(R_1(i_1;n+i_q+j_1)\right)\right)P\left((0,i),(n,j_q+j),R_2,2,2\right)+$$
$$+\mu_1\delta\left(R_1(1;n+j_q)\right)P\left((1,0),(n-1;j_q+1),R_1(1;n+j_q),2,1\right),\quad j_q\geqslant 1,\quad (27)$$

$$\left(n\mu_2+\mu_{2,1}+\lambda_{1,2}+\lambda_1\right)P\left((0,0),(n,j_q),R_2,1,2\right)=\lambda_{1,2}P\left((0,0),(n,j_q),R_2,1,2\right)+$$
$$+\mu_{1,2}\delta\left(R_1(0;n+j_q)\right)P\left((0,0),(n,j_q),(R_1(0;n+j_q),1,1\right)+$$
$$+\mu_2 P\left((1,0),(n-1,j_q+1),R_2,1,2\right)+$$
$$+n\mu_2 P\left((0,0),(n,j_q+1),R_2,1,2\right),\quad j_q\geqslant 1,\quad (28)$$

$$\left(n\mu_2+\mu_{2,1}+\lambda_{2,1}+\lambda_2\right)P\left((0,0),(n,j_q),R_2,2,2\right)=\lambda_2 P\left((0,0),(n,j_q-1),R_2,1,2\right)+$$
$$+\mu_2 P\left((1,0),(n-1,j_q+1),R_2,2,2\right)+\lambda_{2,1}P\left(0,0\right),\left((n,j_q),R_2,2,2\right)+$$
$$+\mu_{1,2}\delta\left(R_1(0;n+j_q)\right)P\left((0,0),(n,j_q),(R_1(0;n+j_q),2,1\right)+$$
$$+n\mu_2 P\left((0,0),(n,j_q+1),R_2,2,2\right),\quad j_q\geqslant 1.\quad (29)$$

The applications of both types are in the system, but only some (not all) servers are occupied:

$$\left(\lambda_1+\mu_{1,2}+\lambda_{1,2}+(i_s+j_s)\mu_1\right)P\left((i_s,0),(j_s,0),R_1(i_s;j_s),1,1\right)=$$
$$=\lambda_1 P\left((i_s-1,0),(j_s,0),R_1(i_s-1;j_s),1,1\right)+\lambda_{2,1}P\left((i_s,0),(j_s,0),R_1(i_s;j_s),2,1\right)+$$
$$+\mu_{2,1}P\left((i_s,0),(j_s,0),R_2,1,2\right)+(i_s+1)\mu_1 P\left((i_s+1,0),(j_s,0),R_1(i_s+1;j_s),1,1\right)+$$
$$+(j_s+1)\mu_1 P\left((i_s,0),(j_s+1,0),R_1(i_s;j_s+1),1,1\right),$$
$$i_s=\overline{1,n-2},\quad j_s=\overline{1,n-1-i_s},\quad (30)$$

$$\left(\lambda_2+\mu_{1,2}+\lambda_{2,1}+(i_s+j_s)\mu_1\right)P\left((i_s,0),(j_s,0),R_1(i_s;j_s),2,1\right)=$$
$$=\lambda_2 P\left((i_s,0),(j_s-1,0),R_1(i_s;j_s-1),2,1\right)+\lambda_{1,2}P\left((i_s,0),(j_s,0),R_1(i_s;j_s),1,1\right)+$$
$$+\mu_{2,1}P\left((i_s,0),(j_s,0),R_2,2,2\right)+(i_s+1)\mu_1 P\left((i_s+1,0),(j_s,0),R_1(i_s+1;j_s),2,1\right)+$$
$$+(j_s+1)\mu_1 P\left((i_s,0),(j_s+1,0),R_1(i_s;j_s+1),2,1\right),$$
$$i_s=\overline{1,n-2},\quad j_s=\overline{1,n-1-i_s},\quad (31)$$

$$\left(\lambda_1+\mu_{2,1}+\lambda_{1,2}+(i_s+j_s)\mu_2\right)P\left((i_s,0),(j_s,0),R_2,1,2\right)=$$

$$= \lambda_1 P((i_s-1,0),(j_s,0),R_2,1,2) + \lambda_{2,1} P((i_s,0),(j_s,0),R_2,2,2) + \\ + \mu_{1,2} P((i_s,0),(j_s,0),R_1(i_s;j_s),1,1) + (i_s+1)\mu_2 P((i_s+1,0),(j_s,0),R_2,1,2) + \\ + (j_s+1)\mu_2 P((i_s,0),(j_s+1,0),R_2,1,2), \quad i_s=\overline{1,n-2}, \quad j_s=\overline{1,n-1-i_s}, \quad (32)$$

$$(\lambda_2+\mu_{2,1}+\lambda_{2,1}+(i_s+j_s)\mu_2) P((i_s,0),(j_s,0),R_2,2,2) = \\ = \lambda_2 P((i_s,0),(j_s-1,0),R_2,2,2) + \lambda_{1,2} P((i_s,0),(j_s,0),R_2,1,2) + \\ + \mu_{1,2} P((i_s,0),(j_s,0)R_1(i_s;j_s),2,1) + (i_s+1)\mu_2 P((i_s+1,0),(j_s,0),R_2,2,2) + \\ + (j_s+1)\mu_2 P((i_s,0),(j_s+1,0),R_2,2,2), \quad i_s=\overline{1,n-2}, \quad j_s=\overline{1,n-1-i_s}. \quad (33)$$

