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Contents

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Diagnostic graphs for distributed radio direction finding system

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Abstract. We consider a distributed over large area radio direction finding system (RDFS) and its hardware. The possibility of its diagnosis based on Preparata-Metze-Chien, Barsi-Grandoni-Maestrini and Russel-Kime models is studied. An approach is suggested to the construction of diagnostic graphs for the general case of topology and structure of the distributed RDFS. An example of construction of diagnostic graphs for the existing system is presented.

Keywords: diagnostics, diagnostic model, radio system, communication network, communication channel, radio direction finding, system topology, graph.

1. Introduction

Despite the existence of global navigation satellite positioning systems, radar and radio direction finding continue to be widely used at the present time, because in remote and underdeveloped areas radars and undetectable automatic direction finders (ADF) are among the main flight facilities and main means of monitoring of mobile [1].

Distributed on the landscape radio direction finding system (RDFS), which has the topology type multilevel star network consists of equipment of the local dispatching center (LDC), communication channels and unattended radio terminals (URT), which can be removed from the LDC over distances up to several hundred kilometers. The general structure of the distributed over the landscape RDFS and its hardware is shown in Figure 1.

In URT hardware [2] signals from Doppler antenna and phased array antenna (PAA) arrive at the analog automatic direction finder (ADF) and the radio locator (RL), respectively. The digital signal processing (DSP) block forms flows of bearing data and radar data, which are then transmitted via channeling equipment (CE) both to the main communication channel (CC) and to the reserve one (CCr). For the reason that URT is unattended, it is necessary to provide it with an autonomous energy supply system, which also requires self-diagnosis. Similarly CE of LDC [3] receives data streams from several URTs via CCs and feeds data into the data processor, afterwards the data is displayed on the monitor and is recorded in the storage subsystem.

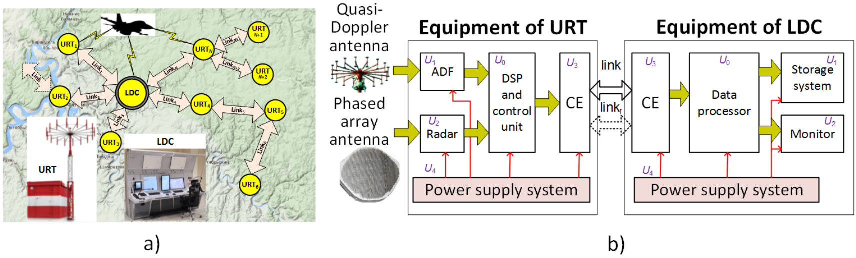


Figure 1. Distributed over area RDFS (a) and its hardware structure (b)

Since the distributed RDFS is a complex telecommunications, computing radio electronic system, to ensure its reliable operation it is necessary to establish the periodic and systematic diagnosis of technical condition of parts and components of its equipment. It is reasonable to ensure such a diagnosis through an automated system based on the model for diagnosis of distributed systems [4].

2. Diagnosis models of distributed systems

The basic model for the representation of a self-diagnosable technical system is the graph model proposed by Preparata, Metze and Chien (PMCh) [5]. According to this model a system is represented in the form of a directed graph $G(U, E)$ without loops with binary weights on arcs. The set of vertices U corresponds to the system modules, and the set of arcs E corresponds to the verifying connections. A weight r_{ij} of an arc $e_{ij} \in E$ is determined by the result of testing the inspected unit u_j by the inspecting one u_i and is equal to 0(1) if u_i considers u_j operable (malfunctioning). The set of all test results of units is called the syndrome of the system. The test results for a unit u_j is valid only in the case of operability of a unit u_i, if the testing unit is malfunctioning, than the test output value is invalid ($r_{ij} = x, x \in \{0, 1\}$). In order to ensure the unambiguity of the test results there should be not more than a certain number of $\tau(G)$ simultaneously malfunctioning units in the system. The number $\tau(G)$ is a system diagnostic factor [6].

The system is called parallel diagnosable (or single-step diagnosable) with respect to $t, t \leq \tau(G)$ faulty units if all the malfunctioning units are unambiguously determined without replacement, provided that their number does not exceed t. If under the same condition the replacement of failed units is allowed in the process of diagnosis, then the system is called sequentially t-fault diagnosable. Sequential diagnosis is also called k-step diagnosis or diagnosis with repairs. To maintain parallel diagnosability of a system it is required that the system should contain n units, $n \geq 2t+1$, and each unit should be inspected by at least t other units.

The mentioned above necessary conditions are also sufficient for systems where there are no pairs of mutually verifiable modules. An additional necessary and sufficient condition for a system to be t-diagnosable in the absence of this restriction is as follows: for any integer p, $0 \leq p < t$ and each $X \subset U$, $|x| = n - 2t + p$ it holds that $|\Gamma^{+}(X)| > p$, where $\Gamma^{+}(u_i) = \{u_i|(u_i, u_j) \in E\}, \Gamma^{+}(X) = \{\cup\Gamma^{+}(u) - X\}, X \subset U$, $|X|$ — the power of set X.

In PMCh model a class of optimal parallel-diagnosable systems is introduced with a regular structure (so-called $D_{\delta t}$ structure) of verifying connections in which an arc from u_i to u_j exists only if $j - i = \delta m($ mod $n)$, where $m = \overline{1, t}$ and δ, n — are relatively prime to each other numbers.

The possibility of building sequentially t-fault diagnosable systems with $n + 2t - 2$ verifying connections at $n \geq 2t + 1$ is proven. The described above PMCh model and the results of its study are fundamental [6].

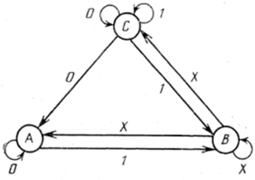
According to PMCh model we represent a system consisting of n interrelated units as a connected directed graph $G(U, V)$ without loops. The set of vertices $U = \{u_1 \dots u_n\}$ of the graph is a set of system units that can check the operability of each other. $V = \{v_{ij}|i, j = \overline{1, n}\}$ — is the set of arcs of the graph that indicate diagnostic checks on a set of units. A label d_{ij} of an arc v_{ij} of the graph, i.e., the "opinion" of a verifying unit u_i about the technical condition of an inspected unit u_j is defined as follows:

$$d_{ij} = \begin{cases} 0, & if\, u_i\, and\, u_j\, are\, operable \\ 1, & if\, u_i\, is\, operable\, and\, u_j\, is\, malfunctioning \\ X, & in\, other\, cases \end{cases}$$

where $X \in \{0, 1\}$. For every unit u_i a set of labels of outgoing arcs is called the syndrome of that unit. Hence the syndrome of the system can be represented as a matrix D, whose rows are syndromes of units.

Define $\tau(G)$ as a diagnosability measure of the system, i.e. the maximum possible number of simultaneously failed units under which the system remains diagnosable. Then the system is called a one step t-fault diagnosable if all $t \leq \tau(G)$ failed units are unambiguously determined without replacing separate units. The possibility of decoding of a system's syndrome exists only if the necessary condition – system diagnosability – holds: $n \geq 2t+1$ and each unit is verified by at least t other units. Denote: $|A|$ — power of set A. Hereafter without special mention we use the graph theory terminology. Systems with diagnosability measure $\tau(G) = [n/2]$, where $[n/2]$ — the largest integer less than $n/2$, are optimal in terms of self-diagnosability. These include systems with a fully- connected structure of verifying links without loops of length $l = 2$, $D_{\delta t}$ - systems. If a system has a non-optimal organization of verifying links, i.e. $\tau(G) < [n/2]$, and the number of malfunctioning units $t > \tau(G)$, then the sufficient condition for t-diagnosability doesn't hold for it. Violation of this condition implies the existence of syndromes decoding of which does not detect all t failed units in the system. This violation may be due to objective reasons, when, for example, the diagnosability measure does not meet the requirement imposed on the number of simultaneous failures in the system.

Analysis of a real syndrome of a system [7] with the use of a number of symptoms, clearly indicating the units belonging to a set of failed or operable ones, in general case allows to reduce the number of units with uncertain technical state. According to the "opinion" of obviously operable units and due to the wrong (0) "opinion" concerning the apparently malfunctioning units one can further reduce the degree of uncertainty of the technical state of the system. A graph G of the system can be represented as a generalized graph (Figure 2a), where A — is a set of obviously operable units, B — a set of obviously faulty units, C — a set of units in uncertain technical state.



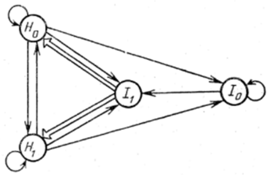


Figure 2. The principle of diagnostic graph construction for basic models

Fig. 2a shows that units of the set C could not be tested by units of the set A, and testing them by units of the set B gives uncertain results. Therefore, the technical condition of units of the set C can be determined using only its own syndrome. Number t_C of malfunctioning units in C is defined as $t_C = t - t_B$ where t_B — power of the set of obviously failed units, $t < [n/2]$. In general, $0 \leq t_C \leq |C| - 1$ and at $t_C > \left\lfloor \frac{|C|}{2} \right\rfloor$ the sufficient diagnosability condition of the entire system is violated.

In the framework of the model in fig. 2a an assumption is introduced that each malfunctioning unit has one particular "opinion" about all tested by it units. At that, it can be claimed that units, which syndromes correspond to rows of matrix D with different values in them, are apparently operable. We exclude from the matrix D syndromes of obviously operable units and units, tested by them. The question arises as how, using the reduced matrix D_c, i.e. matrix containing only syndromes with the same values, to distinguish the remaining t_c failed units? The set I of operable units consists of subsets I_1 and I_0, which produce test results, respectively 1 and 0. In the same way we divide the set H of failed units to subsets H_1 and H_0. On account of the necessary condition (diagnosability) each failed unit is checked by at least one operable unit. Consequently, units of the set I_1 test all units of the set H. It follows from the definition of an operable unit that units of the set I_1 do not test units of its set and set I_0. The set C of units in uncertain state, which corresponds to the reduced matrix D_c, for this model can be represented by a generalized graph in fig. 2b. The double arrow denotes the mandatory availability of test checks.

In the Barsi-Grandoni-Maestrini model (BGM Model) [8], which is a modification of the PMCh model, a failed verifying unit always records the failure of a verified unit and therefore sets a "1" value, in contrast to the PMCh model, where in a similar situation both values "0" and "1" could be obtained. On the basis of this difference, the PMCh model is symmetric and the BGM model is asymmetric. There exists a large class of systems that can be represented by an asymmetric diagnostic model, but under high reliability requirement, it is desirable to use the PMCh model. The peculiarity of the BGM model allows to achieve the t-diagnosability for $t \leq n-2$ under parallel diagnosis and reduce the number of test connections in sequentially diagnosable systems to $|E| = [(n-1)/2] + 1$, where $[a]$ — the smallest integer not less than a. In the cited papers an assumption is used that every test check is produced by one unit and is intended for testing of only one unit a.

The Russel-Kime model (RK model) [9], particular cases of which are PMCh and BGM models, is represented by a quadruple (Φ, T, F, G), where $\Phi = \{f_1, f_2, \ldots, f_n\}$ — is a set of considered single failures ; $T = \{t_1, t_2, \ldots, t_p\}$ — set of tests (a test can be represented by any combination of procedures used to determine the operability of a unit); $F = \{F^1, F^2, \ldots, F^{2^n}\}$ — a set of collections of failures (each collection of failures $F^b \in F$ is defined as a set of single failures that may simultaneously exist in the system); G — summary table of failures of size $2^n \times p$ (an element $G_j^k = 0, 1$, or x, if for a collection of failures F^k a test t_j always passes, fails or has unpredictable results). Diagnostic graph D of a system S is a labeled directed graph that has a vertex for each fault out of Φ and a directed arc from the top f_i to the top f_j if and only if the presence of failures f_i in the system makes invalid at least one complete test for f_j. A test t_k is considered complete for a failure f_l, if it always detects a single failures f_i.

Thus, all basic diagnostic models for distributed systems contain graph representations, reflecting verifying links between the nodes of the system.

3. Construction of diagnostic graphs for the general case of distributed RDFS

Figure 3 shows the diagnostic graphs of the extended topology RDFS and its generalized structure, consisting of a local dispatching center and $N+2$ unattended terminals, some of which are connected with the LDC directly and the rest — through the terminals. Their explanation is given in table 1. The nodes of the graph topology $G_1(U, W)$ correspond to the

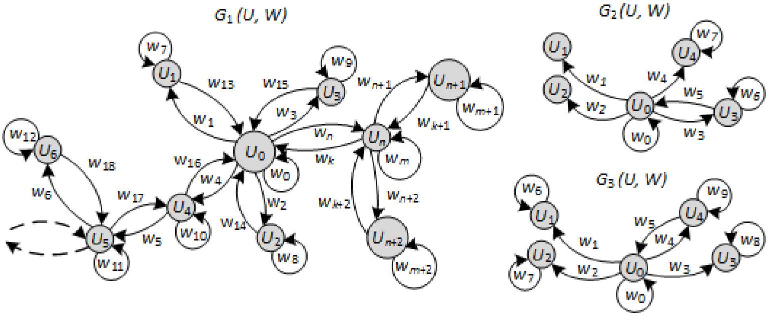


Figure 3. Diagnostic graphs for the RDFS with extended topology and generalized structure

URT and the LDC, and the arcs between the nodes correspond to the testing relations and connection links. Here u_i — is a testing unit, $u_(j)$ — unit being tested, "-" — lack of diagnosability, "w" - diagnostic link. Diagnostic graphs $G_2(U, W)$ and $G_3(U, W)$, which are subgraphs of $G_1(U, W)$, are built for the LDC and URT equipment in accordance with their structure (fig. 1b). Nodes of graphs $G_2(U, W)$ and $G_3(U, W)$ correspond to the components of the URT and the LDC and arcs correspond to the diagnostic links and interface connections.

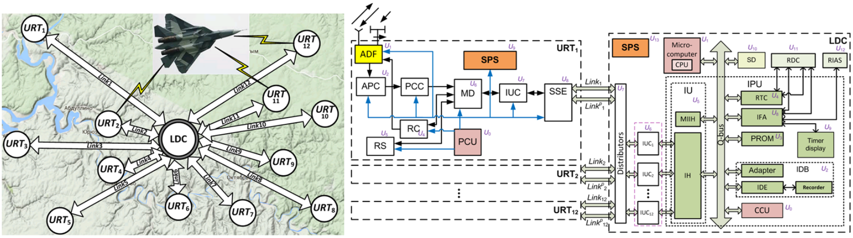
Table 1

Diagnosability of components of a distributed RDFS in general case

$G_1(U,V)$	u_i/u_j	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_n	u_{n+1}	u_{n+2}	$G_2(U,V)$	$\frac{u_i}{u_j}$	u_0	u_1	u_2	u_3	u_4
LDC	u_0	w_0	w_1	w_2	w_3	w_4			w_n			DSP&CU	u_0	w_0	w_1	w_2	w_3	w_4
URT_1	u_1	w_{13}	w_7									ADF	u_1					
URT_2	u_2	w_{14}		w_8								RL	u_2					
URT_3	u_3	w_{15}			w_9							CE	u_3	w_5			w_6	
URT_4	u_4	w_{16}				w_{10}	w_5					Power	u_4					w_7
URT_5	u_5					w_{17}	w_{13}	w_6				$G_2(U,V)$	$\frac{u_i}{u_j}$	u_0	u_1	u_2	u_3	u_4
URT_6	u_6						w_{18}	w_{12}				Processor	u_0	w_0	w_1	w_2	w_3	w_4
URT_n	u_n	w_k							w_m	w_{n+2}	w_{n+2}	Recorder	u_1		w_6			
URT_{n+1}	u_{n+1}								w_{k+1}	w_{m+2}		Monitor	u_2			w_7		
URT_{n+2}	u_{n+2}								w_{k+2}		w_{m+2}	CE	u_3	w_5			w_8	
												Power	u_4					w_9

4. Example of diagnostic graphs for the existing system

The hardware equipment of a distributed over area RDFS [10] consists of the equipment of the local dispatching center (LDC) which collects bearing information through communication channels (CC) from 12 unattended radio terminals (URT), connected to the LDC directly (figure 4).



a) b)

Figure 4. Example of a distributed on the landscape RDFS (a), and its hardware structure (b)

Structural diagram of URT and LDC interaction is shown in Fig. 5b.

Automatic Direction Finder (ADF) receives signals from the quasi-doppler antenna and generates a corresponding corner bearing quadrature voltage, from which analogue phase converter (APC) forms a bearing value as a phase shift. Phase code converter (PCC) generates a bearing angle code out of the phase shift. Remote signaling (RS) device is a receiver, which indicates breakdowns (body temperature, fire, smoke) to the URT and performs the following functions: receives and stores in its buffer the status signals form ADF and URT hardware, generates information existence signals and transmits the RS digital code and control signals. Remote control (RC) device performs remote on/off switching of the ADF, reception and processing of ADF information in APC and PCC modules. Matching device (MD) collects code messages from the PCC, RS, RC and peripheral control unit (PCU) into a single data packet, which is transmitted to the links through the interface unit with the channel (IUC) and secondary sealing equipment (SSE). PCU monitors the performance of URT components while in operation and while performing repair or maintenance works. Secondary power system (SPS) provides power to the other elements of the scheme. Non-end URTs are equipped with a retransmitter (RT), which provides data exchange between end URTs and LDC via CCs.

LDC hardware is designed to: collect bearing information from URT via communication channels (CC); with the help of bearing values determine the location of an object at the moment of radio contact of its on-board transmitter; provide control over URT via the CC; perform bearings membership test; display and record the air situation; perform the automated control of the technical state of URT and LDC hardware. LDC hardware consists of a micro-computer with management firmware, service data storage device (SD), information processing unit (IPU), the reference indicator of air situation (RIAS), remote dispatching controller (RDC), interface unit with a channel (IUC) to interact with an available communication system through distributors.

IPU is designed to: collect information on bearing and signals coming from the IUC and input it to the microcomputer; prepare the information on air situation to display it on RIAS; perform URT remote control and reception of signals; record and store the dynamic information displayed on RIAS; store service data in accordance with which the input information is conversed; perform oversight. IPU includes an interface unit (IU), a remote technic controller (RTC), an image forming apparatus (IFA), an information-documenting block (IDB), a configurable PROM and a central control unit (CCU). IU provides interface between a CC and a microcomputer, and control over the map data recording in PROM. IU consists of interfaces hardware (IH) and the map information input hardware (MIIH). IH allows to receive/transmit code messages between the microcomputer and the IUC. IDB provides (i) connection between a recording and storing device and a microcomputer; (ii) conversion of "Common Bus" interface to the "Q-Bus" interface. IDB consists of the information documenting equipment (IDE) and adapter boards. IDE operates in two modes — recording and displaying. IFA is a part of the display equipment and provides: (i) interface between RIAS and a microcomputer, (ii) RIAS backup, (iii) generation of the current time code. PROM is designed for storing programs and constants, according to which the input information is transformed. Output of programs and constants is performed upon request from the microcomputer.

CCU refers to the control and diagnostic equipment and is designed to provide timely information on the technical condition of LDC. The objects of control are CCU, RTC, PROM, IDB, IU. The microcomputer's CPU controls the rest of the LDC. To manage the CPU control and to ensure the monitoring and diagnostic coordination between channeling equipment and communication channels, CCU has a "semi-active" access to the "Q-bus" that allows to automatically transfer data from CCU into the microcomputer for displaying it RIAS and form the microcomputer to the CCU for displaying it on its front panel.

Diagnostic graphs for RDFS of this topology and structure are presented in Figure 5. As can be seen from the graphs and table 2, all of the nodes of the distributed RDFS are self-diagnosable, and all URTs are diagnosed by LDC via CCs. All URT hardware nodes are diagnosed by PCU. PCU, ADF, SSE and SPS are self-diagnosable. The control over LDC equipment is distributed: CCU and CPU are self-diagnosable and they perform diagnosis of each other like in PMCh model; CCU diagnoses RTC and self-diagnosable IDB, PROM, IU; CPU diagnoses the other elements, of which IUC, distributors, SD and SPS are self-diagnosable.

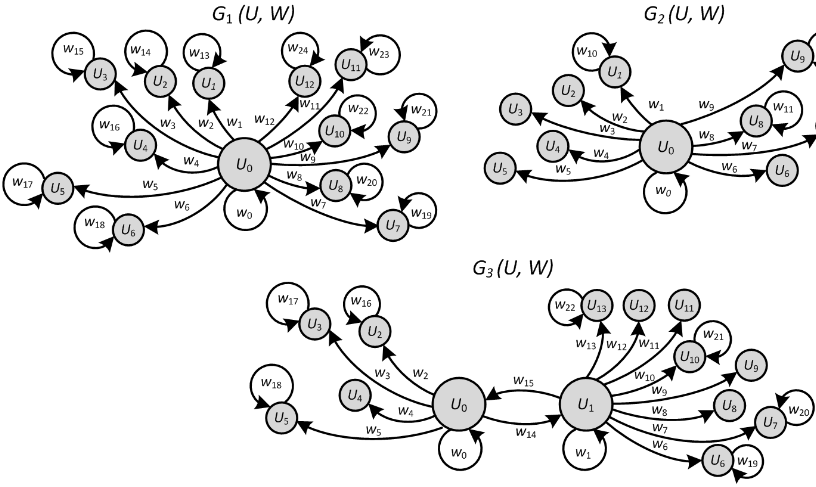


Figure 5. Diagnostic graphs for the distributed RDFS

5. Conclusions

Development of the theory based on Preparata-Metz-Chen, Barsi-Grandoni-Maestrini and Russel-Kime models, allows to build a powerful mathematical tool based on diagnostic graphs for the diagnosis of complex multiprocessor distributed computing and communication systems.

The obtained diagnostic graphs define the principle of diagnosis and are the basis for the development of an automated system of technical diagnostics of a distributed RDFS with given topology and structure. To provide real-distributed radio monitoring systems it is recommended to use truncated versions of PMCh and BGM models due to the cost and complexity restrictions, and the valuation of technical states of equipment units should be carried out on the triple criteria — normal state, deterioration state, failure state.

Some differences between graphs of the existing system and graphs of the RDFS of extended topology and generalized structure and that of PMCh model are due to the complexity and cost restrictions. The fact is, the implementation of inter-module diagnostics tests in most cases requires the use of processing elements and designing specific control algorithms for them, that do not always outweigh the reliability costs.

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References

1. *Aminev D.A., Zhurkov A.P., Silaev V.M.* Overview of U.S. patents for radio direction finding// Materials of XII International Scientific and Practical Conference “Innovations Based on Information and Communication Technologies”, NRU HSE, 2015. pp. 321–324
2. *Aminev D. A., Zhurkov A. P., Kozyrev A. A., Uvaysov S. U.* Algoritmy raboty programmnogo obespechenija mikroprocessornyh sistem kontrolja apparatury pelengatornoj pozicii [The algorithms used in the software of microprocessor systems monitoring equipment direction finding position]. Trudy NIIR, 2014, no. 3, pp. 11–17 (in Russian).
3. *Aminev D. A., Zhurkov A. P., Kozyrev A. A.* Algoritm kontrolja apparatury mestnogo dispetcherskogo punkta nazemnoj lokal’noj radiopelengacionnoj sistemy nabljudenija [The control algorithm for local control point equipment of RDF system] // Trudy NIIR. - M.: -2015, no. 4 - pp. 72–78 (in Russian).
4. *Zhurkov A. P., Aminev D. A., Guseva P. A., Miroshnichenko S. S., Petrosjan P. A.* Analysis of the Possibilities of Self-Diagnosis Approaches to Distributed Electronic Surveillance System. Systems of Control, Communication and Security, 2015, no. 4, pp. 114–122. Available at: http://sccs.intelgr.com/archive/2015-04/06-Zhurkov.pdf (In Russian).
5. *Preparata F. P., Metze G., Chien R. T.* On the connection assignment problem of diagnosable systems // IEEE Trans. Comput. 1967. V. C-16. P. 848–854.
6. *Mikeladze M. A.* Development of basic self-diagnosis models for complex engineering systems // Automation and Remote Control, 1995, 56:5, 611–623
7. *Bogdanov Yu. Yu.* Two diagnostic models of distributed digital systems. Automation and Remote Control, 1986, no. 8, pp. 127–134 (in Russian).
8. *Barsi F., Grandoni F., Maestrini P.* A Theory of Diagnosability of Digital Systems // IEEE Trans. on Comput. 1976. V. C-25. No. 6. P. 585-593.
9. *Russel J.D., Kime C.R.* System Fault Diagnosis: Masking, Exposure and Diagnosability Without Repair // IEEE Trans. on Comput. 1975. V. C-24. No.12. P.1156–1161.
10. *Zhurkov A.P., Aminev D.A.*, Radio direction finding system "NIVA" and requirements to ensure its diagnostic control// Materials of XI International Scientific and Practical Conference “Innovations Based on Information and Communication Technologies” / I. Ivanov and S. Uvajsov (Eds.): NRU HSE, 2014. pp. 453–455.

Table 2

Diagnosability of components of the distributed RDFS

$G_1(U,W)$	$\frac{u_i}{u_j}$	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	
LDC	u_0	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	
URT$_1$	u_1		w_{13}												
URT$_2$	u_2			w_{14}											
URT$_3$	u_3				w_{15}										
URT$_4$	u_4					w_{16}									
URT$_5$	u_5						w_{17}								
URT$_6$	u_6							w_{18}							
URT$_7$	u_7								w_{19}						
URT$_8$	u_8									w_{20}					
URT$_9$	u_9										w_{21}				
URT$_{10}$	u_{10}											w_{22}			
URT$_{11}$	u_{11}												w_{23}		
URT$_{12}$	u_{12}													w_{24}	
$G_2(U,W)$	$\frac{u_i}{u_j}$	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9				
PCU	u_0	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9				
ADF	u_1		w_{10}												
APC	u_2														
PCC	u_3														
RTC	u_4														
RS	u_5														
MD	u_6														
IUC	u_7														
SSE	u_8									w_{11}					
SPS	u_9										w_{12}				
$G_3(U,W)$	$\frac{u_i}{u_j}$	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}	u_{13}
CCU	u_0	w_0	w_{14}	w_2	w_3	w_4	w_5								
CPU	u_1	w_{15}	w_1					w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	w_{13}
IDB	u_2			w_{16}											
PROM	u_3				w_{17}										
RDC	u_4														
AI	u_5						w_{18}								
IUC	u_6							w_{19}							
Distrib.	u_7								w_{20}						
IFA	u_8														
Indicators	u_9														
SD	u_{10}											w_{21}			
RDC	u_{11}														
RIAS	u_{12}														
SPS	u_{13}														w_{22}

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One Problem of the Risk Control

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Abstract. A problem of supply order is considered. It is necessary to determine an order which gives the minimal probability of the upsetting of the supply. A corresponding probabilistic model is elaborated. The reduced gradient method is used for the minimization. Numerical example illustrates the efficiency of the suggested approach.

Keywords: normal distribution, reduced gradient method, planned reward, risk minimization.

1. Introduction

We consider the following problem of the risk. A firm wishes to order some product. The full size of the order equals m^*. For that the firm has the money resources C. There exists n suppliers with numbers $i = 1, ..., n$. The i-th supplier is characterized by the following indices: a_i - maximal size of the supply, c_i - the cost of the product per unit. A product quality is different for the different suppliers and is a random variable. As result, production unit of the i-th supplier gives a random reward R_i. Additionally, R_i has normal distribution with mean r_i and standard deviation σ_i. Vector $R = (R_1, R_2, ..., R_n)$ has multivariate distribution with mean $r = (r_1, r_2, ..., r_n)$ and covariance matrix $\sigma = (\sigma_{i,j})_{n\times n}$ where $\sigma_{i,i} = \sigma_i^2$. The firm is planning getting a reward r^*, at least. It is necessary to determined supply plan $m = (m_1, m_2, ..., m_n)$, which satisfies the request size m^* and the disposed sum C, and gives the minimal probability that gotten reward $S = R_1m_1 + R_2m_2 + ... + R_nm_n$ will be less then r^*.
Let us give a mathematical setting of the described problem. The reward of the firm is the random variable $S = m_1R_1 + m_2R_2 + ... + m_nR_n = m^TR$. We suppose that it has normal distribution with mean and variance, calculated by formulas

$$E(S) = r_1m_1 + r_2m_2 + ... + r_nm_n = r^Tm,$$

$$V(S) = m^T\sigma m = \sum_{i=1}^{n}\sum_{j=1}^{n} m_i\sigma_{i,j}m_j.$$

If $\Phi(z)$ is the cumulative distribution function of the standard normal distribution, then the probability doesn't receive the planned reward r^*

$$P\{S \le r^*\} = \Phi\Big(\frac{r^* - r^T m}{\sqrt{V(S)}}\Big).$$

We must minimize this probability under restrictions:

$$m_1 + m_2 + ... + m_n = m^*,$$

$$m_1 c_1 + m_2 c_2 + ... + m_n c_n \le C,$$

$$0 \le m_i \le a_i, \quad i = 1, ..., n.$$

It is possible to simplify the formulas if to consider weights $w = \left(\frac{m_1}{m^*}, \frac{m_2}{m^*}, ..., \frac{m_n}{m^*}\right)$ instead of $m_1, m_2, ..., m_n$. Setting $C^* = C/m^*$, $\rho = r^*/m^*$, $\alpha_i = c_i/m^*$, and $D(w) = V(S)(m^*)^{-2} = w^T\ \sigma\ w$, have the problem:

Minimize

$$f(w) = P\Big\{\frac{S}{m^*} \le \rho\Big\} = \Phi\Big\{\frac{\rho - r^T w}{\sqrt{D(w)}}\Big\}$$

under restrictions

$$w_1 + w_2 + ... + w_1 = 1,$$

$$w_1 c_1 + w_2 c_2 + ... + w_n c_n \le C^*,$$

$$0 \le w_i \le \alpha_i, \quad i = 1, ..., n.$$

We will solve this problem by reduced gradient method [1].

2. Reduced gradient method

Using vector-matrix notations

$$A = \begin{pmatrix} 1 & 1 & ... & 1 \\ c_1 & c_2 & ... & c_n \end{pmatrix}, \quad \begin{pmatrix} 1 \\ C^* \end{pmatrix}, \quad w = (w_1\ \ w_2\ \ ...\ \ w_n)^T,$$

we rewrite two upper equations as

$$Aw = b.$$

Let us declare w_i and w_j, $i \ne j$, as basic variables, the remaining - non-basic ones, and denote $w_B = (w_i, w_j)$, $w_N = \{w_1, ..., w_n\} - (w_i, w_j)$ so $w = (w_B\ \ w_N)$. Let B be the submatrix of A corresponding to the basic variables, and N be the analogous matrix for non-basic variables. We suppose that the basic is such, that matrix B is nonsingular matrix. Then we have

$$Bw_b + Nw_N = b,$$

$$w_b = B^{-1}(b - Nw_N) = \bar{b} - \bar{N}w_N,$$

$$where\ \ \bar{b} = B^{-1}b,\ \ \bar{N} = B^{-1}N.$$

As the basic variables dependent on non-basic variables only, we have for the aim function

$$\bar{f}(w_N) = f(w_B, w_N) = f(B^{-1}(b - N\omega_N)w_N).$$

The gradient is calculated with respect to the chain rule [2]:

$$\frac{\partial}{\partial w_N}\bar{f}(w_N) = \frac{\partial}{\partial w_N}f(w_B, w_N)|_{w_B = \bar{b}B^{-1}w_N} + \left(\frac{\partial}{\partial w_N}w_B\right)\frac{\partial}{\partial w_B}f(w_B, w_N) =$$

$$\frac{\partial}{\partial w_N}f(w_B, w_N)|_{w_B = \bar{b} - \bar{N}w_N} + (-B^{-1}N)^T\frac{\partial}{\partial w_B}f(w_B, w_N)|_{w_B = \bar{b} - \bar{N}w_N}.$$

The gotten vector is called the reduced gradient $u_N(w)$:

$$u_N(w) = \frac{\partial}{\partial w_N} == \bar{f}(w_N) =$$

$$\frac{\partial}{\partial w_N}f(w_B, w_N)|_{w_B = \bar{b} - \bar{N}w_N} - \bar{N}^T\frac{\partial}{\partial w_B}f(w_B, w_N)|_{w_B = \bar{b} - \bar{N}w_N}.$$

Now the gradient optimization works as follows. Let current value $w = (w_B, w_N)$ be gotten. We go to direction $y = (y_B, y_N)$, where y_N is determined as follows:

$$y_i = 0 \quad if \quad > 0 \quad and \quad w_i = 0, \quad or \quad u_i < 0 \quad and \quad w_i = \alpha_i,$$

$$y_i = -u, \quad otherwise,$$

and for the basic variables $y_B = -\bar{N}w_N$.
If $y_N = 0$, then the optimum is reached. Otherwise we find value $\theta > 0$, which minimizes function $g(w) = Pr(w + \theta y)$. The non-negation condition requests that

$$\theta \le \theta_{max} = \begin{matrix} min \\ y_i < 0 \end{matrix}\left\{-\frac{w_i}{y_i}\right\}.$$

This one-dimension procedure gives optimal value θ^* on the interval $(0, \theta_{max})$. Two cases are possible here.
1)$\theta^* < \theta_{max}$. The procedure is continued with new point $\omega' = \omega + \theta^* y$

and the same base B.
2) $\theta^* < \theta_{max}$. In this case one basic variable, let with number s, is cancelled. If it is non-basic variable then the base doesn't change and all is repeated. If it is a basic variable then the base is changed. Any non-basic variable with number t is introduced on the base, if new matrix B will be nonsingular.

3. Reduced gradient of the aim function

We have:

$$\frac{\partial}{\partial w_i}\Phi\Big(\frac{\rho - r^T w}{\sqrt{D(w)}}\Big) = \frac{\partial}{\partial w_i}\int\limits_{-\infty}^{\frac{\rho - r^T w}{\sqrt{D(w)}}} \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}z^2\Big)dz =$$

$$= \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}(\frac{\rho - r^T w}{\sqrt{D(w)}})^2\Big)\frac{\partial}{\partial w_i}(\frac{\rho - r^T w}{\sqrt{D(w)}}) =$$

$$= \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}(\frac{\rho - r^T w}{\sqrt{D(w)}}^2)\Big)\cdot$$

$$\cdot\Big(-\frac{1}{\sqrt{D(w)}}r_i - (\rho - r^t w)\frac{1}{2}D(w)^{-3/2}\frac{\partial}{\partial w_i}(\sum_{k=1}^{n}\sum_{j=1}^{n} w_k \sigma_{k,j} w_j)\Big) =$$

$$= \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}(\frac{\rho - r^T w}{\sqrt{D(w)}})^2\Big)\cdot$$

$$\cdot\Big(-\frac{1}{\sqrt{D(w)}}r_i - (\rho - r^T w)\frac{1}{2}D(w)^{-3/2}2\sum_{j=1}^{n}\sigma_{i,j}w_j\Big).$$

Therefore the full gradient is of the form

$$\nabla f(w) = \frac{\partial}{\partial w}f(w) = \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}(\frac{\rho - r^T w}{\sqrt{D(w)}})^2\Big)\cdot$$

$$\cdot\Big(-\frac{1}{\sqrt{D(w)}}r - (\rho - r^T w)D(w)^{-3/2}\sigma w\Big).$$

The sub-gradient $\frac{\partial}{\partial w_N}f(w)$ is the following:

$$\frac{\partial}{\partial w_N}f(w) = \frac{1}{\sqrt{2\pi}} exp\Big(-\frac{1}{2}(\frac{\rho - r^T w}{\sqrt{D(w)}})^2\Big)\cdot$$

$$\Big(-\frac{1}{\sqrt{D(w)}}r_N - (\rho - r^T w)D(w)^{-3/2}\sigma w_N\Big),$$

where r_N and w_N are subvectors of r and w, corresponding to non-basic variables.
The sub-gradient $\frac{\partial}{\partial w_B} f(w)$ is calculated analogously by means of change r_N and w_N by r_B and w_B.
Also the reduced gradient is calculated.

4. Numerical example

Below there is presented relative initial data, corresponding to unit production: the disposed financial sum $C^* = 15$, the number of the suppliers $n = 5$, the vector of the costs per unit of the production $c = (9\ \ 12\ \ 15\ \ 18\ \ 21)$, the vector of reward means $r = (10\ \ 15\ \ 20\ \ 25\ \ 30)$. A restrictions on maximal sizes of the supply $\{a_i\}$ is absent. Rewards from various suppliers are multivariate random vector with standard deviation $\sigma = (4\ \ 7\ \ 9\ \ 12\ \ 17)$. Minimal desired reward $r^* = 25$.
Iterative procedure begins with uniform distributed orders $w = (0.2\ \ 0.2\ \ 0.2\ \ 0.2\ \ 0.2)$. Matrix A and vector b from (6)are the following:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 9 & 12 & 15 & 18 & 21 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 15 \end{pmatrix}.$$

Initially the basic variables are w_1 and w_2, so

$$B = \begin{pmatrix} 1 & 1 \\ 9 & 12 \end{pmatrix}, \quad N = \begin{pmatrix} 1 & 1 & 1 \\ 15 & 18 & 21 \end{pmatrix},$$

$$B^{-1} = \begin{pmatrix} 4 & -1/3 \\ -3 & 1/3 \end{pmatrix}, \quad \bar{N} = B^{-1}N = \begin{pmatrix} -1 & -2 & -3 \\ 2 & 3 & 4 \end{pmatrix}.$$

Table 1 below contains stepwise results of the gradient optimization: the current vector of the variables w, corresponding aim's function value P, and further the numbers i and j of basic variables, the step length along gradient θ for the next step.

We see that minimal probability equals 0.717 and is reached by vector $(0.5\ \ 0\ \ 0\ \ 0\ \ 0.5)$, but it is a local minimum only. The less value of the probability is reached for the degenerative case when the third supplier is used only, namely vector $(0\ \ 0\ \ 1\ \ 0\ \ 0)$ gives probability 0.711. As the sequence of basic solutions $(\epsilon\ \ 0\ \ 1-2\epsilon\ \ 0\ \ \epsilon)$ and $(0\ \ \epsilon\ \ 1-2\epsilon\ \ 0\ \ \epsilon)$ trends to $(0\ \ 0\ \ 1\ \ 0\ \ 0)$ when ϵ tends to 0, we can get probability P arbitrarily close to 0.711.

Table 1

Protocol of gradient optimization

Step	1	2	3	4	5	7	8
w_1	0.200	0.247	0.436	0.464	0.492	0.500	0.500
w_2	0.200	0.136	0.048	0.009	0.009	0.000	0.000
w_3	0.200	0.201	0.055	0.056	0.000	0.000	0.000
w_4	0.200	0.203	0.001	0.002	0.002	0.000	0.000
w_5	0.200	0.213	0.459	0.468	0.496	0.500	0.500
P	0.851	0.846	0.730	0.719	0.717	0.717	
i	1	1	1	1	2	2	2
j	2	5	2	3	3	3	3
θ	0.02	0.02	0.01	0.01	0.01	0.01	0.01

Now we suppose that restrictions on maximal sizes of the supply from the third and fifth suppliers exist: $\alpha_3 = 0.8, \alpha_5 = 0.4$. In this case vector $(0.10 \quad 0.8 \quad 0 \quad 0.1 \quad 0)$ gives optimal solution with probability $P(0.10 \quad 0 \quad 0.8 \quad 0 \quad 0.10) = 0.750$. Other solution grows worse: $P(0.4 \ 0 \ 0.2 \ 0 \ 0.4) = 0.756$, $P(0 \ 0.5 \ 0 \ 0.5 \ 0.10) = 0.764$.

The situation is changes for various values of the desired reward r^*. For example, if restrictions on maximal sizes of the supply α_i absents and the firm wish cover its own expenses at least, i.e. $r^* = C^* = 15$, then the optimal solution is $(0.169 \quad 0.226 \quad 0.224 \quad 0.197 \quad 0.184)$ or $(0.182 \quad 0.208 \quad 0.216 \quad 0.216 \quad 0.178)$. This solution insures the probability of the expenses covering 0.145. It is close to uniform distributed order $w = (0.2 \ \ 0.2 \ \ 0.2 \ \ 0.2 \ \ 0.2)$, which gives probability 0.149. The previous optimal solutions $(0.5 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.5)$ and $(0 \quad 0 \quad 1 \quad 0 \quad 0)$ give worse results 0.283 and 0.289, correspondingly.

More involved situation arises, if we take into account dependence of the rewards for various suppliers.

5. Conclusions

Considered examples show that it is impossible to foresee a solution, that insures the minimal risk. The probability theory gives us such possibilities.

References

1. *Minoux M.* Programmation mathematique. Theorie et algorithmes. — Bordas, 1989.

2. *Turkington D. A.* Matrix Calculus & Zero-One Matrices. Statistical and Econometric Applications. — Cambridge University Press, 2002.

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Efficiency of Redundant Multipath Transmission of Requests Through the Network to Destination Servers

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Abstract. It is not uncommon that delay-sensitive requests cannot be processed repeatedly in case of delivery failures, especially in real-time systems. Hence, there is a strong need to enhance reliability of requests' delivery to destination nodes (servers). It is therefore proposed to use redundant transmission of requests, when copies of a request are distributed concurrently through the network over multiple routes to a number of similar servers. However, the resulting increase in the initial flow of requests leads to the rise of the network load and the average residence time and possibly to the excess of the ultimate residence time. In the research, the usefulness of redundant distribution of requests through the network was estimated, with the maximized probability of successful delivery and the minimized average residence time. Thus, the range of efficiency of redundant multipath transmissions was defined. It was also found that there existed the optimal redundancy order for a given set of parameters, namely the intensity of the flow of incoming requests and the bit error rate.

Keywords: reliability, request, redundancy, multipath routing, queueing systems.

1. Introduction

Constantly growing complexity of distributed computing systems, which is ahead of the pace of increasing reliability of storage, processing, and transmission devices, gives rise to the need for developing methods to ensure high reliability, availability, and security of distributed resources for data processing and storage [1–8]. High availability of distributed resources is particularly important for real-time systems, which are critical to delay-sensitive requests, and provided with redundant network elements and redundant data transmission and processing. In delay-sensitive systems, sending and serving requests twice or more times is often impossible or pointless. As a result, methods to introduce redundancy in such systems turn out to be topical and deserving attention in current paper.

Redundant data transmission is proposed to rely on distributing copies of a request (a packet) through the network over multiple paths (routes) to a number of similar servers, for example within the same cluster. Traditionally, one route is supposed to be the main, the "best" path from some point of view, i. e. based on the given criterion, or metrics: bandwidth, average residence time, router and link load, and so on. Other routes are considered as alternate ones, used in case the main path fails.

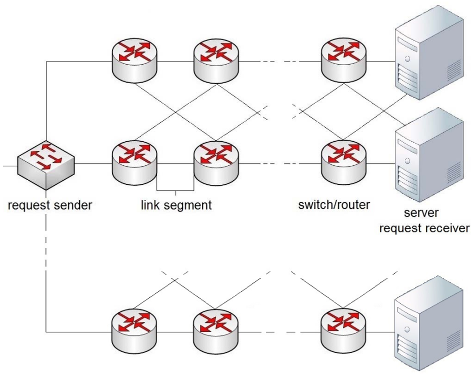

Figure 1. A possible network configuration for redundant data transmission.

Another approach is presented in [9], where requests are being sent over numerous paths concurrently. There is no need to choose the "best" path because all of them are regarded to be acceptable based on some metrics, with no ranking. On the one hand, redundant distribution of requests through the network causes the increased network load and higher average residence time as well as possible excess of the ultimate residence time. On the other hand, sending copies of a request over multiple paths results in the enhanced chance that at least one copy will be delivered successfully. The goal of current research is to estimate the efficiency of redundant distribution of requests through the network when repeated transmission is impossible, and therefore to discover if there is a way to resolve this technical contradiction.

2. System Design and Calculations

Current study focuses on transmitting numerous copies of a request from the node under consideration – the request sender – to multiple request receivers (servers) over a number of routes, as shown in Fig. 1. It is assumed that there are n acceptable paths to deliver a request to one of the servers; in this research, all these paths are considered disjoint. Each path i is set of d_i switches or routers K_{ij} and d_i+1 link segments j. Each node K_{ij} is a single-channel non-preemptive M/M/1 queueing system with the infinite queue [10, 11].

Any errors and faults of switches, routers, link segments, and destination servers prevent requests from being delivered and processed. Let us consider the bit error rate B_{ij} as a measure of transmission errors for the link segment j of the path i. It is possible that a request reaches one of the destination servers with no bit errors but finds it busy or unavailable due to faults, temporary shutdown or the state of being overloaded with the already accepted requests. Thus, we take into account the probability of a server's availability P_0, introduced in [9].

The probability of successful transmission over the path i is

$$R_i = \prod_{j=1}^{d_i} (1 - B_{ij})^N,$$

where N is the average data packet length in bits. Provided that the bit error rate is a constant, this formula can be rewritten as follows: $R = (1 - B)^{Nd}$. The probability of error-free transmission of at least one copy of the request over any redundant path is $P_c = 1 - (1 - P_0R)^k$. Here, k represents the number of redundant paths out of n acceptable ones, or the redundancy rate.

Let us estimate the average residence time, or time in the system, starting from the moment when a request is being initially sent by the request sender to the moment when the request is being accepted for processing by the destination server, depending on the intensity of request flow Λ. The delay T_i is defined as the total delay for all nodes of the path i:

$$T_i = \sum_{j=1}^{d_i} T_{ij} = \sum_{j=1}^{d_i} \frac{v_{ij}}{1 - \frac{v_{ij}k\Lambda}{n}},$$

where $v_{ij} = N/L_{ij}$ is the average time of transmission through the node j of the path i, L_{ij} is the speed of transmission.

Efficiency of redundant distribution of requests through the network depends on the average residence time, which is to be minimized, and the probability of error-free transmission, which is to be maximized. The combination of these two criteria produces a new multiplicative criterion $Mult1 = P_c/T$.

For the case when the ultimate residence time t_0 is known, let us define another multiplicative criterion – $Mult2 = P_c(t_0 - T)$, which describes the average stock time between the ultimate and the average residence time.

Last but not least, let us estimate the efficiency of redundant transmission based on the following additive criterion, with the normalized residence time: $Add = P_c + \frac{t_0 - T}{t_0 - T_{min}}$, where t_0 is the ultimate residence time, while T_{min} is the minimal residence time.

The results of calculations, shown in Fig. 2, are derived from the following values: N=2048, L=100 Mbit/s (consequently, v=2.148x10^{-5}, s), n=10, d=5, P_0=0.9, $B = 10^{-7}$, $\Lambda_1 = 9\text{x}10^3$, $\Lambda_2 = 1\text{x}10^4$, $\Lambda_3 = 2\text{x}10^4$, $\Lambda_4 = 3\text{x}10^4$. Curves 1-4 in each of three graphs correspond to Λ_1 –Λ_4.

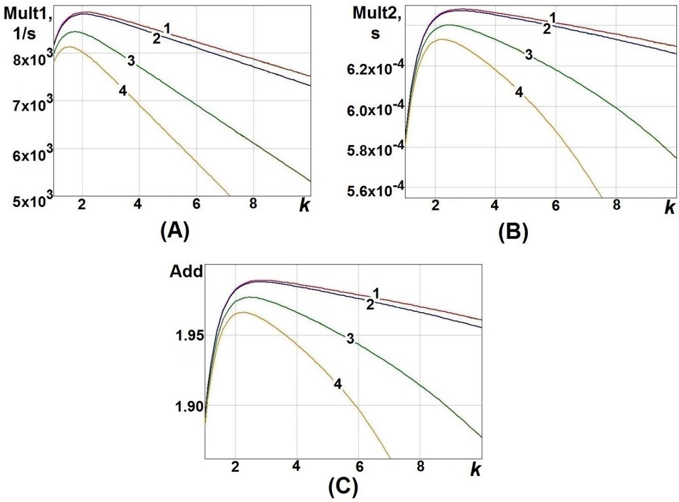

Figure 2. Efficiency of redundant transmission for the multiplicative criteria Mult1 (A) and Mult2 (B) and for the additive criterion Add (C).

3. Conclusion

Fig. 2 gives an indication about the efficiency of redundant transmission of requests through the network, depending on the intensity of request flow. It is clearly seen that introducing redundancy in the process of data transmission appears efficient, according to all three criteria, and that one can calculate the optimal redundancy rate, based on the maximal values of the criteria of k.

References

1. *Bogatyrev V.A.* Protocols for dynamic distribution of requests through a bus with variable logic ring for reception authority transfer Automatic Control and Computer Sciences, vol. 33, No. 1, 1999, pp. 57-63.
2. *Bogatyrev V.A.* On interconnection control in redundancy of local network buses with limited availability Engineering Simulation, Vol. 16, No 4, 1999, pp. 463-469.
3. *Bogatyrev V.A.* An interval signal method of dynamic interrupt handling with load balancing Automatic Control and Computer Sciences, vol. 34, No. 6, 2000, pp. 51-57.
4. *Bogatyrev V.A.* Fault tolerance of clusters configurations with direct connection of storage devices Automatic Control and Computer Sciences, V. 45, No. 6, 2011, pp. 330—337.

5. *Bogatyrev V.A.* Exchange of Duplicated Computing Complexes in Fault tolerant Systems Automatic Control and Computer Sciences, Vol. 46, No. 5, 2011, pp. 268—276.
6. *Bogatyrev V.A., Bogatyrev S.V., Golubev I.Y.* Optimization and the Process of Task Distribution between Computer System Clusters Automatic Control and Computer Sciences, No. 3, 2012, pp. 103-111.
7. *Bogatyrev V.A., Bogatyrev A.V.* Functional Reliability of a Real-Time Redundant Computational Process in Cluster Architecture Systems Automatic Control and Computer Sciences, Vol. 49, No. 1, 2015, pp. 46-56.
8. *Arustamov S.A., Bogatyrev V.A., Polyakov V.I.* Back Up Data Transmission in Real-Time Duplicated Computer Systems. Proceedings of the First International Scientific Conference Intelligent Information Technologies for Industry (IITI–16), Advances in Intelligent Systems and Computing. Springer, 2016, Vol. 2, pp. 103-109.
9. *Bogatyrev V. A., Parshutina S. A.* Redundant Distribution of Requests Through the Network by Transferring Them Over Multiple Paths // Communications in Computer and Information Science, 2016, Vol. 601, pp. 199-207.
10. *Aliev T. I.* The Synthesis of Service Discipline in Systems with Limits // Distributed Computer and Communication Networks. 18th International Conference, DCCN 2015. – Moscow, Russia, October 19—22, 2015. Revised Selected Papers, pp. 151-156.
11. *Aliev T.I., Rebezova M.I., Russ A.A.* Statistical Methods for Monitoring Travel Agencies Automatic Control and Computer Sciences, Vol. 49, No. 6, 2015, pp. 321-327.

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Efficiency of Redundant Service with Destruction of Expired and Irrelevant Request Copies in Real-Time Clusters

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Abstract. Possible ways of increasing the probability of timely and faultless execution of delay-sensitive requests in real-time clustered computing systems, when multiple copies of requests are created and served in different cluster nodes, are investigated. The proposed models of queuing and functional reliability prove the existence of scope of efficiency for service disciplines with redundant handling of copies of requests, when the probability of their prompt and error-free servicing can be increased significantly, despite the rise of the load in the nodes. It is examined how the arrangement of redundancy and the redundancy order affects timely and reliable servicing of requests, with possible faults, failures, and errors in the nodes. It is shown that destruction of the overdue copies, whose waiting in the queue exceeds a given ultimate time, and the copies which become irrelevant after one of them has been executed produces an essential enhancement of efficiency of the system.

Keywords: cluster, real-time, reliability, queuing systems, request copies.

1. Introduction

Reliability, security, and efficiency of the processes of handling and sending data in information and communication systems and networks are highly dependent on timely and faultless transmission of requests over redundant communication channels and execution of those requests by servers within given clusters [1–7]. It is especially important in real-time systems, dealing with delay-sensitive data.

In distributed computing systems with multipath routing and redundant communication channels, the probability of successful delivery and servicing of a request can be increased in case multiple copies of the request are sent over different routes concurrently and served by different servers of a given cluster [8]. However, such redundant transmission of copies of requests leads to the rise of the total intensity of the flow of requests and possibly to the growth of the residence time of those copies in the system.

The choice of design solutions, used to build redundant information and communication systems, should rely on modeling for multi-criteria evaluation of the efficiency of applying redundancy in such systems. An analytical model to assess the (ultimate) residence time of requests is to be developed based on the existing solutions [9–11] and to underlie a simulation model needed to estimate the efficiency of redundant transmission and execution of requests.

2. Object and Purposes of Research

The object of current research is a group of n servers which comprise the cluster created in order to arrange redundant servicing of (copies of) requests, arriving from the multilevel network (Fig. 1). Redundant execution of copies of requests by the servers under consideration and redundant transmission of those copies through the network to the servers serve as a basis for providing timely and faultless servicing of at least one copy of each request.

Let us consider a distributed computing system in which copies of requests can be transmitted in the redundant manner, i. e. over k out of n possible routes (paths) concurrently to k servers belonging to the cluster. The request is regarded to be served successfully if at least one copy of it has been transmitted with no distortion to one of the servers in the cluster and the server has accepted and executed that copy of request within a given period of time. To enhance the performance of the system, the possibility to destroy irrelevant copies of requests, waiting in the queue, was introduced.

The aim of the study is to develop a simulation model and tools to support the design process of highly reliable distributed computing systems. The model is to provide the basis for analyzing the efficiency and selecting the optimal design solutions, used to create networks with redundant transmission and execution of requests. The research focuses on the assessment of efficiency and usefulness of redundant distribution of requests through the network and their redundant servicing in the cluster.

3. Analytical Model Development

Consider the reliability model for the redundant computational process, like in [9], given absolutely reliable nodes and ideal control when, by the time of executing the request, without extra delays, information about the correctness (validity) of the computations is generated.

Let us analyze the redundancy of the computational process in k nodes when the condition of timeliness for computations in the system is fulfilled at least in one of k nodes, i. e. the delay of the request in the queue is less than some limiting value t_0 at least in one of k nodes.

Taking into account the increase in the computational load for the redundant computations in k nodes, the probability that the waiting time of requests will not exceed the limit t_0 in some (particular) node is evaluated as

$$r = 1 - \frac{\Lambda v k}{n} \exp\left(- t_0\left(v^{-1} - \frac{\Lambda k}{n}\right)\right),$$

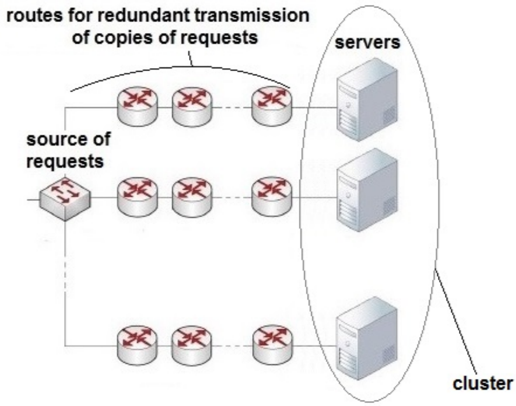

Figure 1. The structure of a distributed computing system with redundant transmission and servicing of copies of requests

where the computational load of the node is $\rho = \Lambda v k / n$.

Taking into account the computational load generated by redundant computations [9], the stationary distribution of the waiting time of requests in the M/M/1 queuing system is calculated as

$$r(t) = 1 - \frac{\Lambda v k}{n} \exp\left(- t\Big(v^{-1} - \frac{\Lambda k}{n}\Big)\right).$$

Assuming that service operations in different nodes are independent, the probability that the delay of the request executed at least in one of k nodes is below the threshold t_0 is written as

$$R = 1 - (1-r)^k = 1 - \left(\frac{\Lambda v k}{n} \exp\left(- t_0\Big(v^{-1} - \frac{\Lambda k}{n}\Big)\right)\right)^k,$$

while the stationary distribution of the waiting time of requests for the redundant execution in k single-channel infinite M/M/1 queuing systems is

$$R(t) = 1 - \left(\frac{\Lambda v k}{n} \exp\left(- t\Big(v^{-1} - \frac{\Lambda k}{n}\Big)\right)\right)^k.$$

For the infinite M/M/1 queuing system, the average waiting time of requests [9], with consideration of the increase in the computational load due to the k-tuple redundancy of requests is calculated as

$$w = \frac{\Lambda v^2 k/n}{1 - (\Lambda v k/n)}.$$

This formula provides the upper estimate of the average waiting time, since it reflects the increase in the computational load due to the redundancy; however, the formula takes no account of the possibility of decreasing the average waiting time owing to the fact that, in one of the nodes, the redundant request can wait less time than in the other nodes.

To obtain more precise estimates, we suggest that simulation models, taking into account service operations not only in the cluster, but also in the communication system, are to be developed.

4. Simulation Model Development

Let us develop a network model to illustrate forwarding requests to the single server and to the group of servers belonging to a given cluster, where those requests are to be served. Delivery of requests might fail due to their loss and bit errors emerging in the communication links, with a certain probability. It is required that requests can be sent over one or multiple routes at a time. For simplicity, we assume that

— the routes are disjoint and predefined;
— there is no loss of requests due to queue overflow;
— no confirmation of delivery of the request is generated;
— the servers have identical performance characteristics;
— each server can handle one request at a time;
— some constant speed of transmission of requests is maintained along the whole path, i. e. within every link segment.

We need to compute the residence time for each request in order to calculate the average residence time and thus to draw the line between those requests which are processed in a timely manner and those requests which expire.

It is expected that parameters of the proposed model, particularly the redundancy order of transmission of requests, can be easily adjusted and the model contains three units (Fig. 2):

— the source of requests with the mechanism which distributes them over multiple predefined routes;
— the routes which can be configured to distort or *lose* requests;
— the servers which handle incoming requests or reject them if they are overdue or irrelevant.

The source of requests consists of the generator of requests (*source*), the buffer for requests produced by the generator (*queue*), the element which delays forwarding requests over the routes (*delay*), and the balancer of requests, which distributes requests among the routes (*selectOutput*).

Each route (*route1*, *route2*, *routeN*) is a set of the elements which are linked between each other and can simulate loss of requests (*loss*), distortion of requests due to bit errors (*damage*), and delays (*delay*).

The servers contain the elements, which buffer requests (*queue*), sort out expired requests (*timeouter*), sort out irrelevant (already handled) requests (*req_ done*), produce delays (*delay*), and destroy requests (*sink*).

The following parameters are to be defined for the source: the intensity of the flow of requests, the redundancy order of transmission, or the number of copies of the request to be sent concurrently, the algorithm for selecting the route for each particular incoming copy of the request, and the algorithm for assigning numbers to the requests and their copies.

The following parameters are to be defined for the route: the number of hops (or transitions on the way from the source to the server); the speed of transmission; the probability of loss of requests; the probability of faultless delivery of requests, which is calculated as $(1 - b)^{Nh}$, where b is the bit error rate for the link segment, N is the number of bits in the packet, and h is the number of link segments. Additionally, we should introduce the formula to compute the delay depending on the size of the packet and the speed of transmission.

The following parameters are to be defined for the server: the queue capacity, the algorithm for sorting out expired requests, the algorithm for sorting out irrelevant requests, and the service time of requests.

Any copy of the request should contain the information about the number to distinguish between different copies of the request, the number of bits in the packet, the time when the source generated the copy, the time when the request (one of its copies) was served, and the route selected to transmit the copy.

5. Timeliness of Results with Redundant Transmission and Redundant Execution of Copies of Requests

On the basis of the proposed model, we explored the probability of timely and faultless distribution of requests through the network and their servicing by one of the servers belonging to the certain cluster as depending on the redundancy order. The simulation process involved applying different values of such parameters as the probability of packet loss within communication channels and the bit error rate, with various values of the ultimate residence time allowed.

We considered that the size of packets varied from 1024 to 4096 bits and the speed of transmission in the channels was 10 Mbit/s. The intensity of the flow of requests generated by the source was 1000 requests per second, while the intensity of their servicing by the server varied from 0,01 to 5 seconds. Both intensities followed an exponential distribution. The simulation model was developed in AnyLogic 7 simulation environment.

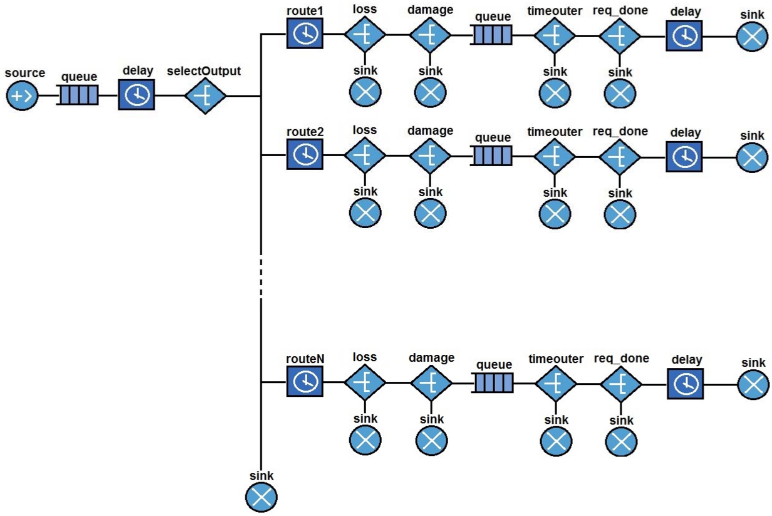

Figure 2. A simulation model illustrating redundant transmission of copies of requests over k routes from the source to at least one of k servers in the cluster

Figure 3 illustrates how the probability of prompt and error-free transmission of requests through the network and their execution in the cluster depends on the redundancy order, when their copies are transmitted through the network over routes with different bit error rate. The increase in redundancy results in the higher probability of the successful delivery and execution of at least one copy of the request but until the redundancy order is less than some value. According to Fig. 3, redundant transmission and execution of requests is efficient if the redundancy order does not exceed a certain threshold value. For example, the optimal redundancy order for the bit error rate b=0,00001 is two, whilst it is three with b=0,0001.

The conducted study proves the efficiency of redundant transmission of (copies of) requests through the network over multiple routes and redundant execution of those (copies of) requests by the servers in the cluster.

6. Conclusions

The proposed simulation model makes it possible to evaluate the efficiency of redundant distribution of requests over multiple routes in the network and redundant (and independent) servicing of those requests by the group of servers belonging to a certain cluster.

The efficiency of redundant transmission and execution of requests is calculated as depending on

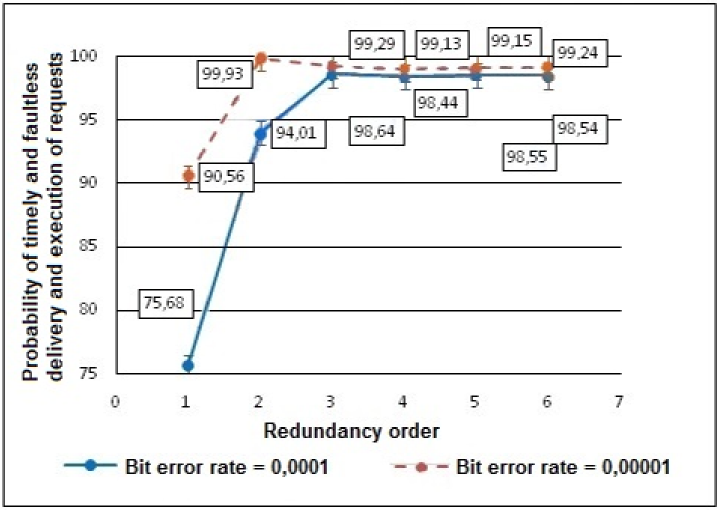

Figure 3. The probability of timely and faultless delivery and execution of requests as depending on the redundancy order in case the bit error rate is 0,0001 (*blue curve*) and 0,00001 (*red curve*)

— the probability of packet loss in the network and the bit error rate,
— the ultimate residence time of requests in the system — computed based on the analytical model considered in current paper, and
— the algorithm for destroying copies of requests in the queue, both expired (because the waiting time of the request has exceeded the ultimate residence time) and irrelevant (due to the fact that a copy of the request has already been processed by one of the servers).

To sum up, the results of current research prove the efficiency of redundant delivery of requests (packets) and their redundant execution by the servers in the cluster. Consequently, the probability of faultless delivery and execution of at least one copy of the request within a given period of time can be increased.

It was found that there exists the scope of efficiency for redundant transmission and servicing of requests. This scope depends on the bit error rate, the ultimate residence time, and the algorithm for destroying expired and irrelevant copies of requests.

The proposed models can be applied in CAD systems and underlie the choice and the optimization process of design solutions, to create fault-tolerant redundant distributed real-time computing systems, dealing with delay-sensitive requests.