The application of the first and the second types are in the system, all servers are occupied, but the buffer is empty:

$$(n\mu_1+\mu_{1,2}+\lambda_{1,2}+\lambda_1\delta(R_1(i_s+1,n-i_s))) P((i_s,0),(n-i_s,0),R_1(i_s;n-i_s),1,1) = \\ = \lambda_1 P((i_s-1,0),(n-i_s,0),R_1(i_s-1;n-i_s),1,1) + \\ + \lambda_{2,1} P((i_s,0),(n-i_s,0),R_1(i_s;n-i_s),2,1) + \mu_{2,1} P((i_s,0),(n-i_s,0),R_2,1,2) + \\ + i_s\mu_1\delta(R_1(i_s+1;n-i_s)) P((i_s,1),(n-i_s,0),R_1(i_s+1;n-i_s),1,1) + \\ + (n-i_s)\mu_1\delta(R_1(i_s;n-i_s+1)) P((i_s,0),(n-i_s,1),R_1(i_s;n-i_s+1),1,1) + \\ +(n-i_s+1)\mu_1\delta(R_1(i_s;n-i_s+1)) P((i_s-1,1),(n-i_s+1,0),R_1(i_s;n-i_s+1),1,1) + \\ + \mu_{2,1} \sum_{i+j=1}^{\infty} \prod_{i_1+j_1=1)}^{j+j} (1-\delta(R_1(i_s+i_1;n-i_s+j_1))) P((i_s,i),(n-i_s,j),R_2,1,2), \\ i_s=\overline{1,n-1}, \quad j_s=n-i_s, \quad (34)$$

$$(n\mu_1+\mu_{1,2}+\lambda_{2,1}+\lambda_2\delta(R_1(i_s,n-i_s+1))) P((i_s,0),(n-i_s,0),R_1(i_s;n-i_s),2,1) = \\ = \lambda_2 P((i_s,0),(n-i_s-1,0),R_1(i_s;n-i_s-1),2,1) + \\ + \lambda_{1,2} P((i_s,0),(n-i_s,0),R_1(i_s;n-i_s),1,1) + \mu_{2,1} P((i_s,0),(n-i_s,0),R_2,2,2) + \\ + i_s\mu_1\delta(R_1(i_s+1;n-i_s)) P((i_s,1),(n-i_s,0),R_1(i_s+1;n-i_s),2,1) + \\ +(n-i_s+1)\mu_1\delta(R_1(i_s;n-i_s+1)) P((i_s-1,1),(n-i_s+1,0),R_1(i_s;n-i_s+1),2,1) + \\ + (n-i_s)\mu_1\delta(R_1(i_s;n-i_s+1)) P((i_s,0),(n-i_s,1),R_1(i_s;n-i_s+1),2,1) + \\ + \mu_{2,1} \sum_{i+j=1}^{\infty} \prod_{i_1+j_1=1}^{j+j} (1-\delta(R_1(i_s+i_1;n-i_s+j_1))) P((i_s,i),(n-i_s,j),R_2,2,2), \\ i_s=\overline{1,n-1}, \quad j_s=n-i_s, \quad (35)$$

$$(n\mu_2+\mu_{2,1}+\lambda_{1,2}+\lambda_1) P((i_s,0),(n-i_s,0),R_2,1,2) = \\ = \lambda_1 P((i_s-1,0),(n-i_s,0),R_2,1,2) + \lambda_{2,1} P((i_s,0),(n-i_s,0),R_2,2,2) + \\ + \mu_{1,2} P((i_s,0),(n-i_s,0),R_1(i_s;j_s),1,1) + (n-i_s)\mu_2 P((i_s,0),(n-i_s,1),R_2,1,2) + \\ + i_s\mu_2 P((i_s,1),(n-i_s,0),R_2,1,2) + (n-i_s+1)\mu_2 P((i_s-1,1),(n-i_s+1,0),R_2,1,1), \\ i_s=\overline{1,n-1}, \quad j_s=n-i_s, \quad (36)$$

$$(n\mu_2 + \mu_{2,1} + \lambda_{2,1} + \lambda_2)\, P\left((i_s, 0), (n - i_s, 0), R_2, 2, 2\right) =$$
$$= \lambda_2 P\left((i_s, 0), (n - i_s - 1, 0), R_2, 2, 2\right) + \lambda_{1,2} P\left((i_s, 0), (n - i_s, 0), R_2, 1, 2\right) +$$
$$+ \mu_{1,2} P\left((i_s, 0), (n - i_s, 0), R_1(i_s; j_s), 2, 1\right) + (n - i_s)\mu_2 P\left((i_s, 0), (n - i_s, 1), R_2, 2, 2\right) +$$
$$+ (n - i_s + 1)\mu_2 P\left((i_s - 1, 1), (n - i_s + 1, 0), R_2, 2, 2\right) + i_s\mu_2 P\left((i_s, 1), (n - i_s, 0), R_2, 2, 2\right),$$
$$i_s = \overline{1, n-1}, \quad j_s = n - i_s. \quad (37)$$