References

1. *Dudin A. N., Sun' B.* A multiserver MAP/PH/N system with controlled broadcasting by unreliable servers // Automatic Control and Computer Sciences. — No. 5, 2009, pp. 32–44.
2. *Bogatyrev V. A.* Exchange of Duplicated Computing Complexes in Fault tolerant Systems // Automatic Control and Computer Sciences. — Vol. 45, No. 5, 2011, pp. 268—-276.
3. *Bogatyrev V. A., Bogatyrev S. V., Golubev I. Y.* Optimization and the Process of Task Distribution between Computer System Clusters // Automatic Control and Computer Sciences. — Vol. 46, No. 3, 2012, pp. 103–111.
4. *Bogatyrev V. A.* Fault Tolerance of Clusters Configurations with Direct Connection of Storage Devices // Automatic Control and Computer Sciences. — V. 45, No. 6, 2011, pp. 330—-337.
5. *Bogatyrev V. A.* An interval signal method of dynamic interrupt handling with load balancing // Automatic Control and Computer Sciences, vol. 34, No. 6, 2000, pp. 51-57.
6. *Bogatyrev V. A.* Protocols for dynamic distribution of requests through a bus with variable logic ring for reception authority transfer // Automatic Control and Computer Sciences. — vol. 33, No. 1, 1999, pp. 57–63.
7. *Bogatyrev V. A., Bogatyrev A. V.* Functional Reliability of a Real-Time Redundant Computational Process in Cluster // Architecture Systems Automatic Control and Computer Sciences. — Vol. 49, No. 1, 2015, pp. 46–56. DOI 10.3103/S0146411615010022.
8. *Bogatyrev V. A., Parshutina S. A.* Redundant Distribution of Requests Through the Network by Transferring Them Over Multiple Paths // Communications in Computer and Information Science, 2016, Vol. 601, pp. 199–207.
9. *Vishnevsky, V. M.* Teoreticheskie osnovy proektirovaniya komputernykh setey [Theoretical Fundamentals for Design of Computer Networks]. — Moscow, Tekhnosfera, 2003 (in Russian).
10. *Aliev T. I.* The Synthesis of Service Discipline in Systems with Limits // Distributed Computer and Communication Networks. 18th International Conference, DCCN 2015. — Moscow, Russia, October 19—22, 2015. Revised Selected Papers, pp. 151–156.
11. *Aliev T. I., Rebezova M. I., Russ A. A.* Statistical Methods for Monitoring Travel Agencies in the Settlement System // Automatic Control and Computer Sciences. — Vol. 49, No. 6, 2015, pp. 321–327.

УДК 621.395

Анализ свойств трафика машина-машина и его влияния на качество обслуживания

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Аннотация. В статье приведены результаты исследования свойств трафика машина-машина (M2M) и его влияния на систему обслуживания. Рассмотрена модель узла обслуживающего смешанный поток трафика, состоящий из потока, создаваемого устройствами M2M и трафика традиционных услуг. Приведен анализ результатов имитационного моделирования обслуживания смешанного потока и анализа параметров качества обслуживания для обоих составляющих смешанного потока.

Ключевые слова: Трафик машина-машина, M2M, система массового обслуживания, регулярный поток, простейший поток, самоподобный поток, коэффициент потерь, качество обслуживания.

1. Введение

Развитие технологий связи и вычислительной техники привели к формированию относительно нового класса источников и потребителей трафика в сетях связи – различного рода автоматических устройств. Их взаимодействие в сети связи получило название машина-машина (M2M) [1]. Хотя информационные взаимодействия такого рода уже давно практиковались в различных областях, например, телеметрия, охранная сигнализация и др. лишь сравнительно недавно началось их стремительное проникновение во все сферы деятельности человека [2]. Это выражается как в появлении специализированных технических устройств, так и программных средств используемых, например, в смартфонах, планшетных ПК и других гаджетах [3]. Уже сегодня в некоторых странах число M2M устройств, подключаемых к сети вязи на правах мобильного терминала соизмеримо с число пользователей подвижной связи [4]. Этот процесс часто соотносят с развитием систем связи определенных в концепции Интернета вещей (IoT) [5]. Порождаемый этими системами трафик в сети связи пока относительно мал (около 5% [6]), по сравнению с безусловным лидером – трафиком видео, однако, тенденции его роста позволяют предположить, что в ближайшем будущем он будет оказывать существенное влияние на функционирование сетей связи [7]. Наряду с ростом трафика M2M следующей особенностью является относительно большое количество источников трафика, которое в перспективе может значительно (в разы) превысить число пользователей подвижной связи [8]. Это приводит к формированию высокой плотности источников трафика, что с учетом того, что эти устройства имеют, как правило, беспроводный интерфейс, требует пересмотра подходов к организации сетей связи и методов обслуживания трафика. В данной работе мы рассмотрим только некоторые особенности M2M трафика, характерные для многих современных мониторинга и систем сбора данных.

2. Постановка задачи

Трафик, производимый большинством систем мониторинга и сбора данных можно условно разделить на три характерных типа: опосредованный, т.е. порождаемый как реакция на некоторые внешние события; детерминированный – производимый устройствами, работающими по жестко заданному расписанию; технологический – необходимый для поддержания функционирования системы [2]. Производимый устройствами M2M трафик может обслуживаться совместно с трафиком других услуг связи, например, базовыми станциями СПС, точками беспроводного ШПД и другими узлами сети. Ввиду того что характер трафика M2M, в общем случае, отличается от трафика других услуг имеет смысл оценить его характеристики и влияние на качество обслуживания. Рассмотрим модель приведенную на рис.1. Модель состоит из источника M2M трафика, который имитирует работу одного или нескольких устройств M2M, источника фонового трафика, условно названного H2H. Оба потока трафика поступают на узел связи, моделируемый системой массового обслуживания с ожиданием и отказами (комбинированная дисциплина обслуживания). Среднее время обслуживания пакета равно t.

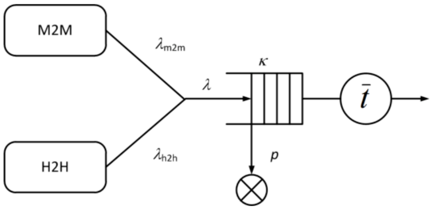

Рис. 1. Модель узла связи, обслуживающего смешанный поток

интенсивность трафика M2M λ_{M2M}, трафика H2H λ_{H2H}, интенсивность смешанного потока $\lambda{=}\lambda_{M2M}{+}\lambda_{H2H}$. С вероятностью p пакет поступает на вход системы, когда все позиции в очереди заняты и получает отказ в обслуживании (теряется). На выходе системы поток имеет интенсивность $\hat{\lambda}$. Свойства смешанного потока на входе системы определяются свойствами обоих потоков, поэтому в общем случае, они отличаются как от свойств H2H так и M2M трафика. Работа такой системы характеризуется показателями качества обслуживания: вероятностью отказов (потерь) пакетов, временем доставки, т.е. временем ожидания в очереди и временем обслуживания. Как правило, для различных услуг, порождающих сетевой трафик имеются различные требования к качеству обслуживания. Процесс обслуживания влияет на свойства обслуженного трафика, который далее поступает на другие сетевые элементы, поэтому также представляют интерес анализ свойств потока на выходе данной системы. Для оценки взаимного влияния потоков трафика имеет смысл оценить показатели качества обслуживания дифференцированно для трафика M2M и H2H.

3. Описание модели системы

Приведенная выше модель СМО, в общем случае имеет вид G/G/1/k, для которой отсутствуют точные аналитические модели оценки вероятности потерь и времени ожидания. В [9] для расчета потерь в системе данного вида при известных распределениях, описывающих входной поток и процесс обслуживания, предлагается использовать метод диффузионной аппроксимации:

$$p = \frac{1-\rho}{1-\rho^{\frac{2}{C_a^2+C_s^2}n_b+1}}\rho^{\frac{2}{C_a^2+C_s^2}n_b} \tag{1}$$

где C_a^2 и C_s^2 – квадратичные коэффициенты вариации соответственно распределений входящего потока и времени обслуживания, n_b – размер буфера, ρ - загрузка системы. Для получения приближенной оценки среднего времени доставки пакета различных потоков воспользуемся выражением [10]

$$T = \frac{\rho\bar{t}}{2(1-\rho)}\left(\frac{\sigma_a^2+\sigma_s^2}{\bar{t}^2}\right)\left(\frac{\bar{t}^2+\sigma_s^2}{\bar{a}^2+\sigma_s^2}\right) \tag{2}$$

где σ_a^2, σ_s^2 - дисперсии интервалов времени между пакетами и времени обслуживания, соответственно, $\bar{a}$ - среднее значение интервала между пакетами, $\bar{t}$ - среднее время обслуживания.

В нашем случае представляет интерес дифференцированная оценка качества обслуживания пакетов H2H и M2M трафика, поэтому имеет смысл исследовать применимость известных приближенных решений для оценки качества обслуживания смешанного потока.

Будем полагать, что поток фонового трафика (H2H) имеет свойства простейшего потока. Данное предположение не всегда приемлемо, но во многих случаях, когда фоновый трафик образован агрегированием достаточно большого количества потоков (потоки от множества различных пользователей или приложений) с различными свойствами и без явного доминирования одного из них, то свойства результирующего потока приближаются к свойствам простейшего потока [11]. Сделаем допущение о том, что трафик M2M представляет собой детерминированный поток, определяемый как периодический процесс отправки данных системы мониторинга.

4. Анализ результатов моделирования

Для исследования зависимостей показателей качества построена имитационная модель с помощью системы имитационного моделирования AnyLogic [12].

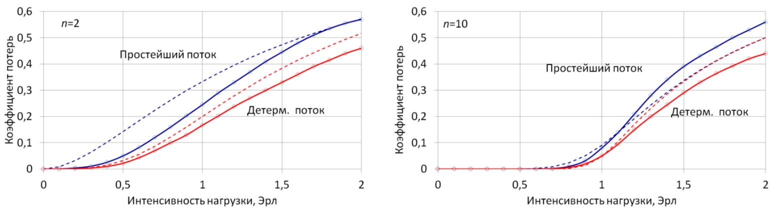

Рис. 2. Зависимость вероятности потерь от величины нагрузки при экспоненциальном распределении времени обслуживания при различной длине буфера (n=2; 10, $\rho \neq 1$)

На рис.1 также показаны зависимости, полученные согласно аппроксимации (1). Результаты моделирования показали, что оценка с помощью (1) дает завышенное значение коэффициента потерь, причем наибольшая ошибка (около 2 раз) имеет место для простейшего потока при средних значениях интенсивности нагрузки. Из приведенных графиков также видно, что коэффициент потерь для заявок детерминированного потока значительно меньше коэффициента потерь для заявок простейшего потока в смешанном потоке. На рис.2 приведены зависимости задержки доставки пакета для простейшего и детерминированного потоков в смешанном потоке от интенсивности нагрузки при различном размере буфера (2 – левый рисунок, 10 – правый рисунок), полученные имитационным моделированием. На рисунках для сравнения приведены оценки, полученные с помощью модели (2) (пунктирная линия). 8

Как видно из результатов моделирования средняя задержка доставки пакета для простейшего потока несколько превышает задержку доставки пакета детерминированного потока, разница значений не превышает 20%. Аналитическая модель (2)(2) для смешанного потока

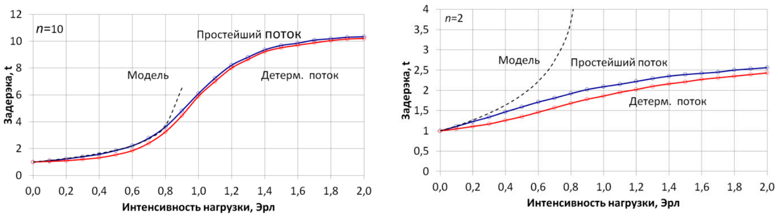

Рис. 3. Зависимость задержки доставки пакета от величины нагрузки при экспоненциальном распределении времени обслуживания при различной длине буфера (n=2; 10, $\rho \neq 1$)

достаточно точно описывает задержку доставки пакета на интервале значений интенсивности в котором потери пакетов блики к нулю (до 0,5 Эрл при длине буфера n=2 и 0,8 Эрл при n=10) нагрузки, причем ее значения наиболее близки к значениям задержки для простейшего потока.

Таким образом, заявки детерминированного потока в заявок обслуживается с более высоким качеством, причем в наибольшей степени это проявляется в росте коэффициента потерь для совместно обслуживаемого трафика.

Для исследования свойств трафика на выходе системы обслуживания исследована зависимость коэффициента Херста [13] от интенсивности нагрузки. На рис.3 приведены результаты имитационного моделирования СМО вида G/M/1/k, на вход которой подавался смешанный поток полученный агрегированием самоподобного и детерминированного потоков. Значение коэффициент Херста входного потока H_{in}=0,77.

С ростом интенсивности нагрузки на входе СМО наблюдается уменьшение коэффициента Херта потока на входе СМО. При малых и средних значениях интенсивности нагрузки на входе от 0 до 0,5 Эрл, коэффициент Херста выходного потока, практически, равен аналогичному параметру входного потока. С ростом интенсивности нагрузки свойства выходного потока, определяются в большей степени законом распределения времени обслуживания, чем свойствами входного потока, что совпадает с результатами исследования [14].

Таким образом при нехватке ресурсов, приводящей к потерям пакетов доминируют потери простейшего потока. Свойства смешанного потока сохраняются после обслуживания узлами сети при относительно малой интенсивности нагрузки и изменяются при средней и большой величине нагрузки.

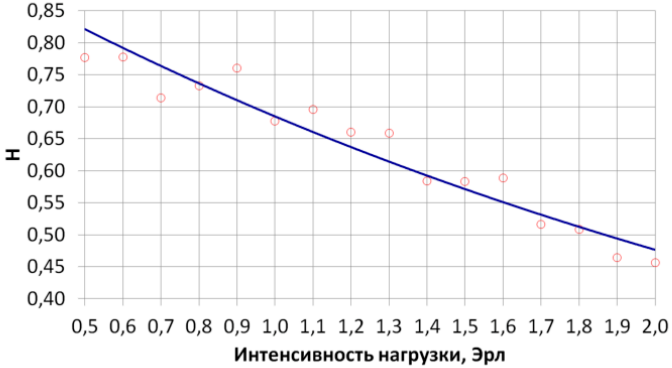

Рис. 4. Зависимость коэффициента Херста потока на выходы системы от интенсивности нагрузки на входе

5. Выводы

1. Результаты моделирования показали, что при обслуживании агрегированного потока параметры качества обслуживания различны для каждого из потоков.

2. Анализ результатов имитационного моделирования обслуживания смешанного потока показал, что вероятность потери пакета детерминированного (регулярного) потока меньше, чем случайного потока. Эта разница возрастает с увеличением интенсивности нагрузки.

3. Описана область применимости известных приближенных моделей для систем G/G/1/k и G/G/1 для описания коэффициента потерь и задержки доставки пакета.

4. Свойства выходного потока близки к свойствам входного потока при малых и средних значениях интенсивности входной нагрузки, при относительно большой интенсивности нагрузки свойства выходного потока определяются в большей степени распределением времени обслуживания.

5. При выборе сетевых решений по совместному обслуживанию регулярного трафика M2M и трафика других услуг следует учитывать большую «устойчивость» трафика M2M к потерям, в то время как случайные потоки в большей степени подвержены потерям при совместном обслуживании.

Благодарности

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Литература

1. *Paramonov A., Koucheryavy A.* M2M Traffic Models And Flow Types In Case Of Mass Event Detection. Lecture Notes in Computer Science. 2014. Т. 8638. С. 294-300.
2. *Парамонов А.И.* Модели потоков трафика для сетей M2M. М.: Электросвязь. 2014. № 4. С. 11-16.
3. *Chornaya D., Paramonov A., Koucheryavy A.* Investigation Of Machine-To-Machine Traffic Generated By Mobile Terminals. International Congress on Ultra Modern Telecommunications and Control Systems and Workshops 6. Сер. "2014 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops, ICUMT 2014"2015. pp. 210-213.
4. *oecd* OECD (2015), OECD Science, Technology and Industry Scoreboard 2015: Innovation for growth and society, OECD Publishing, Paris. $http://dx.doi.org/10.1787/sti_scoreboard-2015-en$.
5. *Боронин П. Н., Кучерявый А. Е.* Интернет вещей как новая концепция развития сетей связи // Информационные технологии и телекоммуникации. — 2014 . — № 3 (7) . — С. 7-30.
6. Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2015–2020 White Paper.
7. *Парамонов А.И.* Управление трафиком машина-машина на основе расписания. Системы управления и информационные технологии. 2014. Т. 56. № 2. С. 84-88.
8. *Киричек Р. В., Парамонов А. И., Прокопьев А. В., Кучерявый А. Е.* Эволюция исследований в области беспроводных сенсорных сетей // Информационные технологии и телекоммуникации. — 2014 . — № 4 (8) . — С. 29-41 .
9. *Н.Б. Зелигер, О.С. Чугреев, Г.Г.* Яновский Проектирование сетей и систем передачи дискретных сообщений. М.: «Радио и связь», 1984 г. – С. 177.
10. *Villy B. Iversen* Teletraffic Engineering Handbook. COM Center Technical University of Denmark Building 343, DK-2800 Lyngby Tlf.: 4525 3648 www.tele.dtu.dk/teletra.
11. *Г.И. Ивченко, В.А. Каштанов, И.Н. Коваленко.* Теория массового обслуживания. М.: «Высшая школа», 1982 г. 255 с.
12. *Ю. Карпов.* Имитационное моделирование систем. СПб. «БЧВ Петербург», 2005 г., 389 с.
13. *О.И. Шелухин, А.М. Тенякшев, А.В. Осин* Фрактальные процессы в телекоммуникациях. М.: «Радиотехника», 2003 г., 479 с.
14. *Мутханна А.С., Выборнова А.И., Парамонов А.И.* Исследование перегрузок во всепроникающих сенсорных сетях. М.: «Электросвязь», № 1, 2016 г., С. 53-59.

UDC 621.395

Machine-to-Machine Traffic Analysis And Its Impact On Quality of Service

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This paper presents some results obtained by simulation model of M2M traffic. We consider a service system in the network which serves aggregated traffic formed by two flows which represents traffic of traditional services and M2M traffic. We consider M2M traffic as a determinated (not random) flow. Results of investigation depict quality of service parameters of the service system for both flows which are different for each one.

Keywords: Machine-to-Machine Traffic, M2M, mass service system, regular flow, selfsimilar flow, losses ratio, quality of service.

UDC 004.67, 004.94

Multichannel Discrete Wavelets and Multiscale Image Processing

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Abstract. Wavelet signal decomposition into two or more subbands is discussed. Two-band decomposition could be implemented using FIR filters with even and odd taps. Scheme for multiband decomposition and restoration is presented. Examples of two, tree, four and five band decomposition and restoration filters are shown. Utilization of multiband wavelet decomposition for image compression and method for quantization of decomposed image subbands are discussed. Improvement of image processing using additional decomposition of high frequency subbands is shown with illustrations on standard test images.

Keywords: discrete wavelet transform, wavelet decomposition, filter bank, wavelet image decomposition, quantization.

1. Introduction

The base of multiscale analysis is the representation of the signal waveform as a summary of coarse approximation and local refinements at various time intervals [1–3].

Standard two-channel discrete signal transform scheme is shown at Fig. 1. Low frequency (LF) and high frequency (HF) finite impulse response (FIR) filters are used. In this case orthogonal FIR filters with both odd and even taps may be used [4].

In the first case (odd number of taps) filter frequency responses are the following:

$$\begin{cases} H(x) = h_0 + 2\sum_{n=1}^{N} h_n \cos(\pi n x), \\ G(x) = g_0 + 2\sum_{m=1}^{M} g_n \cos(\pi m x), \end{cases} \quad 0 \le x \le 1. \tag{1}$$

The samples of input signal $U(k) = 1$ are transformed after LF and HF filters into two sequences:

$$\begin{cases} h_{-N}, h_{-N+1}, ..., h_0, ..., h_{N-1}, h_N; \\ g_{-M}, g_{-M+1}, ..., g_0, ..., g_{M-1}, g_M. \end{cases} \tag{2}$$

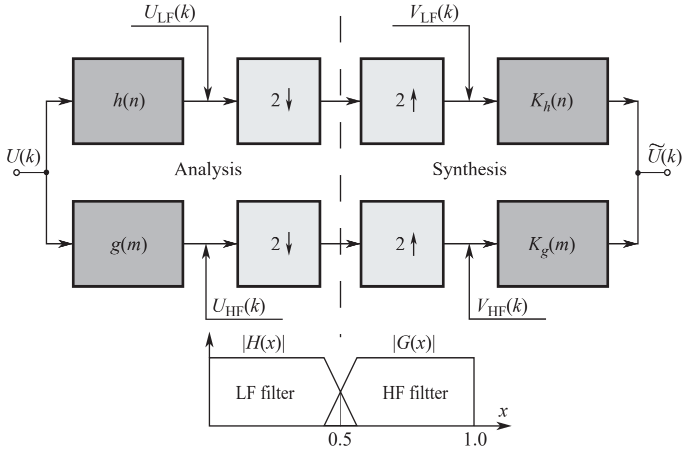


Figure 1. Block diagram of two-channel signal transform system

In the second case (even number of taps):

$$\begin{cases} H(x) = 2\sum\limits_{n=1}^{N} h_{(2n-1)/2}\cos(\pi x(2n-1)/2), \\ G(x) = -2\sum\limits_{m=1}^{M} g_{(2m-1)/2}\cos(\pi x(2m-1)/2), \end{cases} \tag{3}$$

and two sequences are the following:

$$\begin{cases} h_{-N+1/2}, h_{-N+3/2}, \dots, h_{-1/2}, h_{1/2}, \dots, h_{N-3/2}, h_{N-1/2}; \\ -g_{-M+1/2}, -g_{-M+3/2}, \dots, -g_{-1/2}, g_{1/2}, \dots, g_{M-3/2}, g_{M-1/2}. \end{cases} \tag{4}$$

These sequences are decimated two times, i.e. each second sample of the sequences (2) or (4) is discarded.

Two variants of decimation are possible: co-phase discarding of samples in the sequences after LF and HF filters or discarding of HF signal samples with shift regarding to discarding of LF signal samples [4, 5].

During the reverse process of signal synthesis zero samples are inserted instead of discarded ones. Let us denote discrete signals as $H_1(x)$, $G_1(x)$ for the first case of decimation and $H_1(x)$, $G_2(x)$ or $H_2(x)$, $G_1(x)$ for the second case. The case with simultaneous shift of the centers of formed signals $H_2(x)$, $G_2(x)$ is also possible.

Restoration (synthesis) of the signal is possible for example using the following system of equations:

$$\begin{cases} H_1(x)K_h(x) + G_2(x)K_g(x) = 1, \\ H_2(x)K_h(x) + G_1(x)K_g(x) = 1. \end{cases}$$

The determinant of this system may be presented as:

$$det(x) = A_0 + \sum_{\forall} A_k \cos(\pi k x).$$

If all the coefficients A_k, $k \neq 0$, are set to zero and $A_0 = 1$, then

$$\begin{cases} K_h(x) = G_1(x) - G_2(x), \\ K_g(x) = H_1(x) - H_2(x), \\ K_h(0) = K_g(1) = \sqrt{2}, \\ K_h(1) = K_g(0) = 0. \end{cases}$$

Taking into account that $N+M+2$ equations are needed for calculation of unknown values h_n, $0 \leq n \leq N$, and g_m, $0 \leq m \leq M$, additionally odd derivatives of $H(x)$ and $G(x)$ may be set to zero:

$$\begin{aligned} &H^{(2r)}(x)|_{x=0 \text{ or } x=1},\ r = 1, 2, 3, ...; \\ &G^{(2p)}(x)|_{x=0 \text{ or } x=1},\ p = 1, 2, 3, ... \end{aligned}$$

Even derivatives are equal to zero according to definitions of $H(x)$ and $G(x)$.

Examples of $H(x)$, $K_h(x)$, $G(x)$, $K_g(x)$ waveforms are shown on Fig. 2. Waveforms on Fig. 2a corresponds to equation (1) with $N = 5$ and $M = 6$; waveforms on Fig. 2b corresponds to equation (3) with $N = M = 5$.

Strict matching of input and output signals of two-channel transform system $U(k) \equiv \tilde{U}(k)$ corresponds to the equation:

$$\frac{1}{2}[H(x)K_h(x) + G(x)K_g(x)] \equiv 1.$$

2. Multichannel wavelet signal transform

A sequence of medium frequency (MF) filters is used in addition to LF and HF filters in the case of multichannel signal transform system.

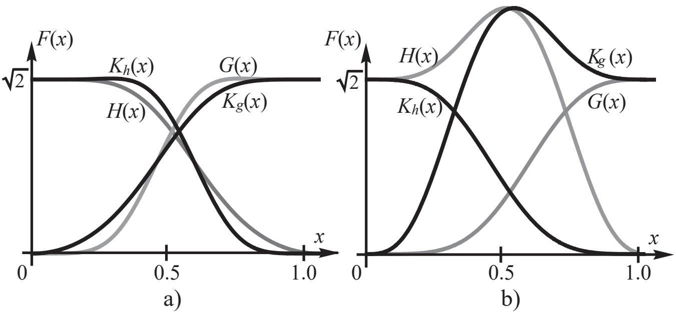


Figure 2. Frequency responses of orthogonal FIR filters with odd (a) and even (b) number of taps

Three-channel system uses MF filter with characteristic frequency response $B(x)$: $B(0) = B(1) = 0$, $B(0.5) \to max$.

Four-channel system uses two additional filters $B_1(x)$, $B_2(x)$: $B_1(0) = B_1(1) = B_2(0) = B_2(1) = 0$, $B_1(1/3) \to max$, $B_2(2/3) \to max$.

Three additional firlters $B_1(x)$, $B_2(x)$, $B_3(x)$ of five-channel system obtain the following characteristic parameters: $B_1(0) = B_1(1) = B_2(0) = B_2(1) = B_3(0) = B_3(1) = 0$, $B_1(1/4) \to max$, $B_2(1/2) \to max$, $B_3(3/4) \to max$, etc.

Block diagram of multichannel signal transform system is shown at Fig. 3.

Equations (1) for LF and HF filter characteristics are extended by equation for MF filter characteristic for construction of three-channel system:

$$B(x) = 2 \sum_{k=1}^{K} b_k \sin(\pi k x).$$

In this case three times decimation (removing of two from every three samples) is performed for the signals after LF, MF and HF filtering. Decimates samples are set to zero during restoration (synthesis) process. The input signal will be restored if restoration filter characteristics $K_h(x)$, $K_b(x)$, $K_g(x)$ meets the following equation system:

$$\begin{cases} H_1(x)K_h(x) + B_1(x)K_b(x) + G_1(x)K_g(x) = 1, \\ H_2(x)K_h(x) + B_2(x)K_b(x) + G_2(x)K_g(x) = 1, \\ H_3(x)K_h(x) + B_3(x)K_b(x) + G_3(x)K_g(x) = 1, \end{cases}$$

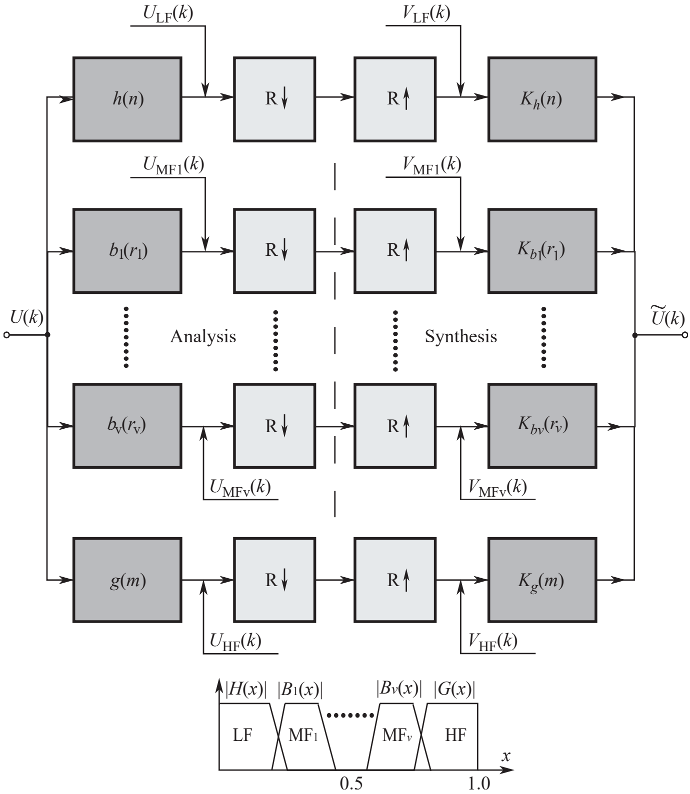


Figure 3. Block diagram of multichannel signal transform system

where H_i, B_i, G_i, $i = 1...3$, denotes corresponding filters with 3 variants of decimation position.

A set of filter banks could be constructed as a solution of this system enhanced similar to two-channel system calculation. Fig. 4 shows an example of three-band transform filter characteristics for $N = 6$, $K = 5$, $M = 6$.

Strict matching of input and output signals of three-channel transform system $U(k) \equiv \tilde{U}(k)$ corresponds to the equation:

$$\frac{1}{3}[H(x)K_h(x) + B(x)K_b(x) + G(x)K_g(x)] \equiv 1.$$

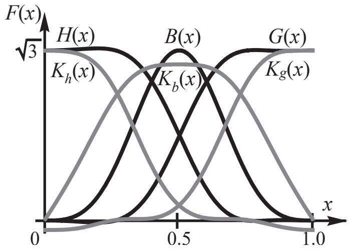


Figure 4. An example of three-channel filter bank frequency responses

The equations (3) for LF and HF filter characteristics are complemented by MF1 and MF2 filter characteristics for construction of four-channel system:

$$B_1(x) = 2\sum_{l=1}^{L} b_{1|(2l-1)/2}\sin(\pi x(2l-1)/2),$$
$$B_2(x) = 2\sum_{p=1}^{P} b_{2|(2p-1)/2}\cos(\pi x(2p-1)/2).$$

Four times decimation (removing of three from every four samples) is performed for the signals after four input filters. Decimates samples are set to zero during synthesis. The input signal will be restored if restoration filter characteristics $K_h(x)$, $K_{b1}(x)$, $K_{b2}(x)$, $K_g(x)$ meets the following equation system:

$$\begin{cases} H_1(x)K_h(x) + B_{1|1}(x)K_{b1}(x) + B_{2|1}(x)K_{b2}(x) + G_1(x)K_g(x) = 1, \\ H_2(x)K_h(x) + B_{1|2}(x)K_{b1}(x) + B_{2|2}(x)K_{b2}(x) + G_2(x)K_g(x) = 1, \\ H_3(x)K_h(x) + B_{1|3}(x)K_{b1}(x) + B_{2|3}(x)K_{b2}(x) + G_3(x)K_g(x) = 1, \\ H_4(x)K_h(x) + B_{1|4}(x)K_{b1}(x) + B_{2|4}(x)K_{b2}(x) + G_4(x)K_g(x) = 1, \end{cases}$$

where H_i, $B_{k|i}$, G_i, $i = 1...4$, denotes corresponding filters with 4 variants of decimation position.

Fig. 5 illustrates the example of four-channel filter bank characteristics for $N = M = 4$, $L = P = 3$.

Strict matching of input and output signals of four-channel transform system $U(k) \equiv \tilde{U}(k)$ corresponds to the equation:

$$\frac{1}{4}[H(x)K_h(x) + B_1(x)K_{b1}(x) + B_2(x)K_{b2}(x) + G(x)K_g(x)] \equiv 1.$$

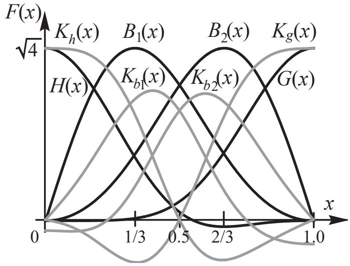


Figure 5. An example of four-channel filter bank frequency responses

The equations (1) for LF and HF filter characteristics are complemented by MF1, MF2 and MF2 filter characteristics for construction of five-channel system:

$$B_1(x) = 2\sum_{l=1}^{L} b_{1|l}\sin(\pi l x),$$
$$B_2(x) = b_{2|0} + 2\sum_{k=1}^{K} b_{2|k}\cos(\pi k x),$$
$$B_3(x) = 2\sum_{p=1}^{P} b_{3|p}\sin(\pi p x).$$

Five times decimation is performed for the signals after five input filters. Decimates samples are set to zero during synthesis. The input signal will be restored if restoration filter characteristics $K_h(x)$, $K_{b1}(x)$, $K_{b2}(x)$, $K_{b3}(x)$, $K_g(x)$ meets the following equation system:

$$\begin{cases} H_1(x)K_h(x)+B_{1|1}(x)K_{b1}(x)+B_{2|1}(x)K_{b2}(x)+B_{3|1}(x)K_{b3}(x)+G_1(x)K_g(x)=1, \\ H_2(x)K_h(x)+B_{1|2}(x)K_{b1}(x)+B_{2|2}(x)K_{b2}(x)+B_{3|2}(x)K_{b3}(x)+G_2(x)K_g(x)=1, \\ H_3(x)K_h(x)+B_{1|3}(x)K_{b1}(x)+B_{2|3}(x)K_{b2}(x)+B_{3|3}(x)K_{b3}(x)+G_3(x)K_g(x)=1, \\ H_4(x)K_h(x)+B_{1|4}(x)K_{b1}(x)+B_{2|4}(x)K_{b2}(x)+B_{3|4}(x)K_{b3}(x)+G_4(x)K_g(x)=1, \\ H_5(x)K_h(x)+B_{1|5}(x)K_{b1}(x)+B_{2|5}(x)K_{b2}(x)+B_{3|5}(x)K_{b3}(x)+G_5(x)K_g(x)=1, \end{cases}$$

where H_i, $B_{k|i}$, G_i, $i = 1...5$, denotes corresponding filters with 5 variants of decimation position.

Fig. 6 illustrates the example of five-channel filter bank characteristics for $N = M = K = 4$, $L = P = 3$.

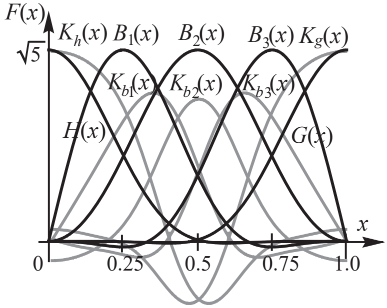


Figure 6. An example of five-channel filter bank frequency responses

Strict matching of input and output signals of five-channel transform system $U(k) \equiv \tilde{U}(k)$ corresponds to the equation:

$$\frac{1}{5}[H(x)K_h(x)+B_1(x)K_{b1}(x)+B_2(x)K_{b2}(x)+B_3(x)K_{b3}(x)+G(x)K_g(x)] \equiv 1.$$

3. Multichannel image decomposition and quantization

Processing of two-dimensional signal $u(i, k)$ during image transform using separable scaling wavelet functions is performed firstly along rows then along columns.

Fig. 7 shows block diagram of such procedure using three-channel filter bank as an example. Nine areas (sequences of discrete samples) are separated and then decimated forming nine subbands. Size of each sequence – $(N/3) \times (M/3)$, where $N \times M$ – image size (Fig. 8).

Each sample of HH subband is quantized using linear quantization scale during luminance (chrominance) processing. So the number of quantization levels J is defined by B – bit depth of sample representation: $J = 2^B$.

Distribution of wavelet transform coefficients in high frequency subbands is approximated with rather high accuracy by one-dimensional Laplace law of probability density:

$$w(x) = \frac{\lambda}{2} e^{-\lambda|x|}.$$

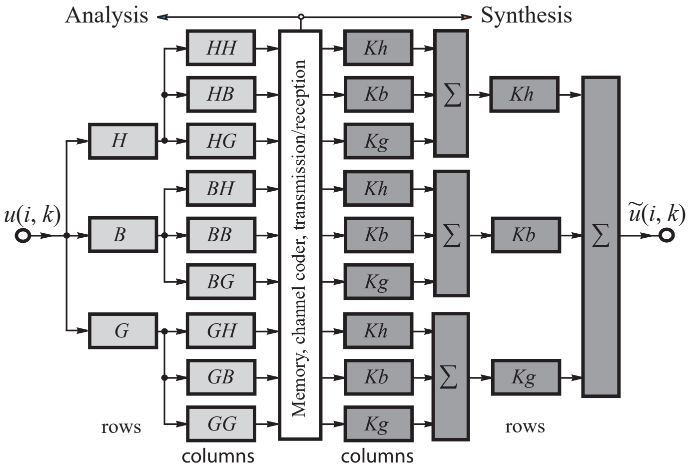


Figure 7. Block diagram of two-dimensional image wavelet transform

where $\lambda = \dfrac{\sqrt{2}}{MSD} = \dfrac{1}{Mod}$, $MSD \cong \sqrt{\dfrac{1}{MN}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} Y_{nm}^2}$,
$Mod \cong \dfrac{1}{MN}\sum_{n=0}^{N-1}\sum_{m=0}^{M-1} |Y_{nm}|$, Y_{nm} – transform coefficient values, $M \times N$ – size of subband.

Threshold levels and quantization levels are determined by solution of equations:

$$\frac{1}{m} = \int_{d_{j-1}}^{d_j} w(x)dx,\ i = 1, ..., m-1,$$

$$r_j = \int_{d_{j-1}}^{d_j} xw(x)dx,\ i = 1, ..., m,$$

where d_j, r_j – j^{th} quantization threshold and level respectively, m – the number of quantization levels.

Intensive investigation of a large number of various images results in a mask for choice of quantization level number N depending on λ parameter, given in Table 1 [4].

The value $\lambda \leq 0.02$ is specific usually for textures (synthetic images) and rarely occur in natural and television images.

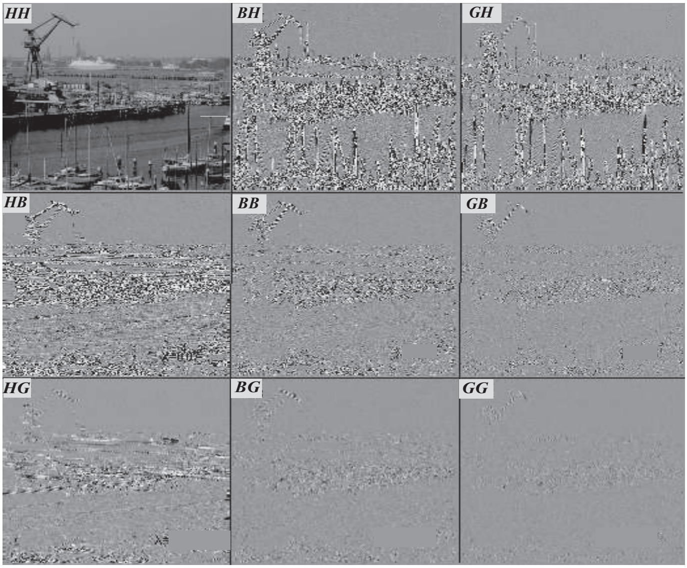


Figure 8. Example of transformed image

Table 1
The quantization level number mask for coefficients of wavelet transform high frequency subbands

λ	N
$\lambda > 1$	0
$0.4 \le \lambda < 1$	3
$0.15 \le \lambda < 0.4$	7
$0.05 \le \lambda < 0.15$	15
$0.02 \le \lambda < 0.05$	31
$\lambda \le 0.02$	63

For the improvement of image transform effectiveness, it is possible to decompose the signal using, for example, three-channel filter bank following by additional decomposition of some high frequency subbands using two-channel wavelet filters. Fig. 9 gives an example of image decomposition into 9, 11, 13 and 16 subbands.

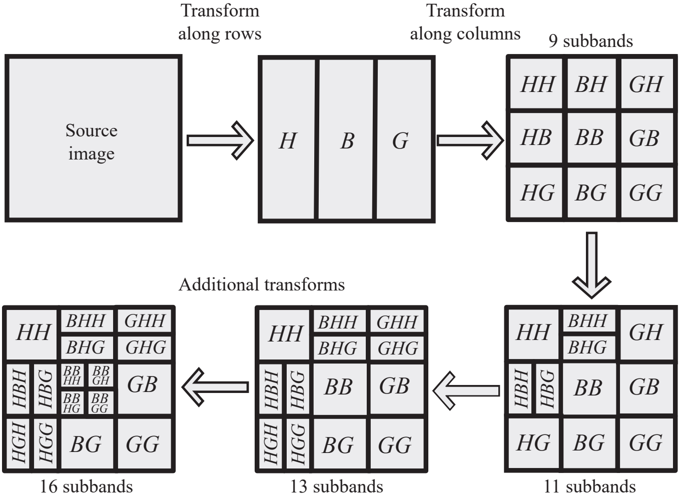


Figure 9. Diagram of wavelet image decomposition into 9 – 16 subbands

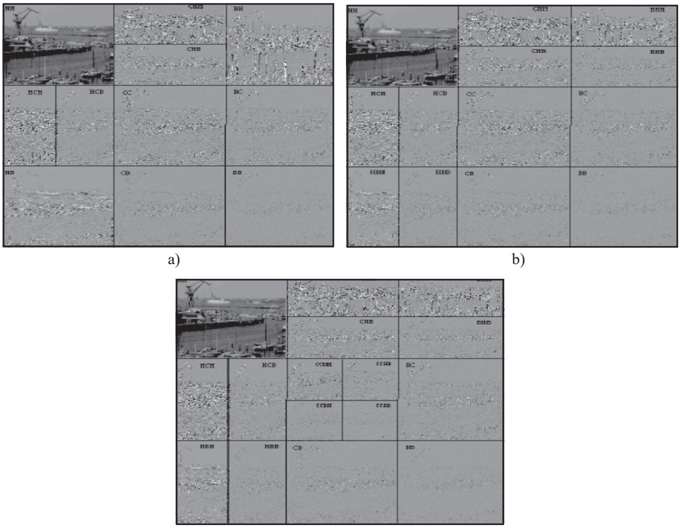
Fig. 10 illustrates wavelet decomposition of test image "Harbour" into 11, 13 and 16 subbands. The values of *HH* subband coefficients are decreased 3 times, the values of all other coefficients are increased 5 times and placed on gray level (128).

Table 2 shows the results of information amount decrease due to image decomposition into 4, 6, 9, 11, 13 and 16 subbands providing that restored image PSNR $\geq$ 37 dB.

Table 2

Information amount decrease due to wavelet image decomposition

Test image	Information amount decrease depending on the number of decomposition subbands					
	4	6	9	11	13	16
"Lenna"	1.68	2.05	2.32	2.51	2.61	2.84
"Barbara"	1.60	1.94	2.16	2.34	2.46	2.63
"Goldhill"	1.68	2.05	2.16	2.29	2.42	2.67
"Harbour"	1.60	1.94	2.32	2.49	2.58	2.81



a) b)

c)

Figure 10. Decomposition of test image “Harbour” into 11 (a), 13 (b) and 16 (c) subbands

The amount of information could be decreased 1.64 times after standard wavelet image decomposition into 4 subbands and following quantization of high frequency subbands. Additional processing of 2 high frequency subbands enable further 25% decrease of the information amount. Wavelet decomposition of image into 9, 11, 13 and 16 subbands enable average decrease of information amount in 35%, 45%, 50% and 60% respectively compared to 4 subband decomposition.

Image decomposition into comparatively large amount of subbands leads to some increase of calculation operations in coding device compared to standard wavelet decomposition into 4 subbands. The number of calculation operations at the decoder side increases negligibly because of few number of non-zero samples in medium and high frequency components.

4. Conclusions

Two-band wavelet decomposition is usually used for signal analysis. Decomposition could be implemented using FIR filters with even or odd taps. It was shown that appropriate decimation is very important in this case. Multiband wavelet decomposition could enrich signal analysis. New multiband filter bank calculation technique was presented with examples of tree-, four- and five-band filter bank characteristics. Calculated filter banks could be efficiently used for wavelet image compression if combined with the technique for quantization of high frequency subbands. The efficiency of decomposition and quantization could be improved by utilization of additional decomposition of high frequency subbands. It was illustrated using evaluation of information amount decrease on standard test images.

Acknowledgments

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References

1. *Daubechies I.* Ten Lectures on Wavelets. — Society for Industrial and Applied Mathematics, Philadelphia, Pennsylvania, 1992.
2. *Blatter Ch.* Wavelets: A Primer. — A K Peters/CRC Press, 1999.
3. *Stark H.-G.* Wavelets and Signal Processing: An Application-Based Introduction. — Springer, 2005.
4. *Dvorkovich V.P., Dvorkovich A.V.* Measurements in video information systems (theory and practice). — Moscow: Technosfera, 2015.
5. *Dvorkovich V.P., Dvorkovich A.V.* Window Functions for Harmonic Analysis of Signals. Second edition. — Moscow: Technosfera, 2016.

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New Methods for Harmonic Analysis of Signals Using Window Functions

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Abstract. New methods for high efficiency window function calculation are described. The methods allow to calculate both well-known classic windows and new ones with needed characteristics. The first method is based on minimization of window spectrum power outside of given frequency interval. The second one is based on minimization of deviation between the window waveform and its spectrum envelope. Modification of Dolph-Chebyshev and Barcilon-Temes window functions is also proposed. This modification improves some characteristic features of the windows.

Keywords: harmonic analysis, window function, discrete Fourier transform (DFT), finite impulse response (FIR) filter.

1. Introduction

The main purpose of signal processing using window functions is the analysis of its characteristics usually in the presence of various interferences. Discrete Fourier transform (DFT) is often used for such tasks. DFT provides decomposition of the analyzed signal with respect to the basis consisting of simple cosine and sine functions. In this case it is assumed that the signal is time limited and its duration is equal to processing interval.

Windows (window functions) are weighting functions that provide the decrease of spectral component leakage due to finite observation interval. Influence of window function leads to substantial decrease of investigated signal discontinuity at the boundaries of its periodic extension, if maximum number of window function derivatives are equal or close to zero at the boundaries of processing interval [1, 2].

Using of window smoothing allows calculation finite impulse response (FIR) filters for any practical purposes for Gibbs effect reduction and improvement of filter characteristics with approximation of complex gain factor and linear phase frequency response (PFR).

Filter banks are used traditionally in perceptual audio codecs. These banks helps to encode efficiently audio signals and to form quantization noise according to evaluated masking curve [3].

Special area of window function utilization is the development of adaptive antenna arrays. The characteristics of these antenna arrays, including direction pattern, are changing automatically for supply of the best or near to best useful signal reception conditions in the presence of permanently varying interference.

After the publication of F.J. Harris [1] most problems of window function construction and utilization were seemed solved. Nevertheless, the authors have proposed principally new approaches to development of window functions: by minimization of spectrum power outside of given frequency interval; by minimization of difference between window waveform and its spectrum envelope. Proposed modernization of Dolph-Chebyshev and Barcilon-Temes windows has a special importance.

2. High efficiency window function synthesis using the minimization of spectrum components outside of given interval

Waveforms of window functions are symmetric with respect to the center of processing interval $-T/2 \le t \le T/2$ and limited in duration by this interval. So such functions could be presented using even and odd cosine basis functions:

$$u_e(x) = \frac{1 + 2\sum_{m=1}^{M} a_m \cos(2\pi m x)}{Sum_e} = b_0 + 2\sum_{m=1}^{M} b_m \cos(2\pi m x), \quad (1)$$

where $Sum_e = 1 + 2\sum_{m=1}^{M} a_m$, $b_0 = \dfrac{1}{Sum_e}$, $b_m = \dfrac{a_m}{Sum_e}$, $b_0 + 2\sum_{m=1}^{M} b_m = 1$, $|x| \le 1/2$,

$$u_o(x) = \frac{2\sum_{k=1}^{K} c_{2k-1} \cos(\pi(2k-1)x)}{Sum_o} = 2\sum_{k=1}^{K} d_{2k-1} \cos(2\pi(2k-1)x), \quad (2)$$

where $Sum_o = 2\sum_{k=1}^{K} c_{2k-1}$, $d_{2k-1} = \dfrac{c_{2k-1}}{Sum_o}$, $2\sum_{k=1}^{K} d_{2k-1} = 1$, $|x| \le 1/2$.

Normed spectrums of these functions are the following:

$$F_e(y) = \mathrm{sinc}(\pi y) + \sum_{m=1}^{M} a_m[\mathrm{sinc}(\pi(y+m)) + \mathrm{sinc}(\pi(y-m))], \quad (3)$$

where $\mathrm{sinc}(z) = \dfrac{\sin(z)}{z}$, $y = \dfrac{\omega T}{2\pi} = fT$ – normed frequency, $|y| < \infty$,

$$F_o(y) = \frac{1}{S_o}\sum_{k=1}^{K} c_{2k-1}[\mathrm{sinc}(\pi(y + \frac{2k-1}{2})) + \mathrm{sinc}(\pi(y - \frac{2k-1}{2}))], \quad (4)$$

where $S_0 = 2\sum_{k=1}^{K} c_{2k-1}\mathrm{sinc}(\pi\frac{2k-1}{2}) = \frac{4}{\pi}\sum_{k=1}^{K}(-1)^{k-1}\frac{c_{2k-1}}{2k-1}$, $|y| < \infty$.

Fourier spectrum (3) and (4) of window functions (1) and (2) is infinite and theoretically couldn't be restricted because window functions are restricted at finite time interval. But for specially selected coefficients a_m and c_{2k-1} the spectrum could be practically restricted by some frequency interval. Variations of some terms of series (3) and (4) will be almost completely compensated by variations of other terms outside that frequency interval.

$F_e(y)$ and $F_o(y)$ functions (denoted further as $F(y)$) have minimum meansquare power of sidelobes outside the interval $[-C, C]$ under the following condition:

$$\int_{-\infty}^{-C} F^2(y)dy + \int_{C}^{\infty} F^2(y)dy = 2\int_{C}^{\infty} F^2(y)dy \Rightarrow \min .$$

For example, maximal sidelobe of window function (1) spectrum $W_{\max}$ gradually decreases with the increase of interval $[-C, C]$: from –26 dB for $C = 1$ to –188 dB for $C = 7$. In spite of considerable variation of C parameter, the window function quality coefficient δ [1] varies slightly from 4.7% for $W_{\max} = -26$ dB to 6% for $W_{\max} = -188$ dB.

Maximal sidelobe of window function (2) spectrum also gradually decreases with the increase of interval $[-C, C]$: from –23 dB for $C = 1$ to –157 dB for $C = 6$, while quality coefficient δ varies slightly from 3.8% for $W_{\max} = -23$ dB to 5.8% for $W_{\max} = -157$ dB.

Signal waveforms with even taps calculated using the described method are shown at Fig. 1a for $C = 2$, $M = 2$ ($W_{\max} = -42$ dB); $C = 6.5$, $M = 6$ ($W_{\max} = -144$ dB). Fig. 1b shows spectra of these signals.

Signal waveforms with odd taps are shown at Fig. 1c for $C = 2$, $K = 2$ ($W_{\max} = -43$ dB); $C = 5$, $K = 5$ ($W_{\max} = -118$ dB). Fig. 1d shows spectra of these signals.

It should be noted that practically all classic window functions (for example, Hamming, Kaiser-Bessel, etc.) could be calculated using the method of minimization of spectrum power outside given frequency interval.

3. High efficiency window function synthesis using the minimization of deviation between the waveform and its spectrum envelope

This algorithm is implemented using the aid of method described above.

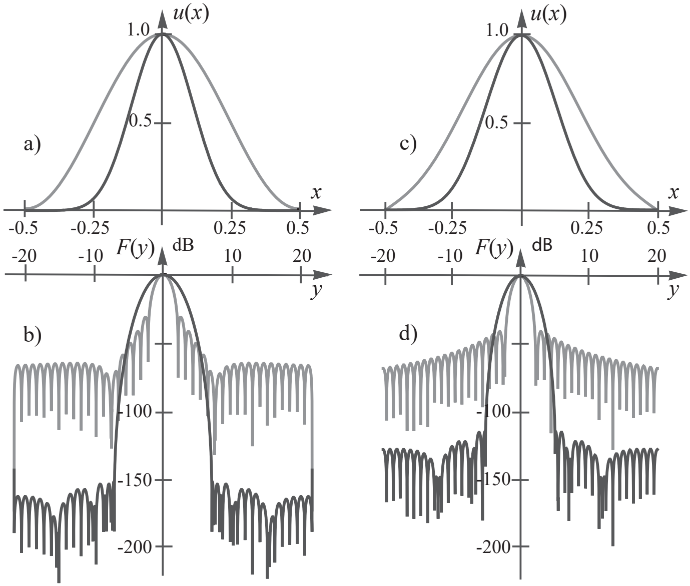

Figure 1. Examples of window calculation using minimization of its spectrum power for even cosine components (a, b) and odd cosine components (c, d)

Let us transform the waveform of window function with even cosine components so that time interval become equal $[-(M+1),(M+1)]$:

$$u_e(y)=\frac{1}{Sum_e}\left[1+2\sum_{m=1}^{M}a_m\cos(\pi my/(M+1))\right],\ |y|<(M+1).$$

$F_e(y)$ and $u_e(y)$ coincide is the points $y=-(M+1)$, $y=0$, $y=(M+1)$ if

$$1+2\sum_{m=1}^{M}(-1)^m a_m=0.$$

Let us fulfill such transform for window with odd cosine components so that time interval coincides with relative interval $[-(K+1/2),(K+1/2)]$:

$$u_o(y)=\frac{1}{Sum_o}\left[2\sum_{k=1}^{K}c_{2k-1}\cos\left(\pi y\frac{2k-1}{2K+1}\right)\right],\ |y|<(K+1/2).$$

In this case $F_o(y)$ and $u_o(y)$ coincide is the points $y = -(K + 1/2)$, $y = 0$, $y = (K + 1/2)$.

The simplest case of such function calculation uses minimization of maximal value of absolute difference $\Delta(y) = |F(y) - u(y)|$:

$$\Delta = \min_{\Lambda(y)}[\max_{y} |F(y) - u(y)|]. \tag{5}$$

Algorithm for equation (5) solution is rather simply implemented using selection of point positions y_k at the interval $|y| < (M + 1)$ (in the first case) or $|y| < (K+1/2)$ (in the second case), where the values of the signal $u(y_k)$ and its spectrum $F(y_k)$ or their derivatives of some orders are equal each other.

Fig. 2 illustrates the characteristics of window functions with even cosine components for $M = 1$, $M = 2$, and $M = 3$. In this case maximum differences between waveforms and spectrum envelope are 2.7%, 0.07% and 0.0025% correspondingly. For $M = 9$ this difference is lower than $0.4 \cdot 10^{-10}\%$.

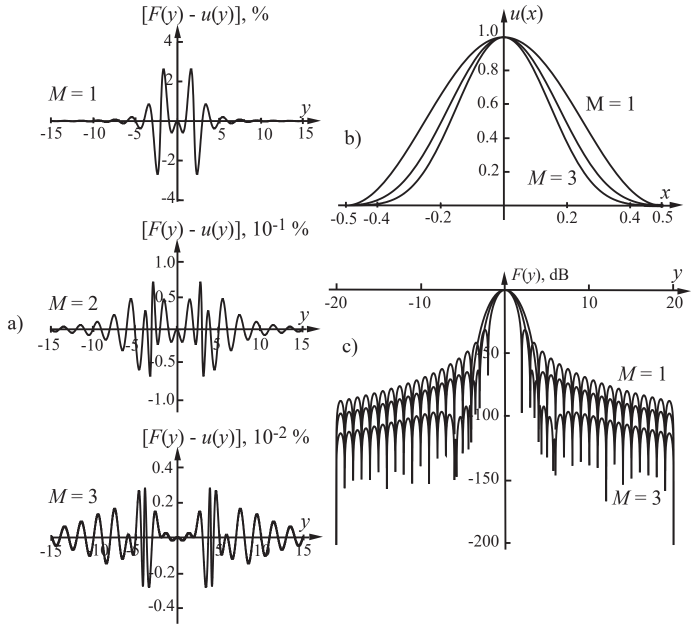

Figure 2. Characteristics of even window functions for $M = 1$, $M = 2$, and $M = 3$

Fig. 3 illustrates the characteristics of window functions with odd cosine components for $K = 1$, $K = 2$, and $K = 3$. In this case maximum differences between waveforms and spectrum envelope are 17%, 0.45% and 0.02% correspondingly. For $K = 10$ this difference is lower than $0.17 \cdot 10^{-10}$%.

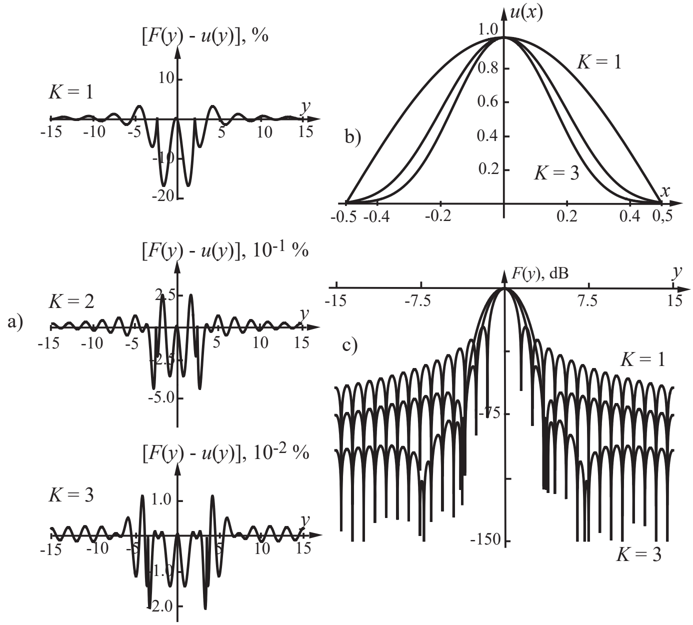

Figure 3. Characteristics of even window functions for $K = 1$, $K = 2$, and $K = 3$

4. High efficiency window function synthesis using the minimization of deviation between the waveform and its spectrum envelope

4.1. Characteristics of Dolph-Chebyshev and Barcilon-Temes window functions

Characteristic feature of Dolph-Chebyshev windows is the equality of all spectrum sidelobe amplitudes. This leads to a significant suppression of sidelobes for a given width of the main lobe, or the minimum width of the main lobe for a given sidelobe level. These windows were developed for analysis of discrete signals with periodic spectrum.

Realization of Barcilon-Temes windows is based on algorithm of minimization of power of spectrum components outside mane lobe. This criterion gives some compromise between the criteria for construction of Dolph-Chebyshev and Kaizer-Bessel windows.

Normalized spectra of Dolph-Chebyshev and Barcilon-Temes windows depends on two parameters (n and h):

$$F_{DCh}(y) = F(y,n,h) = \frac{T_n\left(n\cos\left[\frac{\mathrm{Arccosh(1/h)}}{n}\right]\cos(\pi y)\right)}{T_n\left(n\cos\left[\frac{\mathrm{Arccosh(1/h)}}{n}\right]\right)},$$

$$F_{BT}(y) = F(y,n,h) = \frac{T_n(\gamma\cos(\pi y)) + \frac{\gamma}{R}\cos(\pi y)\cdot U_n(\gamma\cos(\pi y))}{T_n(\gamma) + \frac{\gamma}{R}\cos(\pi y)\cdot U_n(\gamma)},$$

where $y = f\Delta T$ – normalized frequency, ΔT – sampling interval, $\gamma = \mathrm{ch}(R/M)$, $R = \mathrm{Arch}(1/h)$, $h = 10^{-\alpha}$, T_n – Chebyshev polynomial of the first kind of n-th order ($T_n(z) = \cos(n\cdot\arccos(z))$, $z \le 1$), U_n – Chebyshev polynomial of the second kind of n-th order ($U_n(z) = \sin(n\cdot\arccos(z))/(1-z)^{1/2}$, $z \le 1$).

The structure of spectra of both functions are practically very close.

For even value of parameter $n = 2m$ both functions $F(y)$ are periodical and defined at all frequency interval (Fig. 4a):

$$F(y+k) = F(y),\ |y| \le 1/2,\ k = ..., -2, -1, 0, 1, 2, ...$$

For odd value of parameter $n = 2m - 1$ both functions $F(y)$ are also defined at all frequency interval (Fig. 4b):

$$F(y+k) = (-1)^k F(y),\ |y| \le 1/2,\ k = ..., -2, -1, 0, 1, 2, ...$$

Parameter n defines relative frequency of cosine oscillations $ny/2$ at the interval $[-1/2, 1/2]$, and parameter h defines the amplitude of these oscillations.

Fig. 4c illustrates two variants of the function $F(y)$ structure near zero for even value of n; sequence of cosine oscillations $h\cdot\cos(\pi ny)$ is shown below.

If $n = 4m$ then integer number of periods fits into the half interval $[-1/2, 0]$ or $[0, 1/2]$ and the values of function $F(y)$ at interval boundaries are positive and equal to h.

If $n = 4m \pm 2$ then integer number of half-periods fits into the half interval $[-1/2, 0]$ or $[0, 1/2]$ and the values of function $F(y)$ at interval $[-1/2, 1/2]$ boundaries are negative and equal to $-h$.

Fig. 4d illustrates the function $F(y)$ structure near zero for odd value of n. In this case integer number of quarter-periods fits into interval

$[-1/2, 1/2]$ and the values of function $F(y)$ at interval $[-1/2, 1/2]$ boundaries are equal to zero.

The derivatives of these functions at the boundaries of interval $[-1/2, 1/2]$ for $n = 4m \pm 1$ are negative – $F'(1/2) = F'(-1/2) = -\pi nh$, and for $n = 4m \pm 3$ are positive – $F'(1/2) = F'(-1/2) = \pi nh$.

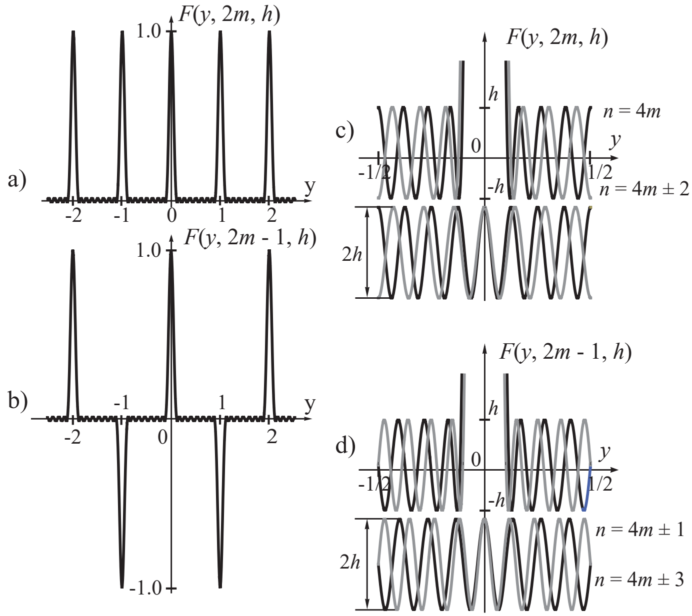

Figure 4. Even and odd Dolph-Chebyshev and Barcilon-Temes functions spectrum structures

It was proved that normalized spectra of Dolph-Chebyshev and Barcilon-Temes windows are identically defined by finite number of cosine functions [2].

For $n = 2m$

$$F(y) \equiv b_0 + 2\sum_{k=1}^{m} b_k \cos(2\pi k y), \tag{6}$$

where $b_0 = \int\limits_0^{1/2} F(y)dy$, $b_k = \int\limits_0^{1/2} F(y)\cos(2\pi k y)dy$, $b_0 + 2\sum\limits_{k=1}^{m} b_k \equiv 1$.

For $n = 2m - 1$

$$F(y) \equiv 2\sum_{k=1}^{m} d_k \cos(\pi(2k-1)y), \tag{7}$$

where $d_k = 2\int\limits_0^{1/2} F(y)\cos(\pi(2k-1)y)dy$, $2\sum\limits_{k=1}^{m} d_k \equiv 1$.

It is rather easy to determine relative waveforms of these windows using equations (6) and (7):

for $n = 2m$

$$u(x) = \mathrm{sinc}(\pi x) + \sum_{k=1}^{m}\frac{b_k}{b_0}[\mathrm{sinc}(\pi(\mathrm{x}-\mathrm{k})) + \mathrm{sinc}(\pi(\mathrm{x}+\mathrm{k}))], \tag{8}$$

for $n = 2m - 1$

$$u(x) = \frac{1}{u(0)}\sum_{k=1}^{m} d_k[\mathrm{sinc}(\pi(\mathrm{x}-\mathrm{k}+\frac{1}{2})) + \mathrm{sinc}(\pi(\mathrm{x}+\mathrm{k}-\frac{1}{2}))], \tag{9}$$

where $u(0) = \frac{4}{\pi}\sum\limits_{k=1}^{m}\frac{(-1)^{k-1}}{2k-1}$.

Equations (6) and (7) defines the waveform of LF filter frequency responses after substitution of relative frequency fT instead of y argument, where $T = 1/(2f_0)$, f_0 – cutoff frequency of LF filter, $0 \le f \le f_0$. Impulse responses of these filters:

$$u_{2m}(t) = \sum_{k=-m}^{m} b_k\delta(|k|T) \quad \text{for } n = 2m,$$

$$u_{2m-1}(t) = \sum_{k=-m}^{m} b_k\delta(\frac{2|k|-1}{2}T) \quad \text{for } n = 2m-1.$$

Dolph-Chebyshev window main sidelobe levels are defined by coefficient α and equals -20α [dB]. For $\alpha = 2.5$, 3, 3.5 and 4 the levels are equal to –50 dB, –60 dB, –70 dB and –80 dB.

Barcilon-Temes window main sidelobe levels for $\alpha = 2.5$, 3, 3.5 and 4 are equal to –52 dB, –62 dB, –72 dB and –82 dB.

4.2. Modification of Dolph-Chebyshev and Barcilon-Temes window functions

The characteristics of Dolph-Chebyshev and Barcilon-Temes window functions could be improved significantly with the use of the following window components:

$$F(y) = \frac{1}{2}[F_{DCh/BT}(y,n,h)+ \\ \frac{F_{DCh/BT}(y,n+2,h)+F_{DCh/BT}(y,n-2,h)}{2}], \tag{10}$$

where $F_{DCh/BT}(y,n,h)$ – standard Dolph-Chebyshev or Barcilon-Temes windows.

Taking into account that waveforms of functions $F_{DCh/BT}(y,n,h)$ and $(F_{DCh/BT}(y,n+2,h)+F_{DCh/BT}(y,n-2,h))/2$ are very similar in the area of main lobes and cosine oscillations near zero are shifted on 180^o, equation (10) constructs new window functions with the main lobe very close to the waveform of $F_{DCh/BT}(y,n,h)$ and sidelobes substantially decreasing from center to boundaries of interval $[-1/2, 1/2]$.