The equations for the case when both types of applications are in the system (on servers and in the buffer):

$$(\lambda_1 \delta\left(R_1(i_s + i_q + 1, n - i_s + j_q)\right) + \lambda_{1,2} + \mu_{1,2} + n\mu_1) \times$$
$$\times P\left((i_s; i_q), (n - i_s; j_q), R_1(i_s + i_q; n - i_s + j_q), 1, 1\right) =$$
$$= \lambda_1 P\left((i_s; i_q - 1), (n - i_s; j_q), R_1(i_s + i_q - 1; n - i_s + j_q), 1, 1\right) +$$
$$+ \lambda_{2,1} P\left((i_s; i_q), (n - i_s; j_q), R_1(i_s + i_q; n - i_s + j_q), 2, 1\right) +$$
$$+ \mu_{2,1} P\left((i_s; i_q), (n - i_s; j_q), R_2, 1, 2\right) +$$
$$+ \mu_{2,1} \sum_{i+j=1}^{\infty} \prod_{i_1+j_1=1}^{j+j} \left(1 - \delta\left(R_1(i_s + i_q + i_1; n - i_s + j_q + j_1)\right)\right) \times$$
$$\times P\left((i_s, i_q + i), (n - i_s, j_q + j), R_2, 1, 2\right) + i_s p_1 \mu_1 \delta\left(R_1(i_s + i_q + 1; n - i_s + j_q)\right) \times$$
$$\times P\left((i_s; i_q + 1), (n - i_s; j_q), R_1(i_s + i_q + 1; n - i_s + j_q), 1, 1\right) +$$
$$+ (n - i_s) p_2 \mu_1 \delta\left(R_1(i_s + i_q; n - i_s + j_q + 1)\right) \times$$
$$\times P\left((i_s; i_q), (n - i_s; j_q + 1), R_1(i_s + i_q; n - i_s + j_q + 1), 1, 1\right) +$$
$$+ (i_s + 1) p_2 \mu_1 \delta\left(R_1(i_s + i_q + 1; n - i_s + j_q)\right) \times$$
$$\times P\left((i_s + 1; i_q), (n - i_s - 1; j_q + 1), R_1(i_s + i_q + 1; n - i_s + j_q), 1, 1\right) +$$
$$+ (n - i_s + 1) p_1 \mu_1 \delta\left(R_1(i_s + i_q; n - i_s + j_q + 1)\right) \times$$
$$\times P\left((i_s - 1; i_q + 1), (n - i_s + 1; j_q), R_1(i_s + i_q; n - i_s + j_q + 1), 1, 1\right),$$
$$i_s = \overline{1, n-1}, \quad i_q + j_q \geqslant 1, \quad (38)$$

$$(\lambda_2 \delta\left(R_1(i_s + i_q, n - i_s + j_q + 1)\right) + \lambda_{2,1} + \mu_{1,2} + n\mu_1) \times$$
$$\times P\left((i_s; i_q), (n - i_s; j_q), R_1(i_s + i_q; n - i_s + j_q), 2, 1\right) =$$
$$= \lambda_2 P\left((i_s; i_q), (n - i_s; j_q - 1), R_1(i_s + i_q; n - i_s + j_q - 1), 2, 1\right) +$$
$$+ \lambda_{1,2} P\left((i_s; i_q), (n - i_s; j_q), R_1(i_s + i_q; n - i_s + j_q), 1, 1\right) +$$
$$+ \mu_{2,1} P\left((i_s; i_q), (n - i_s; j_q), R_2, 2, 2\right) +$$
$$+ \mu_{2,1} \sum_{i+j=1}^{\infty} \prod_{i_1+j_1=1}^{j+j} \left(1 - \delta\left(R_1(i_s + i_q + i_1; n - i_s + j_q + j_1)\right)\right) \times$$
$$\times P\left((i_s, i_q + i), (n - i_s, j_q + j), R_2, 2, 2\right) + i_s p_1 \mu_1 \delta\left(R_1(i_s + i_q + 1; n - i_s + j_q)\right) \times$$
$$\times P\left((i_s; i_q + 1), (n - i_s; j_q), R_1(i_s + i_q + 1; n - i_s + j_q), 2, 1\right) +$$
$$+ (n - i_s) p_2 \mu_1 \delta\left(R_1(i_s + i_q; n - i_s + j_q + 1)\right) \times$$
$$\times P\left((i_s; i_q), (n - i_s; j_q + 1), R_1(i_s + i_q; n - i_s + j_q + 1), 2, 1\right) +$$
$$+ (i_s + 1) p_2 \mu_1 \delta\left(R_1(i_s + i_q + 1; n - i_s + j_q)\right) \times$$

$$\times P\left((i_s+1;i_q),(n-i_s-1;j_q+1),R_1(i_s+i_q+1;n-i_s+j_q),2,1\right)+\\+(n-i_s+1)p_1\mu_1\delta\left(R_1(i_s+i_q;n-i_s+j_q+1)\right)\times\\\times P\left((i_s-1;i_q+1),(n-i_s+1;j_q),R_1(i_s+i_q;n-i_s+j_q+1),2,1\right),\\i_s=\overline{1,n-1},\quad i_q+j_q\geqslant 1,\quad (39)$$

$$(\lambda_1+\lambda_{1,2}+\mu_{1,2}+n\mu_2)\,P\left((i_s;i_q),(n-i_s;j_q),R_2,1,2\right)=\\=\lambda_1 P\left((i_s;i_q-1),(n-i_s;j_q),R_2,1,2\right)+\lambda_{2,1}P\left((i_s;i_q),(n-i_s;j_q),R_2,2,2\right)+\\+\mu_{1,2}\delta\left(R_1(i_s+i_q;n-i_s+j_q)\right)P\left((i_s;i_q),(n-i_s;j_q),R_1(i_s+i_q;n-i_s+j_q),1,1\right)+\\+i_sp_1\mu_2P\left((i_s;i_q+1),(n-i_s;j_q),R_2,1,2\right)+\\+(n-i_s)p_2\mu_2P\left((i_s;i_q),(n-i_s;j_q+1),R_2,1,2\right)+\\+(i_s+1)p_2\mu_2P\left((i_s+1;i_q),(n-i_s-1;j_q+1),R_2,1,2\right)+\\+(n-i_s+1)p_1\mu_2P\left((i_s-1;i_q+1),(n-i_s+1;j_q),R_2,1,2\right),\\i_s=\overline{1,n-1},\quad i_q+j_q\geqslant 1,\quad (40)$$

$$(\lambda_2+\lambda_{2,1}+\mu_{1,2}+n\mu_2)\,P\left((i_s;i_q),(n-i_s;j_q),R_2,2,2\right)=\\=\lambda_2 P\left((i_s;i_q),(n-i_s;j_q-1),R_2,2,2\right)+\lambda_{1,2}P\left((i_s;i_q),(n-i_s;j_q),R_2,1,2\right)+\\+\mu_{1,2}\delta\left(R_1(i_s+i_q;n-i_s+j_q)\right)P\left((i_s;i_q),(n-i_s;j_q),R_1(i_s+i_q;n-i_s+j_q),2,1\right)+\\+i_sp_1\mu_2P\left((i_s;i_q+1),(n-i_s;j_q),R_2,2,2\right)+\\+(n-i_s)p_2\mu_2P\left((i_s;i_q),(n-i_s;j_q+1),R_2,2,2\right)+\\+(i_s+1)p_2\mu_2P\left((i_s+1;i_q),(n-i_s-1;j_q+1),R_2,2,2\right)+\\+(n-i_s+1)p_1\mu_2P\left((i_s-1;i_q+1),(n-i_s+1;j_q),R_2,2,2\right),\\i_s=\overline{1,n-1},\quad i_q+j_q\geqslant 1.\quad (41)$$

Here p_1 — the probability that the first type application is taken from the buffer, p_2 — the probability that the second type application is taken from the buffer.

4. Conclusions

The mathematical model of the system with the allocation of resources to incoming applications and functioning in the random environment is constructed. The system of equations for steady-state probability distribution of the random process, which describes the functioning of the system, is present.

The main task of future research is to present this system of equations in a matrix form and try to apply the well known matrix algorithms [6, 7, 31–33] in order to obtain the steady-state probability distribution in the analytical form.

Also of interest are stationary distributions of applications of each type, the average value of the system resources, the average number of discarded (lost) applications.