The parameters of sum functions could be calculated using (6), (7), (8), (9) with modification of high limit of sums:

$$F(y) \equiv b_0 + 2\sum_{k=1}^{m+1} b_k \cos(2\pi k y) \quad \text{for } n = 2m, \tag{11}$$

where $b_0 = 2\int\limits_0^{1/2} F(y)dy$, $b_k = 2\int\limits_0^{1/2} F(y)\cos(2\pi k y)dy$, $b_0 + 2\sum\limits_{k=1}^{m+1} b_k \equiv 1$;

$$F(y) \equiv 2\sum_{k=1}^{m+1} d_k \cos(2\pi(2k-1)y) \quad \text{for } n = 2m-1, \tag{12}$$

where $d_k = 2\int\limits_0^{1/2} F(y)\cos(2\pi(2k-1)y)dy$, $2\sum\limits_{k=1}^{m+1} d_k \equiv 1$.

It is possible to determine relative waveforms of these windows using (11) and (12):

for $n = 2m$

$$u(x) = \mathrm{sinc}(\pi x) + \sum_{k=1}^{m+1} \frac{b_k}{b_0}[\mathrm{sinc}(\pi(\mathrm{x}-\mathrm{k})) + \mathrm{sinc}(\pi(\mathrm{x}+\mathrm{k}))],$$

for $n = 2m - 1$

$$u(x) = \frac{1}{u(0)} \sum_{k=1}^{m+1} d_k [\mathrm{sinc}(\pi(\mathrm{x} - \mathrm{k} + 1/2)) + \mathrm{sinc}(\pi(\mathrm{x} + \mathrm{k} - 1/2))],$$

where $u(0) = \dfrac{4}{\pi} \displaystyle\sum_{k=1}^{m+1} \frac{(-1)^{k-1}}{2k - 1}$.

The results of normalized spectrum comparison of modified Dolph-Chebyshev (a, b) and Barcilon-Temes (c, d) windows are shown on Fig. 5.

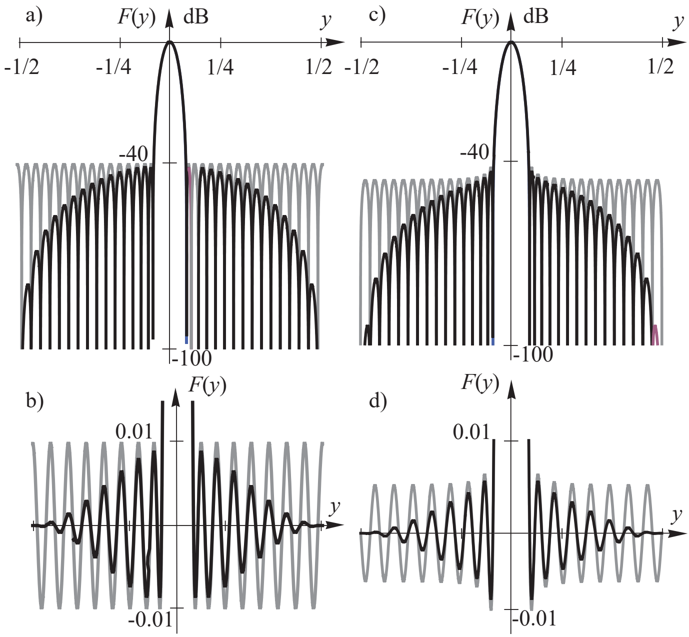

Figure 5. Comparison of modified Dolph-Chebyshev and Barcilon-Temes windows

It is need to note that the maximum level of first sidelobes lower on several dB than that of Dolph-Chebyshev window and the levels of further sidelobes gradually decreases to the boundaries of interval $[-1/2, 1/2]$.

For instance, for $\alpha = 2$, 2.5, 3, 3.5 and 4 the maximum sidelobe levels are equal to –44 dB, –56 dB, –69 dB, –81 dB and –94 dB respectively at $n = 16$.

For modified Barcilon-Temes windows, for $\alpha = 2$, 2.5, 3, 3.5 and 4 the maximum sidelobe levels are equal to –46 dB, –57 dB, –72 dB, –84 dB and –99 dB respectively at $n = 16$.

5. Conclusions

Two new methods for calculation of high efficiency window functions with necessary parameters were presented. The new methods allow to calculate both well-known classic windows and new ones with needed characteristics. The methods are based on minimization of window spectrum power outside of given frequency interval and on minimization of deviation between the window waveform and its spectrum envelope. Dolph-Chebyshev and Barcilon-Temes window functions were also analyzed. The normalized spectra of these windows are identically defined by finite number of cosine functions. Modification of the windows with improved characteristics was proposed.

Acknowledgments

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References

1. *Harris F.J.* On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform. // Proc. IEEE, 1978 — Vol. 66, no. 1 — P. 172-204.
2. *Dvorkovich V.P., Dvorkovich A.V.* Window Functions for Harmonic Analysis of Signals. Second edition. — Moscow: Technosfera, 2016.
3. *Dvorkovich V.P., Dvorkovich A.V.* Measurements in video information systems (theory and practice). — Moscow: Technosfera, 2015.

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Optimal control of $M(t)/M/K$ queues with homogeneous and heterogeneous servers

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Abstract. The paper deals with a multi-server controllable queueing system $M(t)/M/K$ with time-dependent and, in particular, with periodic arrival rates. The models with homogeneous and heterogeneous servers are of interest. In latter case the fastest free server allocation mechanism is assumed and the preemption is allowed. The control problem consists in evaluation of the optimal number of servers during some specified stages and is solved by finite horizon dynamic programming approach. To calculate the transient solutions we use a forth-order Runge-Kutta method for the system with a truncated queue length. The results are compared with corresponding queues operating in a stationary regime. It is shown that the optimal control policies are also time dependent and periodic as arrival rates and heterogeneous systems are superior in performance comparing to the homogeneous ones.

Keywords: time-dependent arrival rate, controllable queueing system, dynamic programming approach, forth-order Runge-Kutta method.

1. Introduction

Many queueing systems are subject to time-dependent changes in system parameters. This feature is very important to cover the problems with seasonality and periodicity of stochastic processes. Particularly it happens with an arrival rate which is used for modelling of arrivals of calls and inquires at call centres, arrival of packets at routers of the telecommunication systems, of time changing air traffic at airports, different arrival rates of trucks to the warehouses, goods depot or seaports and so on. A very good literature overview on this subject can be found in [4]. This paper surveys and classifies the results on performance evaluation approaches for time-dependent queueing systems and their applications and identifies the links between different approaches. The performance analysis of multi-server queueing system subject to breakdowns was studied in [1]. There are several approaches to analyse such systems. The dynamic behaviour

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of Markovian queueing systems is described by a system of Kolmogorov differential equations (KDEs). Analytical solution of such systems exists only for special cases. Numerical approaches are based on a Runge-Kutta method. The systems with an infinite buffer are normally approximated by using a finite buffer system. The numerical solution of KDEs is used for example for the performance evaluation of a $M(t)/M/1/N$ system in [3].

In many cases the time-dependent system parameters must be combined with some controllable problems. The paper [5] deals with optimal allocation of such resources as beds in emergency departments of a hospital. For the mathematical modelling the queueing system with losses of the type is used. It was shown that the periodic variation of arrival rates makes a hysteretic policy time-dependent and periodic with the same period. To find the optimal decisions dynamic programming is used. The same approach was used in [2] for the multi-server queueing system with a controllable number of homogeneous servers.

The contribution of the present work is an evaluation of the optimal number of servers in multi-server queueing system with homogeneous and heterogeneous servers and time-dependent arrival rate. The discretization of a continuous-time Markov process is performed to apply the Runge-Kutta method and the iterative dynamic programming algorithm over a finite horizon. The decisions are chosen at specified moments of time which divide the observation time interval into so called stages. The number of available servers is assumed to be a constant within each stage. The paper provides comparison analysis of stationary and transient solutions as well as homogeneous and heterogeneous systems.

The rest of the paper is organized as follows. In Section 2 we describe a mathematical model and formulate a optimization problem for transient and stationary case. Section 3 deals with a description of a time-dependent arrival rate. In Section 4 a forth-order Runge-Kutta method is adopted for the model under study. The recursive dynamic programming algorithm is shown in Section 5. Some illustrative numerical examples are discussed in Section 6. Conclusions are given in Section 7.

2. Mathematical model

Consider the controllable multi-server queueing system $M(t)/M/K$ with K servers. This system features Poisson arrival stream with time dependent arrival rate $\lambda(t)$. The servers are assumed to be heterogeneous with servers intensities $\mu_j, j = 1, 2, \ldots, K$. In special case when all intensities are equal, $\mu_j = \mu, j = 1, 2, \ldots, K$, we get the homogeneous system. The control consists in specification of the number of servers $K(t)$ at any decision epoch which will be specified later.

Denote by $N(t)$ the number of customers in the system at time t. The dynamics of the system is described by means of the controllable continuous-time inhomogeneous Markov chain $\{N(t)\}_{t\geq 0}$ with a set of states $E = \{n; n \in \mathbb{N}_0\}$ and set of control actions $A = \{1, 2, \ldots, K\}$.

Define additional cost structure with the following components: c_1 – the waiting cost per unit of time for each customer in the queue, $c_{2,j}$ – the idle state cost per unit of time when the server j is idle. In homogeneous case it is assumed that $c_{2,j} = c_2, j = 1, 2, \dots, K$. The servers are enumerated in such a way that

$$\mu_1 \geq \mu_2 \geq \cdots \geq \mu_K, \quad c_{2,1} \geq c_{2,2} \geq \cdots \geq c_{2,K}. \tag{1}$$

In accordance with the given cost structure, the mean total cost criterion is denoted by

$$J^f(n) = \mathbb{E}^f[\int_0^T c(N(t), K(t), \lambda(t), t)dt | N(0) = n]. \tag{2}$$

Here f is a Markov control policy which depends on the current state and time only, i.e. $K(t) = f(t, n(t))$, the expectation $\mathbb{E}^f$ is taken with respect to the probability distribution $\mathbb{P}^f$ over the state-action sequence under control policy f. The immediate cost function $c(N(t), K(t), \lambda(t), t)$ is defined as

$$\begin{aligned} c(N(t), K(t), \lambda(t), t) = c_1 &\sum_{k=K(t)+1}^{\infty} (k - K(t)) 1_{\{N(t)=k\}} \\ &+ \sum_{k=0}^{K(t)} \sum_{j=k+1}^{K(t)} c_{2,j} 1_{\{N(t)=k\}}. \end{aligned} \tag{3}$$

The substitution of (3) into (2) yields the relation for the mean total cost in form

$$\begin{aligned} J^f(n) &= \int_0^T \eta(n, K(t), \lambda(t), t)dt \\ &= \int_0^T \Big[c_1 \bar{Q}(n, K(t), \lambda(t), t) + \sum_{k=0}^{K(t)} \sum_{j=k+1}^{K(t)} c_{2,j} \pi_k(n, K(t), \lambda(t), t)\Big] dt. \end{aligned} \tag{4}$$

The first term by c_1 at the right hand side of (4) stands stands for the mean number of customers in the queue at time t with $K(t)$ servers and initial state $N(0) = n$, the second term stands for the mean idle state costs. We wish to minimize the functional $J^f(n)$ over all control policies and find optimal policy f^* that achieves the minimal cost $J^*(n)$, i.e.

$$J^*(n) := J^{f^*}(n) = \min_f J^f(n).$$

The solution of proposed optimization problem can be performed numerically. To realize some iterative algorithm the continuous time model must be converted to a discrete one. We divide a time interval $[0, T]$ into I equally spaced periods. The mean total cost functional in this case can be rewritten a follows,

$$J^f(n) = \sum_{i=1}^{I} \eta(n, K(i), \lambda(i), i) \tag{5}$$
$$= \sum_{i=1}^{I} \Big[c_1 \bar{Q}(n, K(i), \lambda(i), i) + \sum_{k=0}^{K(i)} \sum_{j=k+1}^{K(i)} c_{2,j} \pi_k(n, K(i), \lambda(i), i)\Big],$$

where $K(i)$ is a number of servers at period i, $\bar{Q}(n, K(i), \lambda(i), i)$ is a mean number of customers in the queue at period i, $\pi_k(n, K(i), i)$ – probability of k customers in the system with $K(i)$ servers at period i with initial state n.

The transient solution of the problem will be compared with a stationary one. In this case the long-run average cost per unit of time

$$\eta(K, \lambda(t)) = c_1 \bar{Q}(K, \lambda(t)) + \sum_{k=0}^{K} \sum_{j=k+1}^{K} c_{2,j} \pi_k(K, \lambda(t)) \tag{6}$$

must be minimized over $K(t)$ for any fixed value $\lambda(t)$. The substitution of the stationary state probabilities of the infinite buffer system into (6) yields the relation

$$\eta(K(t), \lambda(t)) = \Big[c_1 \prod_{j=1}^{K(t)} \frac{\lambda(t)}{\sum_{k=1}^{j} \mu_k} \frac{\lambda(t) \sum_{k=1}^{K(t)} \mu_k}{(\lambda(t) - \sum_{k=1}^{K(t)} \mu_k)^2} \tag{7}$$
$$+ \sum_{k=0}^{K(t)} \sum_{j=k+1}^{K(t)} c_{2,j} \frac{\lambda(t)^k}{\prod_{l=1}^{k} \sum_{j=1}^{l} \mu_j}\Big] \pi_0(K(t), \lambda(t)),$$

$$\pi_0(K(t), \lambda(t)) =$$
$$\Big[\sum_{k=0}^{K(t)-1} \frac{\lambda(t)^k}{\prod_{l=1}^{k} \sum_{j=1}^{l} \mu_j} + \prod_{j=1}^{K(t)} \frac{\lambda(t)}{\sum_{k=1}^{j} \mu_k} \frac{\sum_{k=1}^{K(t)} \mu_k}{\sum_{k=1}^{K(t)} \mu_k - \lambda(t)}\Big]^{-1}.$$

3. Arrival Rate

We have chosen a similar arrival rate $\lambda(t)$ as in the paper from [2]. The authors have studied there incoming and service of airplanes of an airport

Table 1

Values for the arrival rate $\lambda(t)$

Time in hour	Input intensity
1-5	$8.75 + 4.25\ Cos(\frac{t}{1.6})$
6	4.5
7	5.0
8	5.5
9	6.5
10-17	7.0
18-21	10.0
22-24	13.0

modelled via a $M(t)/M/K/N$ queuing system. In the queueing system under study the condition

$$\lambda(t) < \sum_{j=1}^{K} \mu_j \tag{8}$$

is a necessary one, since there is no cost relation for customers who get rejected. That means that the maximum number of server K can handle the average arrival rate of users. The data for $\lambda(t)$ is given in Table 1.

The arrival rate will be divided into three equidistant stages. It means that each stage lasts eight hours, which is a normal working shift cycle. At the beginning of a stage, the number of customers n in the system is known. This value will be called initial state of the current stage. A natural question, one can ask is, how many servers (in this context workers) should be hired at current and following stage(s) so that the expected costs are minimized. Obviously the number of necessary operators are depending on the initial state n.

4. Fourth-Order Explicit Runge-Kutta Method

Since there is no way to solve the system of Kolmogorov forward equations

$$\pi'(t) = \pi(t)A(t) \tag{9}$$

analytically a numerical algorithm is needed to get an approximate solution. For this task we have used the standard fourth order explicit Runge-Kutta procedure which is a widely used one-step method. It considers differential equations of the form

$$y'(t) = f(t, y(t)) \quad \forall t \in (0, T)$$

with given initial condition

$$y(t_0) = y_0.$$

Algorithm 1 *The explicit fourth-order Runge-Kutta method.*
Step 1. Computation of five parameters $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ *and* κ*:*

$$\begin{aligned}
\kappa_1 &= f(t_n, y_n)\Delta t \\
\kappa_2 &= f(t_n + \frac{\Delta t}{2}, y_n + \frac{\kappa_1}{2})\Delta t \\
\kappa_3 &= f(t_n + \frac{\Delta t}{2}, y_n + \frac{\kappa_2}{2})\Delta t \\
\kappa_4 &= f(t_n + \Delta t, y_n + \kappa_3)\Delta t \\
\kappa &= \frac{1}{6}(\kappa_1 + 2\kappa_2 + 2\kappa_3 + \kappa_4)
\end{aligned}$$

Step 2. Evaluation of y_{n+1} *by the recursive relation,*

$$y_{n+1} = y_n + \kappa.$$

For solving the Kolmogorov forward equations (9) interpret $f(t)\pi(t)$ as $f(\pi(t), t)$, choose a appropriate step size Δt and apply this Runge-Kutta method directly on the function $f(\pi(t), t)$. Obviously the error between the calculated and the real solution gets less if one selects a smaller step size. On the other hand a greater step size means less computing time. We have chosen $\Delta t = 0.005$. This value for the step size seems to have a reasonable balance between computing time and computation error.

To solve the system (9) we use a truncation of the buffer capacity by assuming that N is the maximum allowable number of customers in the system.

5. Optimisation Problem

Let $T = 24h$ be an observation cycle and a finite horizon for the dynamic programming. The decision epochs occur each $8h$, hence we get $S = 3$ stages with $\frac{I}{S}$ periods i within each stage s. A strategy at a decision epoch $d = (S - s)I/S + 1$ which depends on a current stage s is donated by $f(d, n)$, where n stands as before for the initial state. A strategy is equal for any period i from the interval

$$d \leq i \leq d + \frac{I}{S} - 1.$$

Denote by $V_n(s)$ the optimal cost function for s stages left which we refer to as value function:

$$V_n(s) = \min_f \mathbb{E}^f [\sum_{i=1}^{I} c(N(i), K(i), \lambda(i), i) | N(s) = n], \; s = 1, 2, \ldots, S, \; n \in E.$$

The minimum must be taken over tail policies

$$(f(1, K(1)), f(I/S{+}1, K(I/S{+}1)), \ldots, f((S{-}1)I/S{+}1, K((S{-}1)I/S{+}1)).$$

Obviously, $V_n(S) = J^*(n)$.

Algorithm 2 *The finite horizon dynamic programming:*

Step 1. Backward recursion: $V_n(0) = 0, n \in E$, *and*

$$V_n(s) = \min_{1 \le k \le K} \Big\{ r(n, k, s) + \sum_{m=0}^{N} p_{nm}(k, s) V_m(s-1) \Big\},$$

where $r(n, k, s)$ *is the total average cost per stage,*

$$r(n, k, s) = \sum_{i=d}^{d+\frac{I}{S}-1} \eta(n, k, \lambda(i), i), \; s = 1, 2, \ldots, S,$$

and the transition probabilities between the stages are defined as

$$p_{nm}(k, s) = \mathbb{P}\Big[N(d + I/S) = m \Big| N(d) = n, f(d, n) = k \Big], \quad (10)$$

$$s = 2, \ldots, S.$$

Step 2. Any Markov policy f^* *that satisfies*

$$f^*(d, n) = \arg\min_{1 \le k \le K} \Big\{ r(n, k, s) + \sum_{m=0}^{N} p_{nm}(k, s) V_m(s-1) \Big\} \quad (11)$$

is an optimal control policy.

The values $p_{nm}(k, s)$ in (10) have to be interpreted in the following way. They stand for the probability to be in state m at the beginning of the next stage under the condition that the initial state of the previous stage was n.

Algorithm 3 *The following basic steps are involved into the computation procedure:*

Step 1. *Compute the state probabilities* $\pi(n, k, \lambda(i), i)$ *for each* n, k *and* i *via the Runge-Kutta fourth order method.*
Step 2. *Compute the cost function* $\eta(n, k, \lambda(i), i)$ *for each* n, k *and* i.
Step 3. *Compute* $r(n, k, s)$ *for each* n, k *and* s *by accumulating* $\eta(n, k, \lambda(i), i)$ *over all periods* i *within the corresponding stage.*
Step 4. *Compute the transition probabilities* $p_{nm}(k, s)$ *for each* n, m, k *and* s *via Runge-Kutta fourth order method.*
Step 5. *Evaluation of the optimal strategy for any* s *and* n *by means of Algorithm 2.*

The number of servers $k^* = f(s, n)$ defined by (11) for which the expression in the right hand side of (2) is minimal is called the best or optimal strategy at stage s given the initial state is n.

Remark 1 Notice that in this queueing model rejecting of customers is not a valid option. If, for example, the initial state n at the current stage is the capacity of the buffer plus 2, the best strategy cannot be one server. However if the one choose the waiting room capacity high enough, restrict n up to this value and condition (8) is clearly fulfilled there will be no dropping of users.

6. Numerical Realisation and Results

The main goal in this paper is to compare the operating costs for the $M(t)/M/K$ queue between homogeneous and heterogeneous servers when optimal policies are used. Further the difference between transient and stationary solution will be contrasted for both philosophies. For this computations the following assumptions are used:

1. The maximum number of servers is $K = 6$.
2. The buffer capacity is $N - K = 10$.
3. The waiting cost $c_1 = 10$.
4. The service rate of the server in homogeneous case is $\mu = 4$.
5. The service intensities $\mu_j, j = 1, 2, \ldots, K$, of heterogeneous servers are listed in table 2.
6. The idle state cost in homogeneous case is $c_2 = 2.1$.
7. The idle state costs $c_{2,j}, j = 1, 2, \ldots, K$, for heterogeneous servers are listed in the table 2.

To get comparability with the homogeneous case we have chosen the service rates and operator idle costs for heterogeneous servers so that they satisfy the following conditions,

$$\sum_{j=1}^{K} \mu_j = K\mu, \quad \sum_{j=1}^{K} \frac{c_{2,j}}{\mu_j} = \frac{c_2}{\mu}$$

together with the ordering (1). That means that the server 1 is the fastest one but has the highest mean standing costs. Followed by server 2 and

Table 2

Values of μ_j and $c_{2,j}$ for heterogeneous servers

Server	Idle state cost	Service rate
1	$c_{2,1} = 0.4371$	$\mu_1 = \frac{480}{49}$
2	$c_{2,2} = 0.9\ c_{2,1}$	$\mu_2 = \frac{\mu_1}{2}$
3	$c_{2,3} = 0.85\ c_{2,2}$	$\mu_3 = \frac{\mu_1}{3}$
4	$c_{2,4} = 0.8\ c_{2,3}$	$\mu_4 = \frac{\mu_1}{4}$
5	$c_{2,5} = 0.75\ c_{2,4}$	$\mu_5 = \frac{\mu_1}{5}$
6	$c_{2,6} = 0.7\ c_{2,5}$	$\mu_6 = \frac{\mu_1}{6}$

so forth. This is a quite reasonable assumption because a faster operator needs more resources. If this server is idle more costs are generated as for a slower one. In heterogeneous case the fastest free server policy is used for the allocation mechanism. If more then one operator is free and a new customer enters the queue the fastest free server will be entrusted with this task. This must be considered in the calculation of $\eta(n, k, \lambda(i), i)$. To calculate the best stationary policy the long-run average cost $\eta(k, \lambda(i))$ must be minimized for $k, 1 \leq k \leq K$. For homogeneous servers the service intensities μ_j as well as idle state costs $c_{2,j}$ in homogeneous case must be set to be equal as was discussed before.

In Figures 1 and 2 we illustrate the mean number of customers in the buffer $\bar{Q}(n, K(t), \lambda(t), t)$ in homogeneous and heterogeneous cases for different number of servers $K(t)$. The queue length for $K(t) = 4, 5, 6$ servers are not shown in these figures, since the values are very small (especially when heterogeneous servers are used). In Figure 1 and 2 the initial state $N(t) = n$ at time $t = 0$ is set to be zero. That means the queueing system is at start empty. The mean number of waiting customers, which is needed in (5), is calculated via the formula

$$\bar{Q}(n, K(i), \lambda(i), i) = \sum_{k=K(i)+1}^{N} (k - K(i))\pi_k(n, K(i), \lambda(i), i), \qquad (12)$$

where $K(i)$ is fixed in period i, k is a state at period i and N is the maximum number of customers in the system. The state probabilities in this expression are the solution of the system (9) performed by the fourth-order Runge-Kutta method for each given number of servers $K(i)$ at each period i.

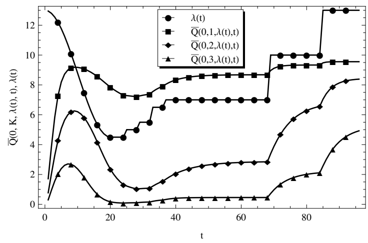

Figure 1. $\bar{Q}(n, K, \lambda(t), t)$ for homogeneous system

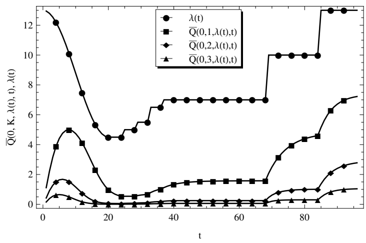

Figure 2. $\bar{Q}(n, K, \lambda(t), t)$ for heterogeneous system

Figures 3 and 4 illustrate how the mean queue length (12) differs from the stationary solution in the homogeneous and heterogeneous case which can be calculated by expression from (7). Again the mean queue length for 4, 5 and 6 servers are not shown because of the small values. The continuously plotted lines belong to the transient and the dashed lines to the stationary solution.

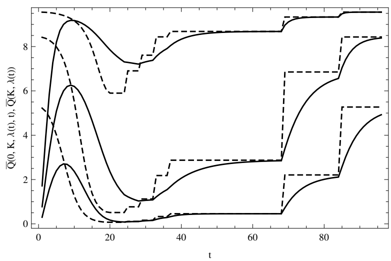

Figure 3. Transient / stationary $\bar{Q}(n, K, \lambda(t), t)$ / $\bar{Q}(K, \lambda(t))$ for system with hom. servers

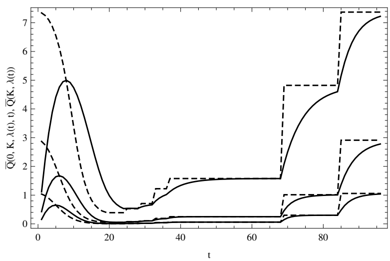

Figure 4. Transient / stationary $\bar{Q}(n, K, \lambda(t), t)$ / $\bar{Q}(K, \lambda(t))$ for system with het. servers

These pictures illustrate clearly the behaviour of the queuing system. When the transient solution is off and $\lambda(t)$ is a constant value for a certain period it converges to the stationary result as time goes by. This is not astonishing because a stationary queuing system can be interpreted as a long running transient system with a constant arrival rate.

To compare the minimum expected costs between homogeneous / heterogeneous servers in stationary / transient case simply evaluate

$\eta(n, k^*, \lambda(i), i)$ and $\eta(k^*, \lambda(i))$ for each period i and corresponding optimal number of servers k^*. The calculated values of the optimal policy are listed in Table 3. To get more server variety in the transient solutions one can increase the waiting room capacity and adjust c_1 and c_2.

Figures 5, 6, 7 and 8 show the minimum expected costs $\eta(n, K(i), \lambda(i), i)$ in homogeneous and heterogeneous case. The first one deals with a stationary case. Here the initial state n is without any significance. The second, third, fourth one is dedicated to the transient solution of the optimizing problem with initial state n at each stage equal to 0, 5 and 10. In the following pictures the stages should be interpreted severally.

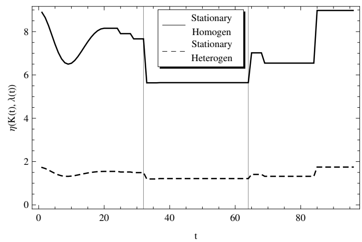

Figure 5. $\eta(K(t), \lambda(t))$ in stationary case

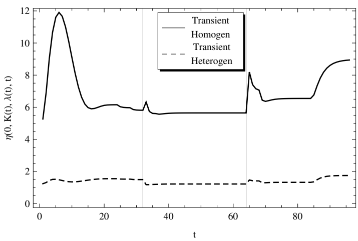

Figure 6. $\eta(n, K(t), \lambda(t), t)$ in transient case for $n = 0$

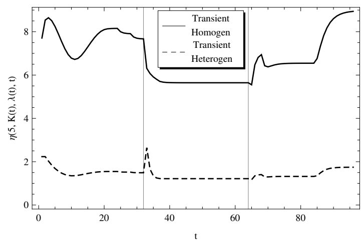

Figure 7. $\eta(n, K(t), \lambda(t), t)$ in transient case for $n = 5$

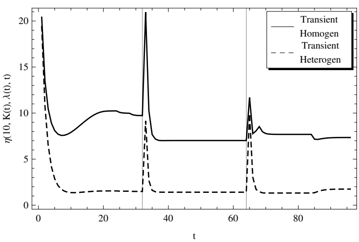

Figure 8. $\eta(n, K(t), \lambda(t), t)$ in transient case for $n = 10$

As one would expect, the queuing system with heterogeneous servers is superior in terms of running costs. A similar gap to the homogeneous costs as in Figure 6, 7 and 8 can be seen for different initial states n at the end of every stage.

The Figures 9, 10, 11, 12 deal with homogeneous and number 13, 14, 15 and 16 with heterogeneous operators. They picture the minimum expected costs $\eta(n, K(i), \lambda(i), i)$ and $\eta(K(i), \lambda(i))$ which were induced in the stationary and transient case respectively for chosen initial states.

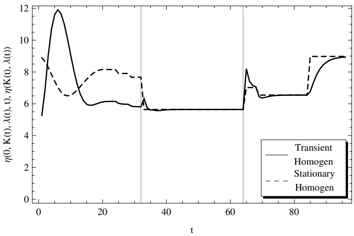

Figure 9. $\eta(n, K(t), \lambda(t), t)$ with homogeneous servers for $n = 0$

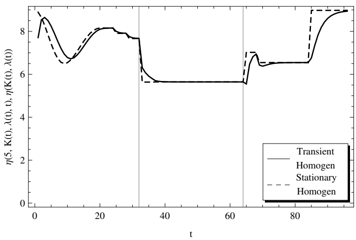

Figure 10. $\eta(n, K(t), \lambda(t), t)$ with homogeneous servers for $n = 5$

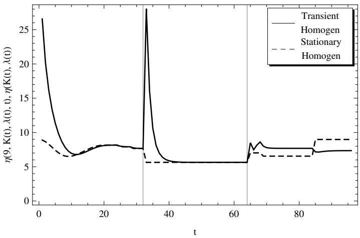

Figure 11. $\eta(n, K(t), \lambda(t), t)$ with homogeneous servers for $n = 9$

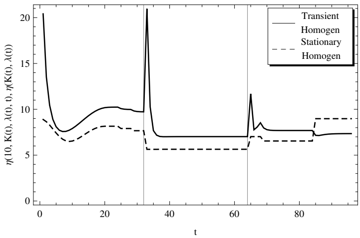

Figure 12. $\eta(n, K(t), \lambda(t), t)$ with homogeneous servers for $n = 10$

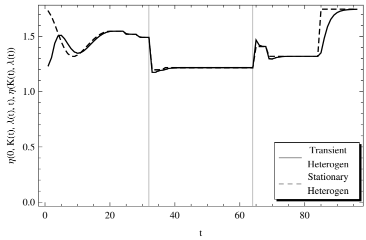

Figure 13. $\eta(n, K(t), \lambda(t), t)$ with heterogeneous servers for $n = 0$

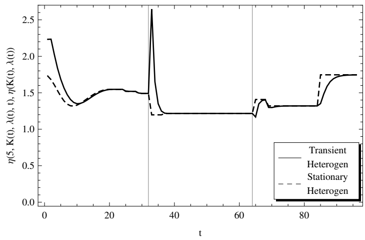

Figure 14. $\eta(n, K(t), \lambda(t), t)$ with heterogeneous servers for $n = 5$

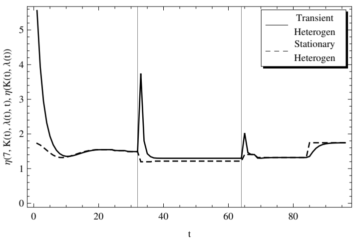

Figure 15. $\eta(n, K(t), \lambda(t), t)$ with heterogeneous servers for $n = 7$

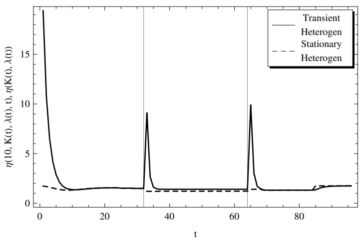

Figure 16. $\eta(n, K(t), \lambda(t), t)$ with heterogeneous servers for $n = 10$

At a close look at Figures 9 up to 16 one can see that the costs induced by the transient queuing system converges to the expenses of the stationary model if and only if the computed optimal policies for this stage are the same.

7. Conclusions

In this paper we have provided performance analysis of the Markovian controllable queueing system with a time-dependent arrival rate. The control policy prescribes the number of allowable servers and is time-dependent as well. The transient and stationary analysis for homogeneous and heterogeneous systems with preemption is provided. It is shown that the optimal policy differs in transient and stationary case. For the same control policy the corresponding cost functions take very close values. It is confirmed that the heterogeneous queueing systems are superior in performance comparing to the homogeneous case.