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References

1. P. P. Bocharov, C. D'Apice, A. V. Pechinkin, S. Salerno, Queueing Theory, VSP, Utrecht, Boston, 2004.
2. G. P. Basharin, Yu. V. Gaidamaka, K. E. Samouylov, Mathematical Theory of Teletraffic and its Application to the Analysis of Multiservice Communication of Next Generation Networks, Automatic Control and Computer Sciences 47 (2) (2013) 62–69. doi:10.3103/S0146411613020028.
3. S. Trivedi Kishor, Probability and Statistics with Reliability, Queuing, and Computer Science Applications, Second Edition, John Wiley & Sons, 2016.
4. N. Chee-Hoc, S. Boon-Hee, Queueing Modelling Fundamentals with Applications in Communication Networks. 2nd Edition, John Wiley & Sons, 2008.
5. M. F. Neuts, A Versatile Markovian Point Process, Journal of Applied Probability 16 (4) (1979) 764–779. doi:10.2307/3213143.
6. M. F. Neuts, Matrix Geometric Solutions in Stochastic Models: An Algorithmic Approach, Johns Hopkins University Press, Baltimore, 1981.
7. M. F. Neuts, Matrix-Analytic Methods in Queuing Theory, European Journal of Operational Research 15 (1) (1984) 2–12. doi:10.1016/0377-2217(84)90034-1.
8. M. F. Neuts, Structured Stochastic Matrices of M/G/1 Type and Their Applications, Marcel Dekker Inc., New York, 1989.
9. W. Fisher, K. S. Meier-Hellstern, The Markov-Modulated Poisson Process (MMPP) Cookbook, Performance Evaluation 18 (2) (1993) 149–171. doi:10.1016/0166-5316(93)90035-S.
10. A. Andersen, B. Nielsen, A Markovian Approach for Modelling Packet Traffic with Long-Range Dependence, IEEE Journal on Selected Areas in Communications 16 (5) (1998) 719–732. doi:10.1109/49.700908.
11. L. Muscariello, M. Mellia, M. Meo, M. A. Marsan, R. L. Cigno, Markov Models of Internet Traffic and a New Hierarchical MMPP Model, Computer Communications 28 (16) (2005) 1835–1852. doi:10.1016/j.comcom.2005.02.012.
12. A. M. Andronov, V. M. Vishnevsky, Markov-Modulated Continuous Time Finite Markov chain as the Model of Hybrid Wireless Communication Channels Operation, Automatic Control and Computer Sciences 50 (3) (2016) 125–132. doi:10.3103/S0146411616030020.
13. A. Ometov, D. Kozyrev, V. Rykov, S. Andreev, Yu. Gaidamaka, Ye. Koucheryavy, Reliability-Centric Analysis of Offloaded Computation in Cooperative Wearable Applications, Wireless Communications and Mobile Computing 2017 (2017) 9625687. doi:10.1155/2017/9625687.
14. V. Rykov, D. Kozyrev, Analysis of Renewable Reliability Systems by Markovization Method, in: Lecture Notes in Computer Science, Analytical and Computational Methods in Probability Theory, ACMPT 2017, Germany, Heidelberg, Springer-Verlag, 2017, pp. 210–220. doi:10.1007/978-3-319-71504-9_19.
15. V. Rykov, D. Kozyrev, E. Zaripova, Modeling and Simulation of Reliability Function of a Homogeneous Hot Double Redundant Repairable System, in: ECMS 2017 Proceedings, European Council for Modeling and Simulation, European Council for Modelling and Simulation, Budapest, Hungary, May 23–26, 2017, pp. 701–705. doi:10.7148/2017-0701.
16. A. P. Pechinkin, R. Razumchik, Approach for Analysis of Finite M2—M2—1—R with Hysteric Policy for SIP Server Hop-by-Hop Overload Control, in: Proceedings – 27th European Conference on Modelling and Simulation, ECMS 2013, European Council for Modelling and Simulation, European Council for Modelling and Simulation, Alesund, Norway, May 27–30, 2013, 2013, pp. 573–579. doi:10.7148/2013-0573.
17. R. Razumchik, Analysis of Finite $MAP|PH|1$ Queue with Hysteretic Control of Arrivals, in: International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, ICUMI-2016, IEEE Computer Society,

IEEE Computer Society, Lisbon, Portugal, October 18–20, 2016, pp. 288–293. doi:10.1109/ICUMT.2016.7765373.
18. R. Razumchik, M. Telek, Delay Analysis of a Queue with Re-sequencing Buffer and Markov Environment, Queueing Systems 82 (1–2) (2016) 7–28. doi:10.1007/s11134-015-9444-z.
19. R. Razumchik, M. Telek, Delay Analysis of Resequencing Buffer in Markov Environment with HOQ-FIFO-LIFO Policy, in: Lecture Notes in Computer Science, Vol. 10497, 14th European Workshop on Computer Performance Engineering, EPEW 2017, Springer Verlag, Berlin, Germany, September 7–8, 2017, pp. 53–68. doi:10.1007/978-3-319-66583-2_4.
20. V. Klimenok, O. Dudina, V. Vishnevsky, K. Samouylov, Retrial Tandem Queue with BMAP-input and Semi-Markovian Service Process, in: Communications in Computer and Information Science, vol. 700, 20th International Conference on Distributed Computer and Communication Networks, DCCN 2017, Springer Verlag, Moscow, Russian Federation, September 25–29, 2017, pp. 159–173. doi:10.1007/978-3-319-66836-9_14.
21. S. Dudin, A. Dudin, O. Dudina, K. Samouylov, Analysis of a Retrial Queue with Limited Processor Sharing Operating in the Random Environment, in: Lecture Notes in Computer Science, LNCS, Vol. 10372, 15th International Conference on Wired/Wireless Internet Communications, WWIC 2017, Springer Verlag, St. Petersburg, Russian Federation, June 21–23, 2017, pp. 38–49. doi:10.1007/978-3-319-61382-6_4.
22. I. Zaryadov, A. Korolkova, D. Kulyabov, T. Milovanova, V. Tsurlukov, The Survey on Markov-Modulated Arrival Processes and Their Application to the Analysis of Active Queue Management Algorithms, in: Communications in Computer and Information Science, Vol. 700, 20th International Conference on Distributed Computer and Communication Networks, DCCN 2017, Springer Verlag, Moscow, Russian Federation, September 25–29, 2017, pp. 417–430. doi:10.1007/978-3-319-66836-9_35.
23. V. Naumov, K. Samouylov, N. Yarkina, E. Sopin, S. Andreev, A. Samuylov, Lte performance analysis using queuing systems with finite resources and random requirements, in: 7th International Congress on Ultra Modern Telecommunications and Control Systems ICUMT-2015, IEEE Computer Society, IEEE Computer Society, Brno, Czech Republic, October 6–8, 2015, pp. 100–103. doi:10.1109/ICUMT.2015.7382412.
24. K. Samouylov, E. Sopin, O. Vikhrova, S. Shorgin, Convolution algorithm for normalization constant evaluation in queuing system with random requirements, in: AIP Conference Proceedings, Vol. 1863, International Conference of Numerical Analysis and Applied Mathematics 2016, ICNAAM 2016, American Institute of Physics Inc., Rodos Palace HotelRhodes, Greece, September 19–25, 2017, p. 090004. doi:10.1063/1.4992269.
25. V. A. Naumov, K. E. Samuilov, A. K. Samuilov, On the Total Amount of Resources Occupied by Serviced Customers, Automation and Remote Control 77 (8) (2016) 1419–1427. doi:10.1134/S0005117916080087.
26. K. E. Samouylov, E. S. Sopin, S. Ya. Shorgin, Queuing Systems with Resources and Signals and Their Application for Performance Evaluation of Wireless Networks, Informatika i ee Primeneniya 11 (3) (2017) 99–105. doi:10.14357/19922264170311.
27. E. Sopin, O. Vikhrova, K. Samouylov, LTE Network Model with Signals and Random Resource Requirements, in: 9th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, IEEE Computer Society, Munich, Germany, November 6–8, 2017, pp. 101–106. doi:10.1109/ICUMT.2017.8255155.
28. V. Naumov, K. Samouylov, Analysis of Multi-resource Loss System with State-dependent Arrival and Service Rates, Probability in the Engineering and Informational Sciences 31 (4) (2017) 413–419. doi:10.1017/S0269964817000079.
29. E. S. Sopin, O. G. Vikhrova, Probability Characteristics Evaluation in Queueing System with Random Requirements, in: CEUR Workshop Proceedings, vol. 1995, 7th International Conference “Information and Telecommunication Technologies and Mathematical Modeling of High-Tech Systems”, ITTMM 2017, CEUR-WS, Peoples’

Friendship University of Russia (RUDN University) Moscow, Russian Federation, 2017, pp. 85–92.
30. V. V. Tsurlukov, The Mathematical Model of the System with Resources and Random Environment, in: Proceedings of 2nd International School on Applied Probability Theory and Communications Technologies, Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation, October 23—27, 2nd International School on Applied Probability Theory and Communications Technologies, RUDN University, Moscow, 2017, pp. 318–320.
31. S. Chakravarthy, A. S. Alfa, Matrix-Analytic Methods in Stochastic Models, CRC Press, 1996.
32. L. Breuer, D. Baum, An Introduction to Queueing Theory and Matrix-Analytic Methods, Springer Netherlands, Dordrecht, 2005.
33. H. Qi-Ming, Fundamentals of Matrix-Analytic Methods, Springer-Verlag New York, New-York, 2014.

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К анализу системы массового обслуживания с ресурсами, функционирующей в случайном окружении

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Строится математическая модель системы, состоящей из накопителя и нескольких однородных приборов, функционирующей в случайном окружении и предоставляющей поступающим заявкам помимо обслуживания ещё и доступ к ресурсам. Случайное окружение представлено двумя независимыми марковскими процессами, управляющими поступлением заявок в систему и обслуживанием заявок. В систему поступает пуассоновский поток заявок, интенсивность поступления и объем ресурсов, необходимый заявке при обслуживании, определяются состоянием внешнего марковского процесса. Время обслуживания заявок на приборах подчинено экспоненциальному распределению. Интенсивность обслуживания и максимальный объем ресурсов системы определяются состоянием второго внешнего марковского процесса. При окончании обслуживания заявки занятые ею ресурсы возвращаются в систему. В рассматриваемой системе возможны отказы в приёме поступающих заявок из-за нехватки ресурсов, а также возможны потери уже принятых в систему заявок при изменении состояния внешнего марковского процесса, управляющего обслуживанием и предоставлением ресурсов. Построен случайный процесс, описывающий функционирование данной системы. Представлена в скалярной форме система уравнений для стационарного распределения вероятностей построенного случайного процесса. Сформулированы основные задачи для дальнейшего исследования.