References

1. *S. Ghimire, R.P. Ghimire and G.B. Thapa* (2015), Performance evaluation of unreliable $M(t)/M(t)/n/n$ queueing system. British Journal of Applied Science & Technology 7(4), 412-422.
2. *M. Jung and E. S. Lee* (1989), Numerical optimization of a queueing system by dynamic programming. Journal of Mathematical Analysis and Application 141, 84-93.
3. *B.O. Koopman* (1972), Air-terminal queues under time-dependent conditions. Operations Research 20(6), 1089-1114.
4. *J.A. Schwarz, G. Selinka and R. Stolletz* (2016), Performance analysis of time-deendent queueing systems: Survey and classification. Omega. http://dx.doi.org/10.1016/j.omega.2015.10.013.
5. *A. Tirdad, W. K. Grassmann and J. Tavakoli* (2016), Optimal policies of $M(t)/M/c/c$ queues with two different levels of servers. European Journal of Operational Research 249, 1124-1130.

Table 3

Optimal policy f

Initial State at Stage One	Homogen Transient	Homogen Stationary	Heterogen Transient	Heterogen Stationary
0	4	5	6	6
1	4	5	6	6
2	5	5	6	6
3	5	5	6	6
4	5	5	6	6
5	5	5	6	6
6	5	5	6	6
7	5	5	6	6
8	5	5	6	6
9	5	5	6	6
10	6	5	6	6
Stage Two				
0	4	4	4	4
1	4	4	4	4
2	4	4	4	4
3	4	4	4	4
4	4	4	4	4
5	4	4	4	4
6	4	4	5	4
7	4	4	5	4
8	4	4	6	4
9	4	4	6	4
10	5	4	6	4
Stage Three				
0	5	5	6	6
1	5	5	6	6
2	5	5	6	6
3	5	5	6	6
4	5	5	6	6
5	5	5	6	6
6	5	5	6	6
7	5	5	6	6
8	5	5	6	6
9	6	5	6	6
10	6	5	6	6

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Построение ведомственных сетей с применением систем распознавания речи

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Аннотация. В данном докладе рассматривается один из подходов к построению корпоративной сети, который можно применить при реализации ведомственной телефонии и системы передачи данных. Некоторые предложенные решения использования систем распознавания речи. Ряд предложенных подходов опробованы при выполнении некоторых проектов построения ведомственной сети.

Ключевые слова: корпоративные сети, учрежденческие автоматические телефонные станции, системы распознавания речи.

1. Введение

Огромную роль в современном обществе играют новейшие телекоммуникационные комплексы. Поэтому при построении и развитии информационных систем важно использовать последние разработки в этой области. При создании ведомственных сетей применяют различные методы проектирования и поддерживающие их аппаратно-программные средства, основной задачей которых является организация и повышение качества современных систем учета, делопроизводства, защиты служебной информации, а также передача данных, факсимильных сообщений и телефонная связь.

В настоящее время интенсивно развиваются средства телефонной связи, благодаря внедрению разнообразных голосовых сервисов, которые включают в себя автоответчики, голосовую почту и ряд других современных функций. Новейшие учрежденческие автоматические телефонные станции (УАТС) представляют целый набор услуг обслуживания и широкие возможности тарификации. Так, например, голосовая почта позволяет экономить время пользователя, которому нет необходимости дозваниваться в течении длительного времени. В современных корпоративных сетях достаточно остро возникает необходимость создания систем, поддерживающих речевой интерфейс. Это определяется тем, что у различных компаний возрастает потребность увеличения объема голосовой информации, обусловленная спецификой взаимодействия с клиентами. Таким образом, число переговоров и объем голосовой информации может возрастать лавинообразно. Соответственно возрастает и число потребителей. Так, например, сервисные телефонные службы очень часто при возникновении аварийных ситуаций не справляются с огромной массой телефонных звонков.

Следует отметить, что ответы телефонных операторов часто не удовлетворяют запросы потребителя. Необходимо отметить, что наряду с накоплением получаемой информации, требуется ее скорейшая обработка. Чаще всего такие задачи решаются с помощью автоответчиков и перераспределения потока телефонных запросов к различным информационным службам корпораций. Как правило, обработка большого потока голосовой информации осуществляется в CALL - центрах [1].

Основополагающим элементом современного развивающегося предприятия является скоординированная работа всех частей предприятия - центрального офиса и территориально удаленных подразделений. Корпоративная сеть, как правило, является территориально-распределенной, т.е. объединяющей офисы, подразделения и другие структуры, находящиеся на значительном удалении друг от друга. В данной работе рассматривается один из подходов, который можно применить для модернизации ведомственной телефонной сети.

В последнее время требования к IT - инфраструктуре определяются потребностями бизнеса. Особенно важным является создание четкой отлаженной системы с интеграцией передачи данных и корпоративной телефонной связи. Одной из тенденций в развитии ведомственных телекоммуникационных систем стало построение мультисервисных сетей с интеграцией различных услуг. Такие сети обеспечивают взаимодействие разнородных коммуникационных подсистем в общей транспортной среде, когда передача трафика данных, голоса и видео используется единая инфраструктура. При проектировании и реализации ведомственных сетей больших корпораций необходимо уменьшать расходы как на отладку таких систем, так и на администрирование и поддержание работоспособности. Как правило, крупномасштабные ведомственные сети чаще всего представляют собой территориально-распределенные автоматизированные информационные системы, образованные совокупностью компонентов, функционирующих как в государственных органах, так и в бизнес структурах. Они включают в себя разнообразные технические решения и аппаратно-программные средства, являясь, как правило, территориально-распределенными, т.е. включающими подразделения и структуры, удаленные друг от друга. Некоторые аспекты технических решений и фрагменты ведомственных сетей рассмотрены в работах [1-3]. Ниже рассматривается один из подходов, модернизации корпоративной сети, опробованный при модернизации ведомственной интегрированной телекоммуникационной сети (ВИТС) Федеральной таможенной службы (ФТС) России, которая представляет собой корпоративную сеть большой размерности.

Сети такого класса представляют собой комплекс аппаратных и программных средств, предназначенных для обеспечения: процедур оформления документации и информационного обмена; непрерывного, устойчивого, защищенного от внешнего воздействия (рис.1).

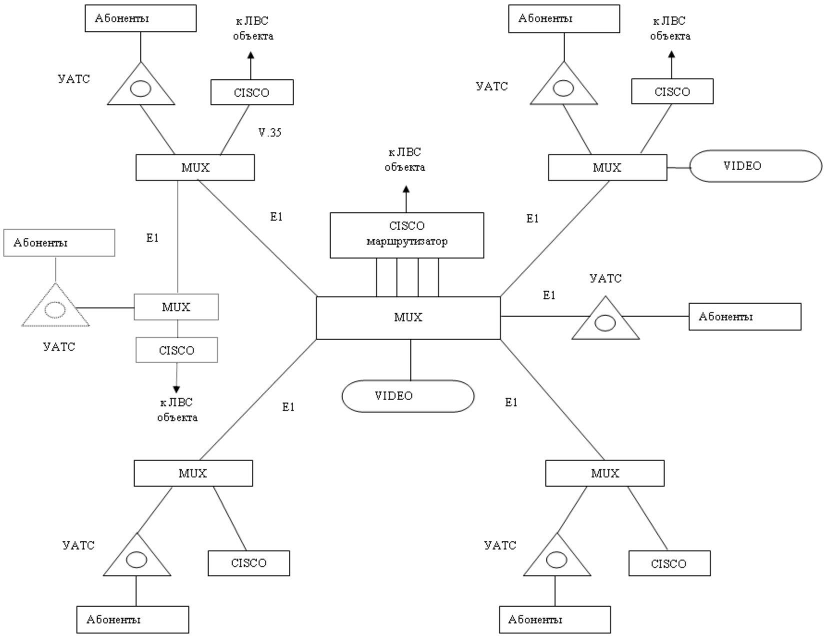

Рис. 1. Фрагмент ВИТС

Основным назначением крупномасштабных ведомственных сетей является обеспечение информационного обмена между территориально распределенными подразделениями. При этом можно выделить следующие составляющие:

— подсистему локальных вычислительных сетей (ПЛВС);
— подсистему передачи данных (ППД);
— подсистему ведомственной телефонной сети (ВТС).

С точки зрения организации информационной системы каждый узел такой ведомственной сети обеспечивает циркуляцию следующих видов информации: трафик данных; телефонный трафик, обеспечивающий голосовую и факсимильную связь; видео-трафик, в частности, видеоконференцсвязь. Более подробно остановимся на построении ППД.

2. Подсистема передачи данных

Подсистема передачи данных является, как правило, географически распределенной телекоммуникационной инфраструктурой, которая предназначена для реализации надежной, оперативной и защищенной передачи информации между подразделениями, а также обеспечения их доступа к централизованным информационным ресурсам.

Данная подсистема представляет собой ведомственную сеть передачи данных, обеспечивающую единое информационное пространство, например, для всех подразделений крупных компаний. Эта сеть располагает точками подключения - телекоммуникационными узлами (ТКУ), расположенными в подразделениях. В этих узлах выполняется сопряжение ЛВС соответствующих подразделений с подсистемой передачи данных.

Подсистема передачи данных обеспечивает транспорт информационных потоков с использованием широко распространенного сетевого протокола IP. Принцип работы подсистемы состоит в передаче IP-трафика между узлами сети наиболее оптимальным способом, определяемым с помощью специализированных протоколов динамической маршрутизации. Механизмы маршрутизации также обеспечивают резервирование в сети за счет использования альтернативных направлений в топологии.

Архитектура подсистемы передачи данных отражает структуру административно-территориального деления корпорации. Целесообразно поэтому разрабатывать систему, имеющую иерархический принцип построения. В соответствии с иерархическим принципом организации в топологии сети передачи данных выделяются два основных уровня иерархии. Верхний уровень образован межрегиональным сегментом, объединяющим центральный телекоммуникационный узел связи и узлы региональных структур. Межрегиональный сегмент представляет собой высокоскоростную транспортную среду, обеспечивающую обмен данными между подразделениями различных регионов. Нижний уровень формируется сегментами объектами непосредственного подчинения. На уровне региональных сегментов осуществляется информационное взаимодействие между объектами, входящими в структуру региональных подразделений. При этом все региональные сегменты имеют доступ к межрегиональному сегменту для взаимного информационного обмена в рамках подсистемы передачи данных. Рассмотрим методологию проектирования системы передачи данных (СПД).

3. Методология проектирования СПД в системе распознавания речи

В общем случае СПД содержит две подсети: терминальную (t) и узловую (U) (рис.2).

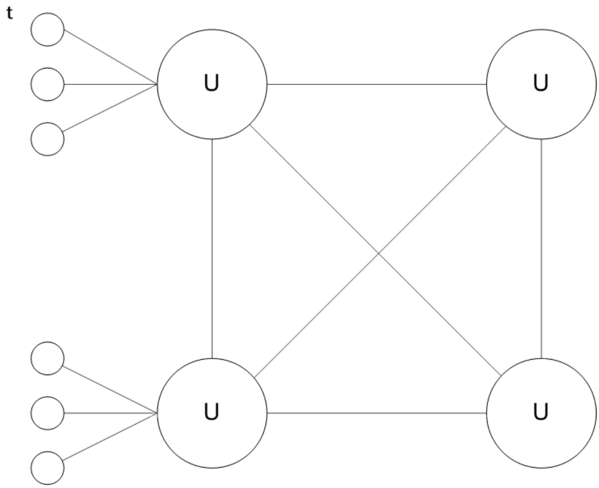

Рис. 2. Пример структуры СПД

Терминалы (t) представляют собой городскую телефонную сеть (сеть мобильных телефонов на рис.2 не показана, хотя она также работает). Рассмотрим некоторую СПД. Обычно можно считать, что топология сети задана.Задача проектирования СПД состоит в оптимизации сети. При этом параметры могут выступать либо как подлежащее определению, либо как подлежащие оптимизации, либо в качестве ограничения. Сформулируем возникающие при этом задачи оптимизации.

1. Определить пропускные способности каналов связи при заданной пропускной способности по критерию минимума среднего времени задержки сообщений.

2. Определить пропускные способности каналов связи при заданном среднем времени задержки по критерию минимума суммарной пропускной способности сети.

Напомним, что пропускная способность системы связана линейно со стоимостью системы. Поэтому, если определить пропускную способность, легко вычисляется стоимость, и наоборот.

Итак, в задаче 1 ограничением является стоимость, а в задаче 2 - время задержки передачи сообщений. К этому можно добавить , что критерии оптимизации в указанных задачах так же различные.

Перейдем к анализу оптимизационных задач.

Задача №1. Формулировка задачи была приведена выше.

Искомые пропускные способности каналов связи представлены формулой (1).

$$C_{i_{\text{опт}}} = \frac{\lambda_i}{\mu_i} + \frac{C(1-\rho)\sqrt{\frac{\lambda_i}{\mu_i}}}{\sum\limits_j \frac{\lambda_j}{\mu_j}} \tag{1}$$

где: $\rho = \frac{\sum\limits_i \frac{\lambda_i}{\mu_i}}{C}$ представляет собой средний по сети коэффициент использования каналов;

λ_i - интенсивность входного потока звуковых сообщений;

μ_i - пропускная способность канала;

С - суммарная пропускная способность СПД. Далее можно вычислить время задержки сообщений в каналах связи

$$T_i = \frac{\sum\limits_j \sqrt{\frac{\lambda_j}{\mu_j}}}{C(1-\rho)\sqrt{\lambda_i \mu_i}} \tag{2}$$

Минимальное среднее по СПД время задержки равно:

$$\overline{T}_{min} = \frac{\left(\sum\limits_i \sqrt{\frac{\lambda_i}{\mu_i}}\right)^2}{\gamma \cdot C \cdot (1-\rho)} \tag{3}$$

где γ - интенсивность входящих в сеть потоков.

Задача №2. В формулировке этой задачи параметр, подлежащий определению, тот же, что в задаче №1, а ограничение и критерий оптимизации поменялись местами. Формулы 1-3 получены, используя теорию массового обслуживания и метод множителей Лагранжа.

Подход, использованный в задаче №1, мог привести либо к чрезмерно малым, либо к чрезмерно большим временам задержки. Однако, желаемое время задержки в практике человеко-машинного диалога известно и составляет примерно 2-3 сек., и, следовательно, может использоваться как ограничение, что и сделано в задаче 2. В качестве результата в задаче №2 получим:

$$C_{i_{\text{опт}}} = \frac{\lambda_i}{\mu_i} + \frac{1}{\gamma} = \sqrt{\frac{\lambda_i}{\mu_i}} \sum_j \sqrt{\frac{\lambda_j}{\mu_j}} \tag{4}$$

Задержки времени передачи телефонных сообщений равны:

$$T_i = \frac{\gamma \overline{T}}{\mu_i \sqrt{\frac{\lambda_i}{\mu_i}} \sum\limits_j \sqrt{\frac{\lambda_j}{\mu_j}}} \tag{5}$$

В этом случае уже не время задержки, а пропускные способности могут выйти из-под контроля и принять недопустимые значения.

Особое положение занимает задача, в которой используется “minimax” критерий.

Сформулируем эту задачу №3.

“Определение пропускных способностей каналов связи при заданной суммарной пропускной способности сети по критерию минимума максимальной задержки в сети”.

Решение этой задачи, уменьшая максимальную из задержек, уравнивает все задержки, а что касается пропускной способности, то она задана. Следовательно, непредсказуемость поведения пропускной способности устраняется.

Запишем минимизируемую функцию:

$$T^{(k)} = \Big[\frac{1}{\gamma}\sum_{i=1}^{N}\lambda_i(T_i)^K\Big]^{\frac{1}{K}} \tag{6}$$

где N — количество каналов связи в сети.

При $K = 1$ получаем рассмотренный выше критерий средней задержки.

При $K = 2$ получим критерий среднеквадратичной задержки.

При $K = \infty$ имеем критерий Чебышева или минимальный критерий.

Действительно, при $K = \infty$ в выражении (6) доминируют слагаемые, соответствующие каналам с большими задержками и именно эти задержки минимизируются. В результате задержки во всех каналах оказываются одинаковыми, а все клиенты оказываются в равном положении.

Как и прежде, используя сообщения теории массового обслуживания и метод множителей Лагранжа, получим:

$$C_i^{(\infty)} = \frac{\lambda_i}{\mu_i} + \frac{1}{\mu_i}\frac{C(1-\rho)}{\sum\limits_j \frac{1}{\mu_j}} \tag{7}$$

Время задержки сообщений во всех каналах равно:

$$T_i^{(\infty)} = \frac{1}{C(1-\rho)}\sum_j \frac{1}{\mu_j} \tag{8}$$

Из (8) видно, что время задержи сообщений в сети равно:

$$\overline{T}^{(\infty)} = \frac{\overline{n}}{C(1-\rho)}\frac{N}{\mu} \tag{9}$$

Здесь $\overline{n}$ равно числу каналов связи, по которым проходит сообщение в сетях с произвольной конфигурацией:

$$\overline{n} = \frac{\sum\limits_{i} \lambda_i}{\gamma}$$

В частности, для сети с топологией типа “звезда” $\overline{n}$ =1, N равно общему числу каналов связи в сети.

Таким образом, в рамках данного доклада, мы рассмотрели некоторые теоретические аспекты проектирования СПД в системе распознавания речи.

4. Заключение

При проектировании ведомственных сетей необходимо учитывать особенности комплексной отладки [4, 5]. В процессе комплексной отладки таких иерархических целевых систем решаются сложные задачи отладки аппаратных и программных компонентов в реальном времени. Необходимо отметить, что для различных аспектов реализации территориально-распределенных ведомственных сетей кроме базовых коммуникационных функций важна дополнительная функциональность. К такой расширенной функциональности можно отнести автоматическую обработку и распределение входящих вызовов, голосовая почта, конференц-связь и др. Современные концепции построения интегрированных информационных сетей предполагает наличие голосового сервиса с системами распознавания речи, что в значительной степени позволяет развивать направление автоматического обслуживания клиентов без участия операторов.

Литература

1. Фархадов М.П., Васьковский С.В., Ревонченкова И.Ф., "Построение интеллектуальных информационных контакт - центров Автоматизация и современные технологии, 2011, №4, с.14-23
2. Васьковский С.В., "Построение современных корпоративных телефонных сетей Датчики и системы, 2009, №9, с.54-56
3. Фархадов М.П., Васьковский С.В., Смирнов В.А., Фархадова М.Э., "Проектирование системы анализа неструктурированной речевой информации Нейрокомпьютеры, 2016, №4, с.25-36
4. Васьковский С.В., Морозов В.П., Зайцев А.А., "Информационное обеспечение систем отладки электронных устройств. Датчики и системы, 2006, № 9, с.18 - 20.
5. Васьковский С.В., Морозов В.П., "Формализация процессов отладки электронных устройств Датчики и системы, 2006, № 10, с. 22 - 24.

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Sensitivity analysis of steady state reliability characteristics of a cold redundant data transmission system to the shapes of lifetime and repair time distributions of its elements

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This report describes one of the approaches to construct a corporate network. The approach can be applied to implement department telephony and data transmission systems. Some of the proposed solutions utilize speech recognition systems. Ourproposed approach was tested in some of recent private network projects.

Keywords: corporate networks, private branch exchanges, speech recognition systems.

УДК 519.218.32

Стационарные характеристики ненадежной системы массового обслуживания с марковским потоком и резервным прибором

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Аннотация. В статье рассматривается система массового массового обслуживания с двумя главными ненадежными приборами и абсолютно надежным резервным прибором. В систему поступает групповой марковкий поток заявок ($BMAP$-Batch Markovian Arrival Process). Поломки двух типов поступают на основные приборы в маркированном марковском потоке ($MMAP$ -Marked Markovian Arrival Process). Времена обслуживания и времена ремонтов имеют распределения фазового типа (PH -Phase type) distributions). Рассматривается стационарный режим функционирования системы, выводится условие существования стационарного режима и основные характеристики производительности системы.

Ключевые слова: ненадежная система массового массового обслуживания; неоднородные приборы; резервный прибор; стационарные характеристики производительности.

1. Введение

Как отмечено в [1], одним из важных направлений создания высокоскоростных и надежных сетей связи является развитие гибридных сетей связи, основанных на лазерных и радио технологиях. Вследствие большой практической значимости гибридных сетей связи вопросы, связанные с их исследованием, рассматриваются во многих работах, опубликованных в последнее время. Некоторые результаты исследований в области гибридных сетей связи изложены в статьях [2]- [4]. В большинстве работ исследуются однолинейные системы с резервным прибором. Настоящая статья является дальнейшим развитием этих исследований на случай двухлинейной системы с ненадежными неоднородными обслуживающими приборами и резервным надежным прибором, которая может использоваться при математическом моделировании гибридной сети связи, состоящей из из трех каналов: FSO (Free Space Optics) канала, миллиметроваго радиоканала и широкополосного радиоканала, функционирующего под управлением протокола IEEE 802.11 и используемого как резервный канал для передачи информации. Специфика основных каналов - FSO и миллиметрового радиоканала - состоит в том, что FSO канал не может передавать данные в условиях плохой видимости (тумана или пасмурной погоды), а миллиметровый радиоканал не может осуществлять передачу во время осадков (дождь, снег и т.д.). В случае, когда выходят из строя оба канала в условиях плохой видимости и осадков, информация передается по резервному широкополосному радиоканалу, который является абсолютно надежным, но обладает гораздо меньшей скоростью передачи по сравнению с основными каналами.

2. Математическая модель

Рассматривается система массового обслуживания с двумя ненадежными приборами, которые моделируют FSO и миллиметровый каналы гибридной сети связи и одним надежным прибором, который моделирует широкополосный радиоканал. Будем называть FSO канал как прибор 1, миллиметровый радиоканал - как прибор 2 и широкополосный радиоканал - как прибор 3.

Запросы на обслуживание поступают в $BMAP$-потоке. Это значит, что запросы могут поступать в систему в моменты скачков неприводимой цепи Маркова $\nu_t, t \geq 0$ с непрерывным временем и конечным пространством состояний $\{0, 1, \ldots, W\}$. Поведение $BMAP$-потока полностью характеризуется матричной производящей функцией $D(z) = \sum\limits_{k=0}^{\infty} D_k z^k$, $|z| < 1$. Матрица D_k характеризует интенсивности переходов процесса ν_t, сопровождающиеся генерацией группы из k запросов, $k \geq 0$. Матрица $D(1)$ является генератором процесса $\nu_t, t \geq 0$. Интенсивность поступления запросов λ определяется как $\lambda = \boldsymbol{\theta} D'(1)\mathbf{e}$, где $\boldsymbol{\theta}$ – вектор стационарного распределения процесса $\nu_t, t \geq 0$. Вектор $\boldsymbol{\theta}$ является единственным решением системы $\boldsymbol{\theta} D(1) = \mathbf{0}$, $\boldsymbol{\theta}\mathbf{e} = 1$. Более подробное описание $BMAP$ можно найти, например, в [5].

Запрос, поступающий на обслуживание, когда прибор 1 является исправным и свободным, немедленно начинает обслуживаться на этом приборе. Если этот прибор занят или находится на ремонте, запрос начинает обслуживаться на приборе 2. Если один из основных приборов находится на ремонте, а второй занят обслуживанием, то запрос становится в очередь, длина которой неограничена, и выбирается на обслуживание позже согласно стратегии FIFO (первым пришел – первым обслужен) Если оба прибора заняты в момент поступления запроса, последний становится в очередь. Если оба прибора находятся на ремонте в момент поступления запроса, он начинает обслуживаться на приборе 3.

Времена обслуживания заявок на всех приборах имеют PH (Phase type) распределения. Полагаем, что процесс обслуживания на j-ом,

$j = 1, 2, 3$, приборе имеет PH-распределение с неприводимым представлением $(\boldsymbol{\beta}_j, S_j)$ и управляющим процессом $m_t^{(j)}$, $t \geq 0$, с пространством состояний $\{1, \ldots, M_j, M_j + 1\}$, где состояние $M_j + 1$ является поглощающим. Это означает следующее. Время обслуживания интерпретируется как время, за которое цепь Маркова $m_t^{(j)}$ достигнет поглощающего состояния $M_j + 1$. Переходы цепи в пространстве состояний $\{1, \ldots, M_j\}$ задаются субгенератором S_j, а интенсивности переходов в поглощающее состояние задаются вектором $\boldsymbol{S}_0^{(j)} = -S_j\mathbf{e}$. Когда обслуживание начинается, состояние процесса $m_t^{(j)}$ выбирается из пространства состояний $\{1, \ldots, M_j\}$ на основании вероятностного вектора-строки $\boldsymbol{\beta}_j$.

Как было сказано выше, поломки приборов зависят от погодных условий. Считаем, что поломки приходят в $MMAP$ (Marked Markovian Arrival Process)-потоке с управляющим процессом η_t, $t \geq 0$, принимающем значения в множестве $\{0, 1, ..., V\}$. Этот $MMAP$ задается $(V + 1) \times (V + 1)$ матрицами H_0, H_1, H_2. Поломки (неблагоприятные погодные условия) могут быть двух типов. Поломки первого типа интерпретируются как факторы, вызывающие плохую видимость, и направляются на прибор 1. Поломки второго типа интерпретируются как осадки и направляются на прибор 2. Интенсивности поступлений поломок n-го типа задаются элементами матрицы $H_n, n = 1, 2$. Недиагональные элементы матрицы H_0 задают интенсивности перехода среды, не сопровождающиеся ухудшением видимости или осадками. Подробное описание $MMAP$ можно найти, например, в [6]. Сразу после поступления поломки на j-й прибор на нем начинается ремонт (период неблагоприятных погодных условий). Время, необходимое для ремонта j-го прибора, имеет PH распределение с неприводимым представлением $(\boldsymbol{\tau}_j, T_j)$, $j = 1, 2$. Как отмечалось выше, приход поломки вызывает прекращение обслуживания запроса (если таковое имеет место) основным прибором, на который направляется поломка. Этот прибор уходит на ремонт, в то время как другой основной прибор, если он не на ремонте и не занят, начинает обслуживание запроса заново. Если этот прибор выходит из строя, в то время как другой основной прибор еще не восстановился, то запрос переходит на резервный прибор (прибор 3), где начинает обслуживаться заново. Если во время обслуживания запроса резервным прибором какой-либо из основных приборов восстанавливается, запрос немедленно переходит на этот прибор и начинает обслуживаться заново.

2.1. Цепь Маркова, описывающая функционирование системы

Пусть в момент t

- i_t - число заявок в системе, $i_t \geq 0$,

• $n_t = 0$, если оба основных прибора исправны; $n_t = 0_j$, если оба основных прибора исправны, прибор j обслуживает запрос, а другой основной прибор свободен, $j = 1, 2$; $n_t = 1$, если прибор 1 на ремонте; $n_t = 2$, если прибор 2 на ремонте; $n_t = 3$, если оба прибора на ремонте;

• $m_t^{(j)}$ - состояние управляющего процесса PH - обслуживания на j-м занятом приборе, $j = 1, 2, 3, m_t^{(j)} = \overline{1, M_j}$;

• $r_t^{(j)}$ - состояние управляющего процесса PH -времени ремонта на приборе $j = 1, 2$, $r_t^{(j)} = \overline{1, R_j}$;

• ν_t и η_t - состояния управляющих процессов входящего $BMAP$ потока и $MMAP$ потока поломок соответственно, $\nu_t = \overline{0, W}$, $\eta_t = \overline{0, V}$.

Процесс функционирования системы описывается неприводимой цепью Маркова ξ_t, $t \geq 0$, с пространством состояний

$$\{(0, n, \nu, \eta),\ i = 0, n = \overline{0, 3},\ \nu = \overline{0, W},\ \eta = \overline{0, V}\} \bigcup$$
$$\{(i, 0_j, \nu, \eta, m^{(j)}),\ i = 1, j = 1, 2, n = 0_j, \nu = \overline{0, W}, \eta = \overline{0, V},\ m^{(j)} = \overline{1, M_j}\}$$
$$\bigcup \{(i, 0, \nu, \eta, m^{(1)}, m^{(2)}),\ i > 1, n = 0, \nu = \overline{0, W}, \eta = \overline{0, V},\ m^{(1)} = \overline{1, M_1},$$
$$m^{(2)} = \overline{1, M_2}\} \bigcup \{(i, 1, \nu, \eta, m^{(2)}, r^{(1)}),\ i \geq 1, n = 1,\ \nu = \overline{0, W}, \eta = \overline{0, V},$$
$$m^{(2)} = \overline{1, M_2}, r^{(1)} = \overline{1, R_1}\} \bigcup \{(i, 2, \nu, \eta, m^{(1)}, r^{(2)}),\ i \geq 1, n = 2,\ \nu = \overline{0, W},$$
$$\eta = \overline{0, V}, m^{(1)} = \overline{1, M_1}, r^{(1)} = \overline{1, R_2}\} \bigcup \{(i, 3, \nu, \eta, m^{(3)}, r^{(1)}, r^{(2)}),\ i > 0,$$
$$n = 3,\ \nu = \overline{0, W}, \eta = \overline{0, V}, m^{(3)} = \overline{1, M_3}, r^{(j)} = \overline{1, R_j}, j = 1, 2\}.$$

Далее будем предполагать, что состояния цепи $\xi_t, t \geq 0$, внутри каждого из приведенных подмножеств упорядочены в лексикографическом порядке и таким образом упорядоченные подмножества упорядочены в том порядке, в котором они перечислены выше. Обозначим через $Q_{i,j}$ матрицу интенсивностей переходов цепи из состояний, соответствующих значению i первой (счетной) компоненты, в состояния, соответствующие значению j этой компоненты, $i, j \geq 0$.

Лемма 1. Инфинитезимальный генератор Q цепи Маркова ξ_t, $t \geq 0$, имеет блочную структуру

$$Q = \begin{pmatrix} Q_{0,0} & Q_{0,1} & Q_{0,2} & Q_{0,3} & Q_{0,4} \cdots \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & Q_{1,3} & Q_{1,4} \cdots \\ O & Q_{2,1} & Q_1 & Q_2 & Q_3 \cdots \\ O & O & Q_0 & Q_1 & Q_2 \cdots \\ O & O & O & Q_0 & Q_1 \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

где ненулевые блоки $Q_{i,j}$ записываются в терминах матриц, описывающих интенсивности входящего потока запросов, потока поломок, интенсивности обслуживания и ремонтов.

Как следует их Леммы 1, генератор Q имеет блочную верхне-Хессен-бергову структуру и блоки $Q_{i,j}$ при $i > 2$ зависят от величин i и j только через разность $j - i$. Тогда, согласно определению, данному в [9], можно утверждать, что

Следствие 1. Цепь Маркова $\xi_t, t \geq 0$, принадлежит классу многомерных квазитеплицевых цепей с непрерывным временем (КТЦМ).

Результаты, полученные в [9] для КТЦМ, были использованы для анализа стационарного режима рассматриваемой системы.

3. Стационарное распределение. Характеристики производительности

Теорема 1. Цепь Маркова ξ_t эргодична тогда и только тогда, когда выполняется неравенство

$$\lambda < -\boldsymbol{\pi}_0(S_1 \oplus S_2)\mathbf{e} + \boldsymbol{\pi}_1\mathbf{S}_0^{(2)} + \boldsymbol{\pi}_2\mathbf{S}_0^{(1)} + \boldsymbol{\pi}_3\mathbf{S}_0^{(3)}, \tag{1}$$

где $\boldsymbol{\pi}_0 = \mathbf{x}_0(\mathbf{e}_{\bar{V}} \otimes I_{M_1 M_2})$, $\boldsymbol{\pi}_1 = \mathbf{x}_1(\mathbf{e}_{\bar{V}} \otimes I_{M_2} \otimes \mathbf{e}_{R_1})$, $\boldsymbol{\pi}_2 = \mathbf{x}_2(\mathbf{e}_{\bar{V}} \otimes I_{M_1} \otimes \mathbf{e}_{R_2})$, $\boldsymbol{\pi}_3 = \mathbf{x}_3(\mathbf{e}_{\bar{V}} \otimes I_{M_3} \otimes \mathbf{e}_{R_1 R_2})$, а вектор $\mathbf{x} = (\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ является единственным решением системы линейных алгебраических уравнений, коэффициенты которой определяются в терминах интенсивностей входящего потока запросов, потока поломок, интенсивностей обслуживания и ремонтов.