Ключевые слова: система массового обслуживания, случайное окружение, MMPP, предоставление ресурсов

Литература

1. Queueing Theory / P. P. Bocharov, C. D'Apice, A. V. Pechinkin, S. Salerno. — Utrecht, Boston: VSP, 2004. — 446 p.

2. *Basharin G. P., Gaidamaka Yu. V., Samouylov K. E.* Mathematical Theory of Teletraffic and its Application to the Analysis of Multiservice Communication of Next Generation Networks // Automatic Control and Computer Sciences. — 2013. — Vol. 47, No 2. — Pp. 62–69. — DOI: 10.3103/S0146411613020028.
3. *Trivedi Kishor S.* Probability and Statistics with Reliability, Queuing, and Computer Science Applications, Second Edition. — John Wiley & Sons, 2016. — 830 p.
4. *Chee-Hoc N., Boon-Hee S.* Queueing Modelling Fundamentals with Applications in Communication Networks. 2nd Edition. — John Wiley & Sons, 2008. — 292 p.
5. *Neuts M. F.* A Versatile Markovian Point Process // Journal of Applied Probability. — 1979. — Vol. 16, No 4. — Pp. 764–779. — DOI: 10.2307/3213143.
6. *Neuts M. F.* Matrix Geometric Solutions in Stochastic Models: An Algorithmic Approach. — Baltimore: Johns Hopkins University Press, 1981. — 352 p.
7. *Neuts M. F.* Matrix-Analytic Methods in Queuing Theory // European Journal of Operational Research. — 1984. — Vol. 15, No 1. — Pp. 2–12. — DOI: 10.1016/0377-2217(84)90034-1.
8. *Neuts M. F.* Structured Stochastic Matrices of M/G/1 Type and Their Applications. — New York: Marcel Dekker Inc., 1989. — 512 p.
9. *Fisher W., Meier-Hellstern K. S.* The Markov-Modulated Poisson Process (MMPP) Cookbook // Performance Evaluation. — 1993. — Vol. 18, No 2. — Pp. 149–171. — DOI: 10.1016/0166-5316(93)90035-S.
10. *Andersen A., Nielsen B.* A Markovian Approach for Modelling Packet Traffic with Long-Range Dependence // IEEE Journal on Selected Areas in Communications. — 1998. — Vol. 16, No 5. — Pp. 719–732. — DOI: 10.1109/49.700908.
11. Markov Models of Internet Traffic and a New Hierarchical MMPP Model / L. Muscariello, M. Mellia, M. Meo et al. // Computer Communications. — 2005. — Vol. 28, No 16. — Pp. 1835–1852. — DOI: 10.1016/j.comcom.2005.02.012.
12. *Andronov A. M., Vishnevsky V. M.* Markov-Modulated Continuous Time Finite Markov chain as the Model of Hybrid Wireless Communication Channels Operation // Automatic Control and Computer Sciences. — 2016. — Vol. 50, No 3. — Pp. 125–132. — DOI: 10.3103/S0146411616030020.
13. Reliability-Centric Analysis of Offloaded Computation in Cooperative Wearable Applications / A. Ometov, D. Kozyrev, V. Rykov et al. // Wireless Communications and Mobile Computing. — 2017. — Vol. 2017. — P. 9625687. — DOI: 10.1155/2017/9625687.
14. *Rykov V., Kozyrev D.* Analysis of Renewable Reliability Systems by Markovization Method // Lecture Notes in Computer Science / Analytical and Computational Methods in Probability Theory, ACMPT 2017. — Germany, Heidelberg, Springer-Verlag, 2017. — Pp. 210–220. — DOI: 10.1007/978-3-319-71504-9_19.
15. *Rykov V., Kozyrev D., Zaripova E.* Modeling and Simulation of Reliability Function of a Homogeneous Hot Double Redundant Repairable System // ECMS 2017 Proceedings / European Council for Modeling and Simulation. — Budapest, Hungary, May 23–26: European Council for Modelling and Simulation, 2017. — Pp. 701–705. — DOI: 10.7148/2017-0701.
16. *Pechinkin A. P., Razumchik R.* Approach for Analysis of Finite M2—M2—1—R with Hysteric Policy for SIP Server Hop-by-Hop Overload Control // Proceedings – 27th European Conference on Modelling and Simulation, ECMS 2013 / European Council for Modelling and Simulation. — Alesund, Norway, May 27–30, 2013: European Council for Modelling and Simulation, 2013. — Pp. 573–579. — DOI: 10.7148/2013-0573.
17. *Razumchik R.* Analysis of Finite $MAP|PH|1$ Queue with Hysteretic Control of Arrivals // International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, ICUMI-2016 / IEEE Computer Society. — Lisbon, Portugal, October 18–20: IEEE Computer Society, 2016. — Pp. 288–293. — DOI: 10.1109/ICUMT.2016.7765373.

18. *Razumchik R., Telek M.* Delay Analysis of a Queue with Re-sequencing Buffer and Markov Environment // Queueing Systems. — 2016. — Vol. 82, No 1–2. — Pp. 7–28. — DOI: 10.1007/s11134-015-9444-z.
19. *Razumchik R., Telek M.* Delay Analysis of Resequencing Buffer in Markov Environment with HOQ-FIFO-LIFO Policy // Lecture Notes in Computer Science / 14th European Workshop on Computer Performance Engineering, EPEW 2017. — Vol. 10497. — Berlin, Germany, September 7–8: Springer Verlag, 2017. — Pp. 53–68. — DOI: 10.1007/978-3-319-66583-2_4.
20. Retrial Tandem Queue with BMAP-input and Semi-Markovian Service Process / V. Klimenok, O. Dudina, V. Vishnevsky, K. Samouylov // Communications in Computer and Information Science, vol. 700 / 20th International Conference on Distributed Computer and Communication Networks, DCCN 2017. — Moscow, Russian Federation, September 25–29: Springer Verlag, 2017. — Pp. 159–173. — DOI: 10.1007/978-3-319-66836-9_14.
21. Analysis of a Retrial Queue with Limited Processor Sharing Operating in the Random Environment / S. Dudin, A. Dudin, O. Dudina, K. Samouylov // Lecture Notes in Computer Science, LNCS / 15th International Conference on Wired/Wireless Internet Communications, WWIC 2017. — Vol. 10372. — St. Petersburg, Russian Federation, June 21–23: Springer Verlag, 2017. — Pp. 38–49. — DOI: 10.1007/978-3-319-61382-6_4.
22. The Survey on Markov-Modulated Arrival Processes and Their Application to the Analysis of Active Queue Management Algorithms / I. Zaryadov, A. Korolkova, D. Kulyabov et al. // Communications in Computer and Information Science / 20th International Conference on Distributed Computer and Communication Networks, DCCN 2017. — Vol. 700. — Moscow, Russian Federation, September 25–29: Springer Verlag, 2017. — Pp. 417–430. — DOI: 10.1007/978-3-319-66836-9_35.
23. LTE Performance Analysis Using Queuing Systems with Finite Resources and Random Requirements / V. Naumov, K. Samouylov, N. Yarkina et al. // 7th International Congress on Ultra Modern Telecommunications and Control Systems ICUMT-2015 / IEEE Computer Society. — Brno, Czech Republic, October 6–8: IEEE Computer Society, 2015. — Pp. 100–103. — DOI: 10.1109/ICUMT.2015.7382412.
24. Convolution Algorithm for Normalization Constant Evaluation in Queuing System with Random Requirements / K. Samouylov, E. Sopin, O. Vikhrova, S. Shorgin // AIP Conference Proceedings / International Conference of Numerical Analysis and Applied Mathematics 2016, ICNAAM 2016. — Vol. 1863. — Rodos Palace HotelRhodes, Greece, September 19–25: American Institute of Physics Inc., 2017. — P. 090004. — DOI: 10.1063/1.4992269.
25. *Naumov V. A., Samuilov K. E., Samuilov A. K.* On the Total Amount of Resources Occupied by Serviced Customers // Automation and Remote Control. — 2016. — Vol. 77, No 8. — Pp. 1419–1427. — DOI: 10.1134/S0005117916080087.
26. *Samouylov K. E., Sopin E. S., Shorgin S. Ya.* Queuing Systems with Resources and Signals and Their Application for Performance Evaluation of Wireless Networks // Informatika i ee Primeneniya. — 2017. — Vol. 11, No 3. — Pp. 99–105. — DOI: 10.14357/19922264170311.
27. *Sopin E., Vikhrova O., Samouylov K.* LTE Network Model with Signals and Random Resource Requirements // 9th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops. — Munich, Germany, November 6–8: IEEE Computer Society, 2017. — Pp. 101–106. — DOI: 10.1109/ICUMT.2017.8255155.
28. *Naumov V., Samouylov K.* Analysis of Multi-resource Loss System with State-dependent Arrival and Service Rates // Probability in the Engineering and Informational Sciences. — 2017. — Vol. 31, No 4. — Pp. 413–419. — DOI: 10.1017/S0269964817000079.

29. *Sopin E. S., Vikhrova O. G.* Probability Characteristics Evaluation in Queueing System with Random Requirements // CEUR Workshop Proceedings, vol. 1995 / 7th International Conference "Information and Telecommunication Technologies and Mathematical Modeling of High-Tech Systems", ITTMM 2017. — Peoples' Friendship University of Russia (RUDN University) Moscow, Russian Federation: CEUR-WS, 2017. — Pp. 85–92.
30. *Tsurlukov V. V.* The Mathematical Model of the System with Resources and Random Environment // Proceedings of 2nd International School on Applied Probability Theory and Communications Technologies, Peoples' Friendship University of Russia (RUDN University), Moscow, Russian Federation, October 23—27 / 2nd International School on Applied Probability Theory and Communications Technologies. — Moscow: RUDN University, 2017. — С. 318–320.
31. *Chakravarthy S., Alfa A. S.* Matrix-Analytic Methods in Stochastic Models. — CRC Press, 1996. — P. 396.
32. *Breuer L., Baum D.* An Introduction to Queueing Theory and Matrix-Analytic Methods. — Dordrecht: Springer Netherlands, 2005. — P. 286.
33. *Qi-Ming H.* Fundamentals of Matrix-Analytic Methods. — New-York: Springer-Verlag New York, 2014. — P. 364.

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