Замечание 1. При физической интерпретации условия эргодичности (1) учитываем, что данное условие отражает процесс обслуживания в системе в условиях перегрузки. Рассмотрим физический смысл первого слагаемого в правой части неравенства (1). Компонента $\boldsymbol{\pi}_0(m^{(1)}, m^{(2)})$ вектора-строки $\boldsymbol{\pi}_0$ есть вероятность того, что приборы 1 и 2 исправны и обслуживают запросы на фазах $m^{(1)}$ и $m^{(2)}$ соответственно. Соответствующая компонента вектора-столбца $(\mathbf{S}_0^{(1)} \oplus \mathbf{S}_0^{(2)})\mathbf{e}$ есть суммарная интенсивность обслуживания запросов 1-м и 2-м приборами при условии, что обслуживание на этих приборах находится в фазах $m^{(1)}$ и $m^{(2)}$ соответственно. Тогда произведение $\boldsymbol{\pi}_0(\mathbf{S}_0^{(1)} \oplus \mathbf{S}_0^{(2)})\mathbf{e}$ представляет собой интенсивность выходящего потока в периоды, когда запросы обслуживаются 1-м и 2-м приборами. Аналогично трактуются остальные слагаемые суммы в правой части неравенства (8.4): второе слагаемое есть интенсивность выходящего потока при обслуживании запросов 2-м прибором (1-й прибор находится на ремонте), третье слагаемое -интенсивность выходящего потока при обслуживании запросов 1-м прибором (2-й прибор находится на ремонте), четвертое слагаемое -интенсивность выходящего потока при обслуживании запросов 3-м прибором (1-й и 2-й приборы находятся на ремонте).

Тогда правая часть неравенства (1) выражает суммарную интенсивность выходящего потока запросов в условиях перегрузки. Очевидно, что для существования стационарного режима в системе необходимо и достаточно, чтобы интенсивность входного потока λ была меньше интенсивности интенсивность выходящего потока.

Далее будем считать, что неравенство (1) выполняется. Обозначим через $\mathbf{p}_i$ вектор-строку стационарных вероятностей, соответствующих значению i счетной компоненты цепи Маркова ξ_t, $i \geq 0$. Для вычисления векторов $\mathbf{p}_i$, $i \geq 0$, мы использовали численно устойчивый алгоритм (см. [9]), который был разработан для вычисления стационарного распределения КТЦМ. Вычислив стационарное распределение цепи ξ_t, можно найти ряд важных характеристик производительности рассматриваемой системы. Формулы для вычисления некоторых характеристик приведены ниже.

- Пропускная способность системы (максимальное значение интенсивности потока, который может быть пропущен через систему)

$$\varrho = -\boldsymbol{\pi}_0(S_1 \oplus S_2)\mathbf{e} + \boldsymbol{\pi}_1\mathbf{S}_0^{(2)} + \boldsymbol{\pi}_2\mathbf{S}_0^{(1)} + \boldsymbol{\pi}_3\mathbf{S}_0^{(3)}.$$

- Вероятность того, что с системе находится i запросов $p_i = \mathbf{p}_i\mathbf{e}$.
- Среднее число заявок системе $L = \sum\limits_{i=1}^{\infty} ip_i$.
- Дисперсия числа заявок в системе $V = \sum\limits_{i=1}^{\infty} i^2 p_i - L^2$.
- Вероятность $P_i^{(0)}$ того, что в системе находится i заявок и оба прибора исправны

$$P_0^{(0)} = \mathbf{p}_0 \begin{pmatrix} \mathbf{e}_a \\ \mathbf{0}_{a(R_1+R_2+R_1R_2)} \end{pmatrix},$$

$$P_1^{(0)} = \mathbf{p}_1 \begin{pmatrix} \mathbf{e}_{a(M_1+M_2)} \\ \mathbf{0}_{a(M_2R_1+M_1R_2+M_3R_1R_2)} \end{pmatrix},$$

$$P_i^{(0)} = \mathbf{p}_i \begin{pmatrix} \mathbf{e}_{aM_1M_2} \\ \mathbf{0}_{a(M_2R_1+M_1R_2+M_3R_1R_2} \end{pmatrix}, \ i \geq 2.$$

где $a = (W+1)(V+1)$.

- Вероятность $P_i^{(1)}(P_i^{(2)})$ того, что в системе находится i заявок и только прибор 1 (прибор 2) на ремонте

$$P_0^{(1)} = \mathbf{p}_0 \begin{pmatrix} \mathbf{0}_a \\ \mathbf{e}_{aR_1} \\ \mathbf{0}_{aR_2} \\ \mathbf{0}_{aR_1R_2} \end{pmatrix}, \quad P_0^{(2)} = \mathbf{p}_0 \begin{pmatrix} \mathbf{0}_{a(1+R_1)} \\ \mathbf{e}_{aR_2} \\ \mathbf{0}_{aR_1R_2} \end{pmatrix}.$$

$$P_1^{(1)} = \mathbf{p}_1 \begin{pmatrix} \mathbf{0}_{a(M_1+M_2)} \\ \mathbf{e}_{aM_2R_1} \\ \mathbf{0}_{aM_1R_2} \\ \mathbf{0}_{aM_3R_1R_2} \end{pmatrix}, \quad P_0^{(2)} = \mathbf{p}_1 \begin{pmatrix} \mathbf{0}_{a(M_1+M_1+M_2R_1)} \\ \mathbf{e}_{aM_1R_2} \\ \mathbf{0}_{aM_3R_1R_2} \end{pmatrix}.$$

$$P_i^{(1)} = \mathbf{p}_i \begin{pmatrix} \mathbf{0}_{aM_1M_2} \\ \mathbf{e}_{aM_2R_1} \\ \mathbf{0}_{aM_1R_2} \\ \mathbf{0}_{aM_3R_1R_2} \end{pmatrix}, \quad P_0^{(2)} = \mathbf{p}_0 \begin{pmatrix} \mathbf{0}_{a(M_1M_2+M_2R_1)} \\ \mathbf{e}_{aM_1R_2} \\ \mathbf{0}_{aM_3R_1R_2} \end{pmatrix}, \ i \geq 2.$$

- Вероятность $P_i^{(3)}$ того, что в системе находится i заявок и оба основных прибора на ремонте

$$P_0^{(3)} = \mathbf{p}_0 \begin{pmatrix} \mathbf{0}_{a(1+R_1+R_2)} \\ \mathbf{e}_{aR_1R_2} \end{pmatrix}, \quad P_1^{(3)} = \mathbf{p}_1 \begin{pmatrix} \mathbf{0}_{a(M_1+M_2+M_2R_1+M_1R_2)} \\ \mathbf{e}_{aM_3R_1R_2} \end{pmatrix},$$

$$P_i^{(3)} = \mathbf{p}_i \begin{pmatrix} \mathbf{0}_{a(M_1M_2+M_2R_1+M_1R_2)} \\ \mathbf{e}_{aM_3R_1R_2} \end{pmatrix}, \ i \geq 2.$$

- Вероятность того, что в произвольный момент времени приборы находятся в состоянии n $\quad P^{(n)} = \sum\limits_{i=0}^{\infty} P_i^{(n)}$, $n = \overline{0,3}$.

4. Заключение

Рассмотрена система массового обслуживания, состоящая из двух основных неоднородных приборов и одного резервного прибора. Основные приборы являются высокоскоростными, но ненадежными. Резервный прибор - низкоскоростной, но абсолютно надежный. Находясь в исправном состоянии, основные приборы обслуживают запросы. Когда оба этих прибора находятся на ремонте, запросы обслуживаются резервным прибором. При достаточно общих предположениях относительно потоков запросов и поломок и распределений времен обслуживания и ремонтов поведение системы описано в терминах многомерной квазитеплицевой цепи Маркова. Получено условие существования стационарного режима в системе и ряд важных характеристик ее производительности. Результаты могут быть использованы при проектировании гибридных сетей связи.

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Литература

1. *Vishnevsky V., Kozyrev D., Semenova O.* Redundant queueing system with unreliable servers. // Proc. of the 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops. Moscow. 2014. P. 383-386.
2. *Arnon S., Barry J., Karagiannidis G., Schober R., Uysal M.* Advanced Optical Wireless Communication Systems. — Cambridge University Press, Cambridge, 2012.
3. *Vishnevsky V.M., Semenova O.V., Sharov S.Yu.* Modeling and analysis of a hybrid communication channel based on free-space optical and radio-frequency technologies. Automation and Remote Control. 2013. —Vol. 72. —P. 345-352.
4. *Sharov S.Yu., Semenova O.V.*Simulation model of wireless channel based on FSO and RF technologies. Distributed Computer and Communication Networks. Theory and Applications. 2010. —P. 368-374.
5. *Lucantoni D.M.* New results on the single server queue with a batch Markovian arrival process. Communications in Statistics-Stochastic Models. 1991. —Vol. 7. —P. 1-46.
6. *He Q.M.* Queues with marked calls. Advances in Applied Probability. 1996. — Vol. 28. —P. 567-587.
7. *Neuts M.* Matrix-geometric Solutions in Stochastic Models – An Algorithmic Approach. — Baltimore: Johns Hopkins University Press, 1981.
8. *Graham A.* Kronecker Products and Matrix Calculus with Applications. —Cichester: Ellis Horwood, 1981.
9. *Klimenok V.I., Dudin A.N.* Multi-dimensional asymptotically quasi-Toeplitz Markov chains and their application in queueing theory. Queueing Systems. —2006. —Vol. 54. —P. 245-259.

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On the queue length in the discrete cyclic-waiting system of $Geo/G/1$ type

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Abstract. We consider a discrete time queueing system with geometrically distributed interarrival and general service times, with FCFS service discipline. The service of a customer is started at the moment of arrival (in case of free system) or at moments differing from it by the multiples of a given cycle time T (in case of occupied server or waiting queue). Earlier we investigated such system from the viewpoint of waiting time, actually we deal with the number of present customers. The functioning is described by means of an embedded Markov chain considering the system at moments just before starting the services of customers. We find the transition probabilities, the generating function of ergodic distribution and the stability condition. The model may be used to describe the transmission of optical signals.

Keywords: Queue length, discrete cyclic-waiting system, $Geo/G/1$.

1. Introduction

This paper continues the investigation of a single-server queueing system where an entering customer might be accepted for service at the moment of arrival or at moments differing from it by the multiples of a given cycle time T. As described in [4] such problem was motivated by the transmission of optical signals: optical signals enter a node and they should be transmitted according to the FCFS rule. The information cannot be stored, if it cannot be served at once is sent to a delay line and returns to the node after having passed it. So the signal can be transmitted at the moment of its arrival or at moments that differ from it by the multiples of time required to pass the delay line. The original problem had been raised in connection with the landing of airplanes, later it appeared to be an exact model for the transmission of optical signals where because of the lack of optical RAM the fiber delay lines are used.

First this system was considered from the viewpoint of number of present customers in the case of Poisson arrivals and exponentially distributed service time distribution [3]. By using Koba's results [1, 2] in [4] we investigated the distribution of waiting time for the continuous time model. [5] solved this problem for the discrete time case if the service time had geometrical distribution. Finally, [6] considered the waiting time problem in the case of general discrete service time distribution.

In this paper we investigate the distribution of queue length for the discrete system with geometrical interarrival and general service time distribution.

2. The theorem

We investigate a service system where the service may start at the moment of arrival (if the system is free) or at moments differing from it by the multiples of a given cycle time T (in the case of busy server or waiting queue). The service is realized according to the FCFS discipline. The service process is not continuous: during the "busy period" there are idle intervals required to reach the starting position, during them there is no real service.

Let the service of the nth customer begin at t_n, and let us consider the number of customers at moment just before the service begins. Then the number of customers is determined by the recursive formula

$$N_{t_{n+1}-0} = \begin{cases} \Delta_n - 1, & \text{if } N_{t_n-0} = 0, \\ \\ N_{t_n-0} - 1 + \Delta_n, & \text{if } N_{t_n-0} > 0, \end{cases}$$

where Δ_n is the number of customers arriving at the system for $[t_n, t_{n+1})$. In [3] we showed that these values form a Markov chain.

Theorem. *Let us consider a discrete queueing system in which the interarrival time has geometrical distribution with parameter r, the service time has general distribution with probabilities q_i $(i = 1, 2, \ldots)$. The service of a customer may start upon arrival or (in case of busy server or waiting queue) at moments differing from it by the multiples of a given cycle time T (equal to n time units) according to the FCFS discipline. Let us define an embedded Markov chain whose states correspond to the number of customers in the system at moments $t_k - 0$, where t_k is the moment of beginning of service of the k-th one. The matrix of transition probabilities has the form*

$$\begin{bmatrix} a_0 & a_1 & a_2 & a_3 & \ldots \\ a_0 & a_1 & a_2 & a_3 & \ldots \\ 0 & b_0 & b_1 & b_2 & \ldots \\ 0 & 0 & b_0 & b_1 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

its elements are determined by the generating functions

$$A(z) = \sum_{i=0}^{\infty} a_i z^i = Q_1 + z\frac{r}{1-r}Q_1 + z\sum_{k=1}^{\infty}(1 - r + rz)^{kn} \times$$

$$\times \left\{ \sum_{i=(k-1)n+2}^{kn+1} q_i + \sum_{i=kn+2}^{\infty} q_i(1-r)^{i-kn-1} - \sum_{i=(k-1)n+2}^{\infty} q_i(1-r)^{i-(k-1)n-1} \right\},$$

$$Q_k = \sum_{i=k}^{\infty} q_i(1-r)^i;$$

$$B(z) = \sum_{i=0}^{\infty} b_i z^i = \sum_{k=0}^{\infty} \sum_{j=1}^{n} q_{kn+j}(1-r+rz)^{kn+j} \times$$

$$\times \left\{ \frac{r}{1-(1-r)^n} \frac{1-(1-r)^{j-1}(1-r+rz)^{j-1}}{1-(1-r)(1-r+rz)} (1-r+rz)^{n-j+1} + \right.$$

$$\left. + \frac{r(1-r)^{j-1}}{1-(1-r)^n} \frac{1-(1-r)^{n-j+1}(1-r+rz)^{n-j+1}}{1-(1-r)(1-r+rz)} \right\}.$$

The generating function of ergodic distribution $P(z) = \sum\limits_{i=0}^{\infty} p_i z^i$ *has the form*

$$P(z) = \frac{p_0[zA(z)-B(z)] + p_1 z[A(z)-B(z)]}{z-B(z)},$$

where

$$p_1 = \frac{1-a_0}{a_0} p_0,$$

$$p_0 = \frac{a_0[1-B'(1)]}{a_0 + A'(1) - B'(1)}.$$

The ergodicity condition is

$$\sum_{i=1}^{\infty} q_i \left\lceil \frac{i}{n} \right\rceil < \frac{1}{1-(1-r)^n} \sum_{i=1}^{\infty} q_i (1-r)^{i-1 \pmod{n}}.$$

3. The proof of Theorem

The matrix of transition probabilities is given in the theorem, the generating functions of transition probabilities are found in the following section. Denote the ergodic probabilities by p_i $(i = 0, 1, \ldots)$ and introduce the generating function $P(z) = \sum\limits_{i=0}^{\infty} p_i z^i$. We have

$$p_j = p_0 a_j + p_1 a_j + \sum_{i=2}^{j+1} p_i b_{j-i+1} \qquad (j \geq 1),$$

$$p_0 = p_0 a_0 + p_1 a_0.$$

By using these equations we obtain the generating function

$$P(z) = \frac{p_0 \left[zA(z) - B(z)\right] + p_1 z \left[A(z) - B(z)\right]}{z - B(z)}.$$

This expression includes two unknown probabilities p_0 and p_1 from the desired distribution, but p_1 can be expressed via p_0,

$$p_1 = \frac{1 - a_0}{a_0} p_0,$$

and p_0 can be found from the condition $P(1) = 1$, i.e.

$$p_0 = \frac{a_0[1 - B'(1)]}{a_0 + A'(1) - B'(1)}.$$

By using the corresponding values we obtain

$$a_0 + A'(1) - B'(1) =$$

$$= \frac{1}{1 - (1 - r)^n} \sum_{i=1}^{\infty} q_i (1 - r)^{i-1 \pmod{n}} (1 - r)^{\lceil \frac{i}{n} \rceil n} > 0.$$

Consequently, the numerator must be positive, too; so the condition

$$1 - B'(1) > 0$$

must be fulfilled. This leads to the ergodicity condition $B'(1) < 1$, i.e.

$$\sum_{k=0}^{\infty} \sum_{j=1}^{n} q_{kn+j} [1 + (k + 1)nr] - \frac{nr}{1 - (1 - r)^n} \sum_{k=0}^{\infty} \sum_{j=1}^{n} q_{kn+j} (1 - r)^{j-1} < 1,$$

which can be written in the form as it appears in the theorem.

4. The generating functions of transition probabilities

Concerning the transition probabilities we have to distinguish two cases: at the moment when the service of a customer begins the next one is present or not. First we find the generating function $A(z)$ corresponding to the case when the next customer is not there yet, then we find the generating function $B(z)$ for the case when the next customer is present, too.

4.1. The generating function $A(z)$

This possibility appears at the states 0 and 1. Assume that the service time of first customer is equal to u, the second customer appears v time after starting its service. The probability of event $\{u-v=\ell\}$ is

$$P\{u-v=\ell\} = \sum_{k=\ell+1}^{\infty} q_k(1-r)^{k-\ell-1}r \qquad (\ell=1,2,\ldots).$$

We are interested in the number of customers appearing during intervals whose lengths are the multiples of n, i.e. $[(i-1)n+1, in]$. The generating functions are represented by the following tables: if ℓ changes from 1 till n (we will not write the factors $r(1-r+rz)^n$)

$$\begin{array}{llllll}
q_2 & q_3(1-r) & q_4(1-r)^2 & \ldots & q_n(1-r)^{n-2} & q_{n+1}(1-r)^{n-1} & \ldots\\
 & q_3 & q_4(1-r) & \ldots & q_n(1-r)^{n-3} & q_{n+1}(1-r)^{n-2} & \ldots\\
 & & q_4 & \ldots & q_n(1-r)^{n-4} & q_{n+1}(1-r)^{n-3} & \ldots\\
 & & & & \vdots & \vdots & \\
 & & & & q_n(1-r) & q_{n+1}(1-r)^2 & \ldots\\
 & & & & q_n & q_{n+1}(1-r) & \ldots\\
 & & & & & q_{n+1} & \ldots
\end{array}$$

for the following columns it is continued as

$$\begin{array}{lllll}
q_{n+2}(1-r)^n & q_{n+3}(1-r)^{n+1} & \ldots & q_{2n}(1-r)^{2n-2} & \ldots\\
q_{n+2}(1-r)^{n-1} & q_{n+3}(1-r)^{n} & \ldots & q_{2n}(1-r)^{2n-3} & \ldots\\
q_{n+2}(1-r)^{n-2} & q_{n+3}(1-r)^{n-1} & \ldots & q_{2n}(1-r)^{2n-4} & \ldots\\
\vdots & \vdots & & \vdots & \\
q_{n+2}(1-r)^3 & q_{n+3}(1-r)^4 & \ldots & q_{2n}(1-r)^{n+1} & \ldots\\
q_{n+2}(1-r)^2 & q_{n+3}(1-r)^3 & \ldots & q_{2n}(1-r)^{n} & \ldots\\
q_{n+2}(1-r) & q_{n+3}(1-r)^2 & \ldots & q_{2n}(1-r)^{n-1} & \ldots
\end{array}$$

etc., if ℓ changes from $n+1$ till $2n$ (we omit the factor $r(1-r+rz)^{2n}$)

$$\begin{array}{llllll}
q_{n+2} & q_{n+3}(1-r) & \dots & q_{2n}(1-r)^{n-2} & q_{2n+1}(1-r)^{n-1} & \dots \\
 & q_{n+3} & \dots & q_{2n}(1-r)^{n-3} & q_{2n+1}(1-r)^{n-2} & \dots \\
 & & \dots & q_{2n}(1-r)^{n-4} & q_{2n+1}(1-r)^{n-3} & \dots \\
 & & & \vdots & \vdots & \\
 & & & q_{2n}(1-r) & q_{2n+1}(1-r)^{2} & \dots \\
 & & & q_{2n} & q_{2n+1}(1-r) & \dots \\
 & & & & q_{2n+1} & \dots
\end{array}$$

etc.

Summing up the elements of columns for $r(1-r+rz)^{n}$, $r(1-r+rz)^{2n}$, $r(1-r+rz)^{3n}$, ... we get the coefficients

$$\begin{array}{l}
\sum\limits_{i=2}^{n+1} q_i \frac{1-(1-r)^{i-1}}{1-(1-r)} + \sum\limits_{i=n+2}^{\infty} q_i(1-r)^{i-n-1} \frac{1-(1-r)^{n}}{1-(1-r)}, \\
\sum\limits_{i=n+2}^{2n+1} q_i \frac{1-(1-r)^{i-n-1}}{1-(1-r)} + \sum\limits_{i=2n+2}^{\infty} q_i(1-r)^{i-2n-1} \frac{1-(1-r)^{n}}{1-(1-r)}, \\
\sum\limits_{i=2n+2}^{3n+1} q_i \frac{1-(1-r)^{i-2n-1}}{1-(1-r)} + \sum\limits_{i=3n+2}^{\infty} q_i(1-r)^{i-3n-1} \frac{1-(1-r)^{n}}{1-(1-r)}, \dots
\end{array}$$

and, in general, for $r(1-r+rz)^{kn}$

$$\sum_{i=(k-1)+2}^{kn+1} q_i \frac{1-(1-r)^{i-(k-1)n-1}}{1-(1-r)} + \sum_{i=kn+2}^{\infty} q_i(1-r)^{i-kn-1} \frac{1-(1-r)^{n}}{1-(1-r)},$$

which canceling r is

$$\sum_{i=(k-1)n+2}^{kn+1} q_i + \sum_{i=kn+2}^{\infty} q_i(1-r)^{i-kn-1} - \sum_{i=(k-1)n+2}^{\infty} q_i(1-r)^{i-(k-1)n-1}.$$

Taking into account that the probability of event during the service of a customer a new one does not arrive

$$\sum_{i=1}^{\infty} q_i(1-r)^{i} = Q_1 = a_0,$$

and the probability of zero waiting time is

$$\sum_{i=1}^{\infty} q_i(1-r)^{i-1} r = \frac{r}{1-r} Q_1$$

we obtain the generating function $A(z)$

$$A(z)=\sum_{i=0}^{\infty}a_iz^i=Q_1+z\frac{r}{1-r}Q_1+z\sum_{k=1}^{\infty}(1-r+rz)^{kn}\times$$

$$\times\left\{\sum_{i=(k-1)n+2}^{kn+1}q_i+\sum_{i=kn+2}^{\infty}q_i(1-r)^{i-kn-1}-\sum_{i=(k-1)n+2}^{\infty}q_i(1-r)^{i-(k-1)n-1}\right\}.$$

Its derivative at $z=1$ gives

$$A'(1)=\frac{rQ_1}{1-r}+\sum_{k=1}^{\infty}\sum_{i=(k-1)n+2}^{kn+1}q_i-\frac{Q_2}{1-r}+$$

$$+nr\sum_{k=1}^{\infty}k\sum_{i=(k-1)n+2}^{kn+1}q_i-nr\sum_{k=1}^{\infty}\sum_{i=(k-1)n+2}^{\infty}q_i(1-r)^{i-(k-1)n-1}.$$

4.2. The generating function $B(z)$

At the beginning of service of first customer the second customer is present, too. Let $x=u-\left[\frac{u-1}{n}\right]n$ ($[x]$ denotes the integer part of x), and let y be the mod T interarrival time ($1\le y\le n$). The time elapsed between the starting moments of two successive customers is

$$\left[\frac{u-1}{n}\right]n+y\quad\text{if}\quad x\le y\quad\text{and}\quad\left(\left[\frac{u-1}{n}\right]+1\right)n+y\quad\text{if}\quad x>y.$$

One can easily see that y has truncated geometrical distribution with probabilities

$$P\{y=\ell\}=\frac{(1-r)^{\ell-1}r}{1-(1-r)^n}\qquad(\ell=1,2,\dots,n),$$

the generating function of entering customer for a time slice is $1-r+rz$. The generating functions of entering customers depending on the service time and the mod T interarrival time are given in the tables (the rows correspond to the mod T interarrival and the columns to the service times):

$$\begin{array}{llll}
q_1(1-r+rz) & q_2(1-r+rz)^{n+1} & \dots & q_n(1-r+rz)^{n+1} \\
q_1(1-r+rz)^2 & q_2(1-r+rz)^2 & \dots & q_n(1-r+rz)^{n+2} \\
q_1(1-r+rz)^3 & q_2(1-r+rz)^3 & \dots & q_n(1-r+rz)^{n+3} \\
\vdots & \vdots & & \vdots \\
q_1(1-r+rz)^{n-1} & q_2(1-r+rz)^{n-1} & \dots & q_n(1-r+rz)^{2n-1} \\
q_1(1-r+rz)^n & q_2(1-r+rz)^n & \dots & q_n(1-r+rz)^n
\end{array}$$

the following n columns

$$\begin{array}{llll}
q_{n+1}(1-r+rz)^{n+1} & q_{n+2}(1-r+rz)^{2n+1} & \dots & q_{2n}(1-r+rz)^{2n+1} \\
q_{n+1}(1-r+rz)^{n+2} & q_{n+2}(1-r+rz)^{n+2} & \dots & q_{2n}(1-r+rz)^{2n+2} \\
q_{n+1}(1-r+rz)^{n+3} & q_{n+2}(1-r+rz)^{n+3} & \dots & q_{2n}(1-r+rz)^{2n+3} \\
\vdots & \vdots & & \vdots \\
q_{n+1}(1-r+rz)^{2n-1} & q_{n+2}(1-r+rz)^{2n-1} & \dots & q_{2n}(1-r+rz)^{3n-1} \\
q_{n+1}(1-r+rz)^{2n} & q_{n+2}(1-r+rz)^{2n} & \dots & q_{2n}(1-r+rz)^{2n}
\end{array}$$

etc. Summing up the elements in the columns, then considering these sums shifted by n (i.e. the sums of columns corresponding to the service times $q_j,\ q_{n+j},\ q_{2n+j},\ \dots\ (1 \le j \le n)$) for a concrete deviation j the generating function equals

$$\left\{\frac{r}{1-(1-r)^n}\frac{1-(1-r)^{j-1}(1-r+rz)^{j-1}}{1-(1-r)(1-r+rz)}(1-r+rz)^{n-j+1}+\right.$$

$$\left.+\frac{r(1-r)^{j-1}}{1-(1-r)^n}\frac{1-(1-r)^{n-j+1}(1-r+rz)^{n-j+1}}{1-(1-r)(1-r+rz)}\right\}\sum_{k=0}^{\infty}q_{kn+j}(1-r+rz)^{kn+j},$$

so summing up by j we obtain $B(z)$.

Its derivative at $z=1$ is

$$B'(1)=\sum_{k=0}^{\infty}\sum_{j=1}^{n}q_{kn+j}[(k+1)nr+1]-\frac{nr}{1-(1-r)^n}\sum_{k=0}^{\infty}\sum_{j=1}^{n}q_{kn+j}(1-r)^{j-1}.$$

Remark 1. In [6] and the present paper we characterized the same discrete cyclic-waiting system, so between their characteristics there exists certain connection. One can check the coincidence of stability condition (in the two cases they are written in different forms), between the zero probabilities there is valid the relation

$$p_0^{(w)} = \left(p_0^{(q)} + p_1^{(q)}\right) \sum_{i=1}^{\infty} q_i (1-r)^{i-1},$$

Here the upper index w corresponds to the waiting time, the upper index q to the queue length.

We clarify the meaning of this expression. Consider a moment just before starting the service of a customer and let the system be free or let there be present one customer. The probability of this event is $p_0^{(q)} + p_1^{(q)}$. One starts the service of the actual customer and it takes i time units. The waiting time for the next customer will be zero if during the first $i-1$ time slices no customer enters and on the last time slice either no customer enters (the server becomes free) or a new customer appears. So, it is not important that during this time slice a further customer arrives or not since either the server becomes free or the service of new one can be started on the following time slice, in such sense it will be taken for service without waiting.

Remark 2. Our formulas for the generating functions of transition probabilities, p_0, $P(z)$ and the stability condition in the case of geometrical service time distribution (i.e. it is i time slices with probability $(1-q)q^{i-1}$) give the following formulas.

$P(z)$ has the same form as in the general case, the generating functions of transition probabilities are

$$A(z) = \sum_{i=0}^{\infty} a_i z^i =$$

$$\frac{(1-r)(1-q)}{1-q(1-r)} + z\frac{r(1-q)}{1-q(1-r)} + z\frac{rq(1-r+rz)^n(1-q^n)}{[1-q(1-r)][1-q^n(1-r+rz)^n]},$$

$$B(z) = \sum_{k=1}^{\infty} b_i z^i =$$

$$= \frac{1-(1-r)^n(1-r+rz)^n}{1-(1-r)(1-r+rz)} \frac{r(1-r+rz)}{1-(1-r)^n} +$$

$$+ \frac{1-q^n(1-r)^n(1-r+rz)^n}{1-q(1-r)(1-r+rz)} \frac{rq(1-r+rz)[(1-r+rz)^n-1]}{[1-(1-r)^n][1-q^n(1-r+rz)^n]}.$$

The probability of free state and the ergodicity condition are respectively

$$p_0 = 1 - r - \frac{rq(1-r)[1-q^n(1-r)^n]}{(1-r)^n(1-q^n)[1-q(1-r)]},$$

and

$$\frac{rq[1-q^n(1-r)^n]}{(1-q^n)(1-r)^n[1-q(1-r)]} < 1.$$

References

1. *Koba E.V.* On a GI/G/1 queueing system with repetition of requests for service and FCFS service discipline // Dopovidi NAN Ukrainy. 2000. no. 6. P. 101-103. (in Russian)
2. *Koba E.V., Pustova S.V.* Lakatos queuing systems, their generalization and application // Cybernet. Systems Anal. 2012. V. 48. P. 387-396.
3. *Lakatos L., Szeidl L., Telek M.* Introduction to queueing systems with telecommunication applications. Springer, 2013.
4. *Lakatos L., Efrosinin D.* Some aspects of waiting time in cyclic-waiting systems // Commun. Comput. Inf. Sci. 2013. V. 356. P. 115-121.
5. *Lakatos L., Efrosinin D.* A discrete time probability model for the waiting time of optical signals // Commun. Comput. Inf. Sci. 2014. V. 279. P. 114-123.
6. *Lakatos L.* On the waiting time in the discrete cyclic-waiting system of $Geo/G/1$ type // Commun. Comput. Inf. Sci. 2016. V. 601. P. 86-93.

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Modeling and Analysis of Caching Rules Based on the Popularity of Objects

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Abstract. The paper is devoted to caching of popular multimedia and Web contents in Internet. We study the Cluster Caching Rule (CCR) recently proposed by the authors. It is based on the idea to store only popular contents arising in clusters of related popularity processes. Such clusters defined as consecutive exceedances of popularity indices over a high threshold are caused by dependence in the inter-request times of the objects and, hence, their related popularity processes. We compare CCR with the well-known Time-To-Live (TTL) and Least-Recently-Used (LRU) caching schemes. We model the request process for objects as a mixture of Poisson and Markov processes with a heavy-tailed noise. We focus on the hit probability as a main characteristic of a caching rule and introduce cache effectiveness as a new metric. Then the dependence of the hit probability on the cache size is studied by simulation.

Keywords: Caching, Cluster Caching Rule, TTL, LRU, hit/miss probability, popularity process, clusters of exceedances, inter-request times.

1. Introduction

Nowadays, caching of contents is intensively applied in the Internet to provide multimedia or Web objects on demand to the users with a minimal delay. The idea stems from computer systems where frequently demanded files have to be cached in a short-term memory to accelerate the exchange between the processor and the operative memory. In telecommunication systems this concept is used to keep the requested content in a cache, e.g. at an edge router in mobile edge computing, or a hierarchy of caches. Numerous problems arising from the randomness of the inter-request time (IRT) sequences concern the optimal cache size, cache utilization and occupancy, and the replacement of objects within a cache to provide the fast availability of the requested content. The latter item is characterized by the hit/miss probability, i.e. the probability to find/miss a requested content in the cache.

Among these caching replacement rules, the Least-Recently-Used (LRU) (cf. [1]), the Least-Frequently-Used (LFU) (cf. [2]) and the Time-to-Live (TTL) policy (cf. [3], [4], [5]) are the most popular schemes. Usually, the Independent Reference Model (IRM) that summarizes a number of assumptions is used to simplify the formulation of the hit/miss probability, the cache utilization and occupancy problems. According to IRM it is

assumed that the inter-request times (IRTs) are independent and exponentially distributed (i.e. the request process is a Poisson renewal process), the popularity of contents (or Web objects) and content sizes are constant. The IRM implies a time and space locality regarding the object popularity. It should be noted that normally a non-Poisson renewal process model cannot capture the superposition of request processes that arise in cache networks, cf. [3].
Not much has been done when the IRM model is not appropriate. Then the IRT sequence may be correlated, heavy-tailed and non-stationary. Our first objective is to show how one can handle the caching problem in this case and what is the impact of such conditions on the effectiveness and utilization of a cache. Correlated IRTs are particularly realistic if some content has become very popular and many users are interested in it. Therefore, such correlations generate clusters of peaks of the popularity index. Following [6] we determine the cluster as a conglomerate of consecutive exceedances of the popularity process over a threshold between two consecutive non-exceedances.
We focus on the Cluster Caching Rule (CCR) policy proposed and studied in [7], [8]. Dealing with a single cache we propose an *effectiveness of a cache* as new caching metric. It is defined as total popularity of objects placed in the cache at time t. The second objective is the analysis and comparison of the CCR, the TTL and the LRU rules by a simulation study. Both the CCR and TTL rule use *timers* as tuning knobs for individual objects to stay in the cache, but they apply different arguments. We propose to select the TTL timers depending on the popularity of the cached objects.
The paper is organized as follows. In Section 2 related work is discussed. In Section 3 we propose the effectiveness of a cache. In Section 4 we compare the hit probability of the CCR, LRU and TTL rules depending on the cache size and the TTL timer selection by simulation. The results are summarized in the Conclusion.

2. Related Work

Cache replacement schemes can be split into capacity-driven and TTL-based policies, cf. [9]. The hit (or miss) probability determines the long-term frequency to find (or not to find) a requested object in the cache. The LRU and LFU policies belong to the capacity-driven group since objects are evicted from the cache by arrivals of those objects not yet stored. Modifications of LRU were proposed like persistent-access-caching (PAC) to improve its miss probability, cf. [11].
The CCR policy [7] is related to a popularity oriented, threshold-driven policy. CCR is in a way similar to LFU where only popular objects may be placed in the cache. Caching only frequently referenced objects has also been developed as central processing unit (CPU) approach in [1].
Regarding the stochastic analysis of caching rules for *correlated request*

processes with heavy tails not much research has been done yet. Poisson arrival processes were considered in [12]- [14] with light- and heavy-tailed request rates λ_i, i.e. $\lambda_i \sim c\exp(-\xi i^\beta)$ and $\lambda_i \sim c/i^\alpha$, respectively. The miss probability of the LRU policy was shown to decrease following a power law or exponentially, respectively, for heavy- and light-tailed λ_i as the cache size C tends to infinity. It was derived that the correlation does not impact on the miss probability for unlimited cache size. Markov arrival processes (MAPs) were also used to model correlated requests, cf. [3], since they are closed regarding superposition. Non-stationary and dependent request processes and the average miss probability for the LRU and moderate cache sizes were considered in [15].

Cache utilization determines an important metric and raises several issues. To optimize cache utilization based on TTL policies, [16] proposed to maximize the sum of utilities of all objects regarding the TTL timers. In [7] the mean cache utilization with regard to CCR was considered both for fixed and random object sizes.

3. Effectiveness of CCR Caching

The analysis of real traces has shown that about 70% of contents in caches is requested only once. It translates into an even higher miss ratio of 0.88, cf. [1]. The LRU and TTL cache policies do not prevent to place unpopular contents in the cache. To prevent caching of a large portion of rarely requested objects, we propose to maximize the *effectiveness* of a cache. It is reflected by the new metric

$$e(t) = \sum_{i=1}^{C} p_i(t) 1\!\!\mathrm{I}\{i\text{th object from the catalog is in cache at time t}\}.$$

We assume that all objects $\{o_j \mid j \in M\}, M = \{1, \dots, N\}$ in the catalog have equal size s and $\widehat{C} = Cs$ is the cache size. N denotes the size of the catalog. $p_i(t)$ is the popularity of the ith object o_{j_i} in the cache at epoch t. $e(t)$ indicates the total popularity of all objects $\{o_{j_1}, \dots o_{j_C}\}$ stored in the cache. It holds $j_C \leq C$ since the cache may not be full. According to the CCR policy, the ith object o_{j_i} may be placed in the cache if its popularity $p_i(t)$ at time t exceeds a given threshold u.

As the cache load is provided by clusters of highly popular objects, their indices $p_i(t)$ may belong only to one cluster. This means that the number of cached objects is limited by the cluster size $T_2(u)$ (w.r.t. notations defined in [6], [7]) or more exactly by the maximal cluster size. Regarding the CCR policy, we then get the effectiveness

$$e_u(t) = \sum_{i=1}^{C} p_i(t)\mathbb{P}\{p_i(t) > u | i\text{th object in cluster}\} = \sum_{i=1}^{j} p_i(t)\mathbb{P}\{T_2(u) = j\} \tag{1}$$

where $j \leq C$ is the observed cluster size. In case $j > C$ we can load the rest of those objects in the next cache of a cache hierarchy or increase u to decrease the cluster size. The effectiveness metric $e_u(t)$ is driven by u. We can find such u that provides a maximal value $e_u(t)$ for a fixed time t. To this end, let us assume that the objects' popularity is determined by Zipf's law, i.e. $p_i \sim \chi/i^{\alpha}$, where $\chi > 0$ is a constant. $\alpha > 0$ is the tail index. It shows the heaviness of the tail of the popularity distribution. As the popularity index may change over time, we can take $p_i(t) \sim \chi/i^{\alpha(t)}$. Regarding a sequence of increasing thresholds $\{u_n\}_{n\geq 1}$, the probability of $T_2(u_n)$ derived in [6] satisfies for each $\varepsilon > 0$ and some n_ε and $j_0(n_\varepsilon)$ the following expression $\mid \mathbb{P}\{T_2(x_{\rho_n}) = j\}/(\theta^2 q_n(1-q_n)^{(j-1)\theta}) - 1 \mid < \varepsilon$ for all $n > n_\varepsilon$ and j sufficiently large, i.e $j > j_0(n_\varepsilon)$. Here high quantiles $\{x_{\rho_n}\}$ of the common popularity process of all objects in the catalog w.r.t. the levels $q_n = 1 - \rho_n$, $\rho_n \sim 1/n$ are taken as thresholds $\{u_n\}$. $\theta \in [0, 1]$ is the dependence measure of the popularity process called extremal index [17]. The reciprocal $1/\theta$ approximates the mean cluster size of exceedances over the threshold $u = u_n$. By (1) and an approximation of the Rieman Zeta function for $\alpha(t) > 0, \alpha(t) \neq 1$, we get the total popularity of the j objects placed in the cache of size $\widehat{C} = Cs$ in terms of

$$e_q(t) \approx \theta^2 q(1-q)^{(j-1)\theta} \sum_{i=1}^{j} \frac{\chi}{i^{\alpha(t)}} \approx \chi\theta^2 q(1-q)^{(j-1)\theta} \frac{j^{1-\alpha(t)} - 1}{1 - \alpha(t)}. \tag{2}$$

As the quantile level q represents now the threshold u, one can find $q = 1/(1+(j-1)\theta)$ that maximizes $e_q(t)$. In Fig. 1 $e_q(t)$ is depicted for a fixed time t, i.e. $\alpha(t) = \alpha$. As Zipf's model may fit the popularity not accurately enough for samples of moderate size, we can estimate the popularity of the ith object o_{j_i} at stopping time t by [7]

$$p_i(t) = J_{i,t}/N_t. \tag{3}$$

Here $J_{i,t}$ and N_t denote the number of requests for the ith object o_{j_i} and for all objects $o_j, j \in M$ in the catalog at time t, respectively, that progress in time. The cluster size probability can be evaluated as ratio of the number of requests R_t with popularity exceedances over u to the total number of requests N_t at time t. Then we get from (1) $e_u(t) = \left[R_t/{N_t}^2\right] \sum_{i=1}^{C} J_{i,t}$. An increasing level u induces clusters with smaller sizes. It may lead to the necessity to select a smaller cache size or to a less efficient utilization of the cache.

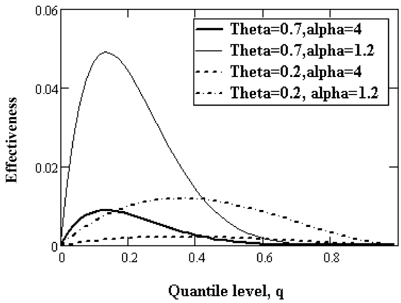

Figure 1. Effectiveness (2) with $C = j = 10$ for the CCR policy against the quantile level q of the extremal index $\theta \in \{0.2, 0.7\}$ and the tail index $\alpha \in \{1.2, 4\}$, where $q \in \{0.137, 0.357\}$ corresponds to the maximal effectiveness.

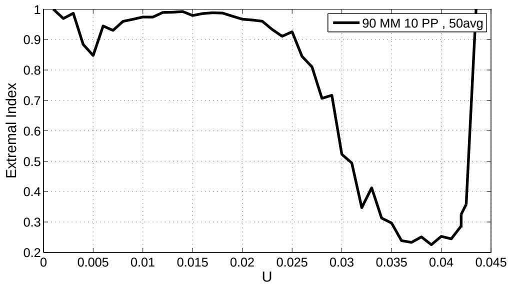

Figure 2. The intervals estimate $\widehat{\theta}$ of the extremal index averaged over 50 samples against the threshold u: the estimate $\widehat{\theta} = 0.22$ corresponds to the stability interval of the plot by threshold u.

4. Comparison of the CCR, LRU and TTL Caching Rules

By simulation we compare the CCR, LRU and TTL caching rules, where the latter TTL scheme is derived from the eviction-reset policy $\mathcal{R}$ in [3]. Following [8] we use a mixture of the Moving Maxima (MM) and the Poisson renewal processes to model a common IRT process regarding all objects of the catalog of different types. The MM process $\{\tau_{i,t}\}$ as IRT model of the ith object type satisfies $\tau_{i,t} = \max_{j=0,\dots,m_i}\{\alpha_j Z_{t-j}\}$, $t \in \mathbb{Z}$, with nonnegative constants $\{\alpha_j\}$ such that $\sum_{j=0}^{m_i} \alpha_j = 1$ and iid standard Fréchet distributed r.v.s $\{Z_t\}$ with distribution function $F(x) = \mathbb{P}\{Z_i \leq u\} = e^{-1/u}$. The distribution of $\tau_{i,t}$ is also Fréchet. The MM process is a

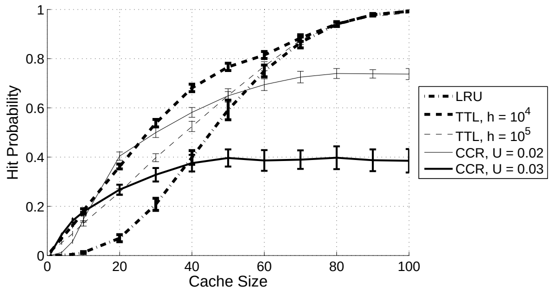

Figure 3. The hit probabilities for the CCR, LRU and the TTL policies averaged over 50 samples against the cache size C, horizontal lines indicate standard deviations.

m_i-dependent Markov chain where m_i determines the popularity duration. The MM process models IRTs of short-term news that are of public interest for a limited time. The Poisson process with intensity λ_i models objects like scientific and culture articles which may attract interest within a long time independently. Each object of equal size $s = 1$ from the catalog has an own $(m_i, \{\alpha_j\})$ or λ_i value as unique IRT model parameter.
The MM processes generate the correlation and the cluster structure of such common IRT process that has been generated here by 90% MM and 10% Poisson renewal processes. In (3) the $J_{i,t}$ is calculated in a cross-window with $N_t = 300$ requests. The number of objects in the catalog was taken as $N = 100$.
We compare the CCR, the LRU and the TTL policies for such simulated IRT processes. For each object o_{j_i} we propose TTL timers $\{t_i\}$ depending on its popularity index $p_i(t)$ and the mean IRT $\mathbb{E}(Y_i)$ of the overall IRT process, i.e. $t_i = h\ \mathbb{E}(Y_i)\ p_i(t)\ , 0 < h < \infty$. h is a scalability parameter. The TTL timers are larger for highly popular objects. t_i is proportional to the popularity of the ith object in $[0, t]$.
In Fig. 2 we estimate the extremal index θ of the popularity process by the intervals estimator proposed in [18]. This allows us to estimate the effectiveness (2) and the cache size as the reciprocal $C = 1/\theta$ equal to the mean cluster size as proposed in [7].
In Fig. 3 we show the hit probability for the TTL, LRU and CCR policies depending on the cache size C for $s = 1$. The hit probability is estimated as the ratio of the number of requests hitting the cache and the total number of requests. For small cache sizes the best hit probability is provided by the CCR scheme with a threshold u corresponding to the stability interval and both the TTL and CCR policy work similar if h and u are relatively small. Small u generates large clusters. Then the CCR stores more objects in the same manner as TTL irrespectively of their popularity processes. If

h and u are small, then the inter-cluster time for large clusters is of similar small scale as the TTL timers. For large caches and long timers TTL is better than CCR. This means a long-term placement of many objects in the large cache which is not effective. For large caches the CCR hit probability reaches a stability level that is lower than the corresponding TTL value due to the limited cluster size and the impossibility to store a larger number of objects than the cluster size. A minimal C corresponding to the stability level of the hit probability may be taken as a sufficient cache size.

5. Conclusion

We have studied the caching of popular contents assuming correlated inter-request time processes and fixed object sizes when the popularity of the stored objects may change over the time. The CCR, LRU and TTL caching rules have been compared by a simulation study. The following results have been obtained: 1) cache effectiveness has been introduced as new quality metric; 2) regarding a TTL determined policy TTL timers based on popularity indices have been proposed; 3) the CCR policy has better hit probability than TTL regarding relatively small cache sizes and thresholds u corresponding to the stability interval of the extremal index plot (u, θ).
Regarding caching in a mobile edge computing environment based on interconnected powerful SBC boards that implement the sketched approach, the adoption of a dynamic version of the proposed CCR policy is a topic of our future research.

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References

1. *Che, H., Tung, Y., Wang, Z.* Hierarchical Web caching systems: modeling, design and experimental results. IEEE JSAC, Vol. 20(7), pp. 1305–1314, 2002.
2. *Lee, D., et al.* LRFU: A Spectrum of Policies that Subsumes the Least Recently Used and Least Frequently Used Policies. IEEE Trans. on Computers, Vol. 50(12), pp. 1352–1362, 2001.
3. *Berger, D. S., Gland, P., Singla, S., Ciucu, F.* Exact Analysis of TTL Cache Networks: The Case of Caching Policies Driven by Stopping Times. The 2014 ACM International Conference on Measurement and Modeling of Computer Systems, SIGMETRICS '14, 595-596, 2014.

4. *Fofack, N. C., Nain, P., Neglia, G., Towsley, D.* Analysis of TTL-based cache networks. The 6th International Conference on Performance Evaluation Methodologies and Tools (VALUETOOLS), pp. 1–10, 2012.
5. *Friecker, C., Robert, P., Roberts, J.* A versatile and accurate approximation for LRU cache performance. In Proceedings of ITC 2012, pp. 1–8, 2012.
6. *Markovich, N. M.* Modeling clusters of extreme values. Extremes, Vol. 17, Issue 1, pp. 97–125, 2014.
7. *Markovich, N. M.* A cluster caching rule in next generation networks. V. Vishnevsky and D. Kozyrev (Eds.), DCCN 2015, CCIS 601, Springer, pp. 305–313, 2016.
8. *Markovich, N. M., Krieger, U. R.* A Caching Policy Driven by Clusters of High Popularity. 7th IEEE International Workshop on TRaffic Analysis and Characterization (TRAC 2016), Paphos, Cyprus, September 5–9, 2016.
9. *Rizzo, L., Vicisano, L.* Replacement policies for a proxy cache. IEEE/ACM Transactions on Networking, Vol. 8, Issue 2, pp. 158–170, 2000.
10. *Breslau, L., Cao, P., Fan, L., Phillips, G., Shenker, S.* Web caching and Zipf-like distributions: evidence and implications. Eighteenth Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM '99), IEEE Proceedings, Vol. 1, pp. 126–134, 1999.
11. *Jelenković, P.R., Radovanović, A.* The persistent-access-caching algorithms. Random Structures Algorithms, Vol. 33, pp. 219–251, 2008.
12. *Jelenković, P.R.* Asymptotic approximation of the move-to-front search cost distribution and least-recently-used caching fault probabilities. Ann. Appl. Prob., Vol. 9, pp. 430–464, 1999.
13. *Jelenković, P.R., Radovanović, A.* Least-recently-used caching with dependent requests. Theoretical Computer Science, Vol. 326(1-3), pp. 293–327, 2004.
14. *Jelenković, P.R., Radovanović, A.* Asymptotic optimality of the static frequency caching in the presence of correlated requests. Operations Research Letters, Vol. 37(5), pp. 307–311, 2009.
15. *Osogami, T.* A fluid limit for a cache algorithm with general request processes. Adv. Appl. Prob., Vol. 42, pp. 816–833, 2010.
16. *Dehghan, M., Massoulie, L., Towsley, D., Menasche, D., Tay, Y.C.* A Utility Optimization Approach to Network Cache Design. arXiv: 1601.06838v1, pp. 1–11, 2016.
17. *Leadbetter, M.R., Lingren, G., Rootzén, H.* Extremes and Related Properties of Random Sequence and Processes. Springer, 1983.
18. *Ferro, C.A.T., Segers, J.* Inference for Clusters of Extreme Values. Journal of the Royal Statistical Society, Series B, Vol. 65, pp. 545–556, 2003.

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Оптические фильтры для адаптивной компенсации хроматической дисперсии в высокоскоростных оптических системах передачи

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Аннотация. Увеличение скорости передачи по оптическому волокну позволяет снизить стоимость передачи бита информации. Но с ростом скорости передачи по оптической линии связи растёт и степень воздействия хроматической дисперсии на сигнал. Наиболее перспективным способом компенсассции дисперсии является прменение адаптивных методов обработки сигналов в оптической области. В статье рассматриваются методы синтеза интегральных адаптивных фильтров, имеющих в своём составе интерферометры Маха-Зендера и кольцевые резонаторы. Производится сравнение нерекурсивных и рекурсивных корректоров по степени сложности их реализации.

Ключевые слова: адаптивные фильтры, рекурсивные фильтры, нерекурсивные фильтры, фазовый корректор, интерферометр Маха-Зендера, кольцевой резонатор, ФЧХ.

1. Введение

В настоящее время происходит активное развитие высокоскоростных волоконно-оптических систем передачи. Основной задачей является увеличение скорости передачи по одному оптическому волокну с целью снизить стоимость передачи бита информации [1]. Один из способов достичь такого результата — уменьшить длительность передаваемого импульса. Однако чем короче импульс, тем сильнее на него действует хроматическая дисперсия — один из важнейших негативных факторов, влияющих на сигнал в оптическом волокне.

Существует множество способов компенсации хроматической дисперсии [2–4]. Наиболее перспективным из них является применение адаптивных фильтров, созданных на компонентах интегральной оптики. Такой способ, в отличие от статических методов компенсации дисперсии в оптической области, позволяет системе самостоятельно адаптироваться к изменению характеристик тракта передачи, что значительно повышает её гибкость. А использование компонентов интегральной оптики позволяет использовать такой метод на высоких скоростях передачи, что затруднительно для похожих решений в электрической области.

Существует два способа реализации оптических фильтров-корректоров: нерекурсивный и рекурсивный. В данной статье рассматриваются особенности каждой из них.

2. Основная часть

Рассмотрим воздействие хроматической дисперсии на сигнал. Распространение импульсов по оптическому волокну может быть записано в виде [5]

$$\frac{\partial \tilde{A}}{\partial z} = -j\beta(\omega)\tilde{A}, \tag{1}$$

где $\tilde{A}(j\omega, z)$ — спектр сигнала на расстоянии z, а $\beta(\omega)$ — постоянная распространения, зависящая от частоты. Решением уравнения (1) является выражение

$$\tilde{A}(j\omega, z) = \tilde{A}(j\omega, 0)e^{-j\beta(\omega)z}$$

Следовательно, передаточная характеристика системы, в которой действует только дисперсия, выглядит следующим образом:

$$H(j\omega) = e^{-j\beta(\omega)z} \tag{2}$$

Для компенсации дисперсионных искажений характеристика фильтра должна быть обратна (2):

$$K(j\omega) = \frac{1}{H(j\omega)} = e^{j\beta(\omega)z} \tag{3}$$

На рис. 1 показана частотная зависимость ФЧХ оптической системы $\beta(\omega)z$. Она содержит существенную линейную часть, которая влияет только на групповую задержку. Поэтому, прежде чем приступать к расчёту фильтра, целесообразно её исключить. Вид ФЧХ без линейной части показан на рис. 2.

2.1. Синтез нерекурсивного фильтра

Для расчёта коэффициентов нерекурсивного фильтра воспользуемся разложением выражения (3) в ряд Фурье. $\beta(\omega)z$ в исходном виде имеет сложную зависимость от частоты [5]. Поэтому нахождение аналитического выражения для коэффициентов ряда представляет сложность. Однако ФЧХ фильтра, как видно из рис. 2, можно аппроксимировать квадратичной функцией. Тогда выражение (3) примет вид:

$$K(j\omega) = e^{j(a\omega^2 + b\omega + c)z}$$

Коэффициенты разложения в ряд Фурье такой функции выражаются через интегралы Френеля:

$$c_n = \frac{e^{j\left(\frac{(bzL+\pi n)^2}{4azL^2} + cz\right)}}{2L\sqrt{az}} \left[C\left(X_2\right) - C\left(X_1\right) + jS\left(X_2\right) - jS\left(X_1\right)\right],$$

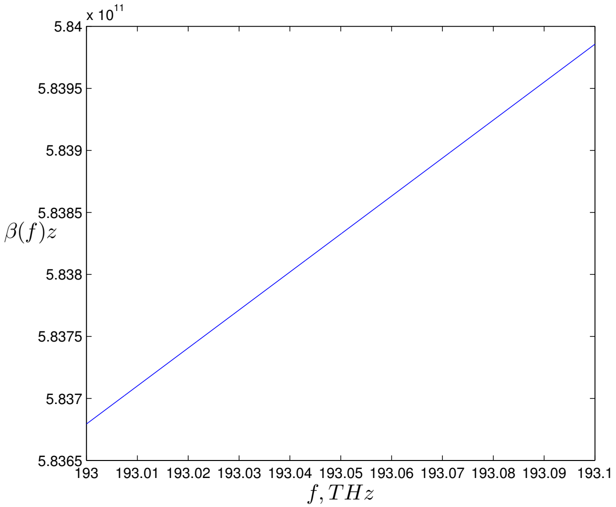

Рис. 1. Частотная зависимость ФЧХ оптической системы на расстоянии 100 км.

где $X_1 = \sqrt{az}\omega_l - \frac{bzL+\pi n}{2L\sqrt{az}}$, $X_2 = \sqrt{az}\omega_h - \frac{bzL+\pi n}{2L\sqrt{az}}$, L — половина диапазона $[\omega_l; \omega_h]$, а $n = [-N; N]$.

Полученные коэффициенты соответствуют коэффициентам нерекурсивного фильтра, показанного на рис. 3. Скорректированный сигнал снимается с центрального отвода. Общее количество коэффициентов равно $2N + 1$. Задержка одного элемента $T = \frac{2\pi}{\omega_h - \omega_l}$.

В интегральной оптике нерекурсивные фильтры строятся в виде последовательного соединения симметричных и асимметричных интерферометров Маха-Зендера [6], как показано на рис. 4. Симметричные интерферометры играют роль электрически управляемых умножителей на коэффициент, асимметричные — линии задержки.

К преимуществам такого фильтра можно отнести устойчивость и сравнительную простоту расчёта коэффициентов. Однако предложенный в статье метод расчёта допустим только для рабочего диапазона частот не более 200 ГГц. В более широком диапазоне ФЧХ имеет значительное отклонение от квадратичной функции, что сказывается на ошибке аппроксимации. В [7] был описан метод, позволяющий минимизировать эту ошибку с сохранением выражения для расчёта коэффициентов в аналитической форме. Кроме того, как будет показано далее, количество коэффициентов у такого фильтра достаточно

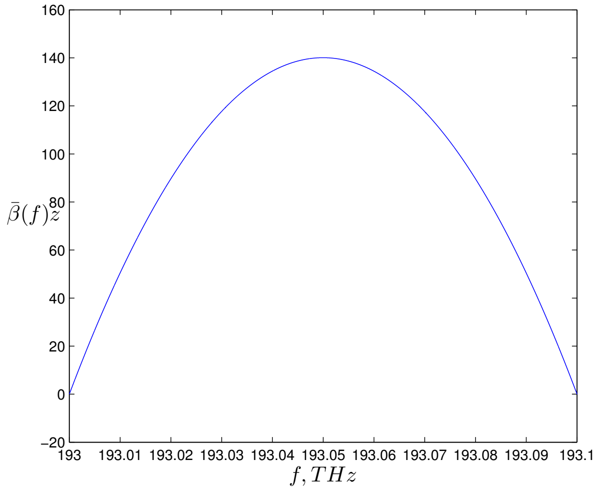

Рис. 2. Нелинейная часть ФЧХ на расстоянии 100 км.

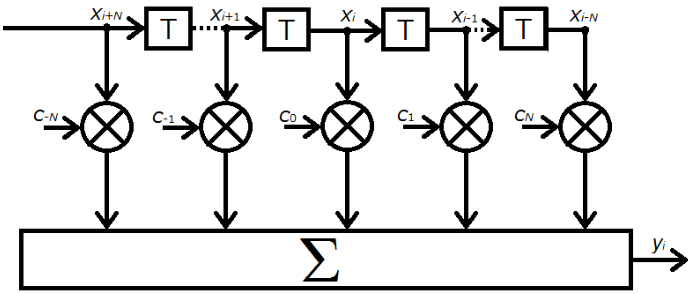

Рис. 3. Структура нерекурсивного фильтра-корректора.

велико, чтобы возникли сложности с физической реализацией на значительных расстояниях (более 500 км).

Альтернативой нерекурсивному фильтру является рекурсивный.

2.2. Синтез рекурсивного фильтра

Так как в выражении (2) модуль передаточной функции равен 1, необходимо скомпенсировать только нелинейность ФЧХ. Такую задачу

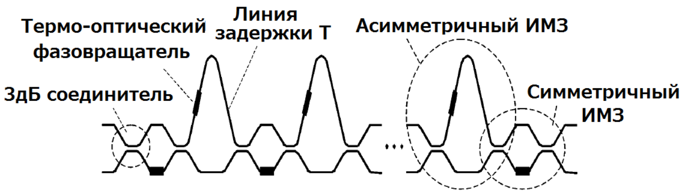

Рис. 4. Нерекурсивный фильтр на основе интерферометров Маха-Зендера.

способен выполнить фазовый фильтр. Его коэффициент передачи:

$$K(j\omega) = \frac{e^{j\omega NT}\left(1+\sum_{k=1}^{N} c_k e^{j\omega kT}\right)}{1+\sum_{k=1}^{N} c_k e^{-j\omega kT}}$$

Рассчитать коэффициенты рекурсивного фазового фильтра c_k можно, решив систему линейных уравнений [8]:

$$\sum_{k=1}^{N} c_k \sin\left[k\omega_i T + \frac{1}{2}\bar{\phi}(\omega_i)\right] = -\sin\frac{1}{2}\bar{\phi}(\omega_i),$$

где ω_i — N граничных частот-подинтервалов, на которые разбит интервал $[\omega_l;\omega_h]$, $\bar{\phi}(\omega_i)$ — нелинейная часть ФЧХ на каждой из частот.

Аналитическое решение данной системы не вызывает принципиальных трудностей, однако при больших N задача может оказаться недопустимо громоздкой. Поэтому в данном случае нужно использовать численные методы. Это приводит к невозможности вычислять коэффициенты фильтра непосредственно из формулы, как в случае с нерекурсивным фильтром. Тем не менее, данную процедуру необходимо выполнить лишь раз, при начальной настройке фильтра.

В интегральной оптике существует несколько способов построения рекурсивных фильтров [6]. Самым простым является последовательное соединение кольцевых резонаторов, как показано на рис. 5. В качестве настраиваемых соединителей обычно используются симметричные интерферометры Маха-Зендера.

Главным преимуществом рекурсивного фильтра-корректора, по сравнению с нерекурсивным, является снижение общего количества элементов за счёт сокращения количества коэффициентов. В таблице 1 приведено общее количество коэффициентов, которое требуется для достижения точности аппроксимации коэффициента передачи (2) 10^{-2} в полосе 193.0..193.1 ГГц для рекурсивного и нерекурсивного фильтров.

Из недостатков рекурсивного фильтра можно отметить отсутствие удобной аналитической формулы для расчёта коэффициентов и необходимость выполнять проверку на устойчивость.

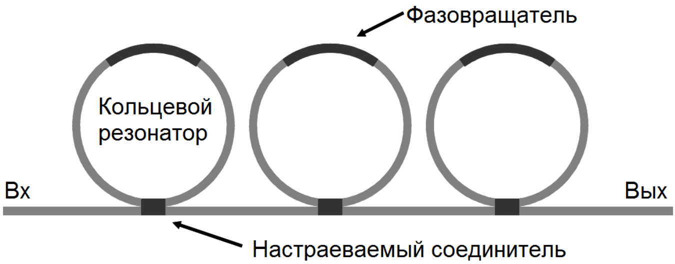

Рис. 5. Рекурсивный фильтр на основе кольцевых резонаторов.

Таблица 1

Сравнение требуемого количества коэффициентов корректоров

Расстояние, км	N, нерекурсивный	N, рекурсивный
10	101	10
50	201	38
100	321	77
200	461	162
500	1101	415
1000	2001	805

3. Заключение

Адаптивная компенсация хроматической дисперсии в оптической области позволяет создавать высокоскоростные оптические системы передачи, способные работать в широком диапазоне условий, и быстро перестраиваться при их изменении. Ключевым компонентом, позволяющим совершать адаптивную подстройку является интегральный интерферометр Маха-Зендера. На основе таких интерферометров, а так же интегральных кольцевых резонаторов, возможно создать корректирующие фильтры, коэффициенты которых могут рассчитываться по предложенным методам. Рекурсивный фазовый корректор имеет выигрыш в количестве элементов более чем в два раза по сравнению с нерекурсивным корректором. Что делает его более привлекательным для использования в оптических системах передачи.

Литература

1. *Величко М. А.* Электронные методы компенсации дисперсии в оптических линиях связи // LIGHTWAVE Russian Edition — 2007. — январь.
2. *Singer A. C. , Shanbhag N. R., Bae H.* Electronic Dispersion Compensation // IEEE Signal Processing Magazine — 2008. —nov.
3. *Riant I., Gurib S., Gourhant J., Sansonetti P., Bungarzeanu C., Kashyap R.* Chirped Fiber Bragg Gratings for WDM Chromatic Dispersion Compensation in Multispan 10-Gb/s Transmission // IEEE Journal of selected topics in quantum electronics — 1999. —No. 5.
4. *Kahlon N. K., Kaur G. G.* Various Dispersion Compensation Techniques for Optical System: A Survey // Open Journal of Communications and Software — 2014. —No. 1.
5. *Agrawal G. P.* Nonlinear Fiber Optics. — Academic Press, 2013.
6. *Madsen C. K., Zhao J. H.* Optical Filter Design and Analysis: A Signal Processing Approach. — Wiley, 1999.
7. *Meltenisov M., Matukhin A.* Analytical model of chromatic dispersion effect in the time domain // 18th International Conference on Advanced Communication Technology (ICACT) / Phoenix Park, Republic of Korea. — pp. 406-409 (2016).
8. *Кисель В. А.* Синтез сигналов и синтез цепей для скоростных систем связи: дис. ... д-ра техн. наук. — Одесса, 1972.

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Optical filters for adaptive compensation of chromatic dispersion in high-speed optical transmission systems

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The increasing of transmission speed in optical fiber allows decreasing transmission cost of one bit of information. But with growth of transmission speed in optical communication line the efficacy of chromatic dispersion to signal grows accordingly. The most promising method of dispersion compensation is the adaptive signal processing in optical area. Methods of synthesis of integrated adaptive filters, which consist of Mach-Zehnder interferometers and ring resonators, are considered in this article. Comparison of infinite impulse response filter and finite impulse response filter by complexity of implementation is performed.

Keywords: adaptive filters, infinite impulse response filters, finite impulse response filters, all-pass filter, Mach-Zehnder interferometer, ring resonator, PFC.

УДК 519.25 + 004.85

Автокодировщики: примеры применения для понижения размерности данных

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Аннотация. Доклад посвящен краткому обзору некоторых видов автокодировщиков (auto-encoders) с иллюстративными примерами их применения для понижения размерности данных.

Ключевые слова: машинное обучение, нейронные сети.

В машинном обучении известен ряд методов линейного и нелинейного понижения размерности данных: в том числе, метод главных компонент (Principal Component Analysis, PCA), ядерный PCA, t-Distributed Stochastic Neighbor Embedding (t-SNE) [1–3].

Вместе с развитием теории нейронных сетей, глубинного обучения (deep learning) [4–6] успешно применяются автокодировщики [7–9] для понижения размерности данных: например, в статье [10] речь идет о сжатии изображений. Существует несколько типов автокодировщиков: разряженные (sparse), стековые (stacked), вариационные (variational) и т.д. Известны реализации автокодировщиков с программированием на различных языках: от C++ до Julia. Например, theanets [11] и Lasagne [12] с программированием на Python, mlpack [13] – на C++.

В данном докладе обсуждаются обзорно некоторые известные автокодировщики применительно к понижению размерности данных на примерах.

Литература

1. *Hastie T., Tibshirani R., Friedman J.* The Elements of Statistical Learning. 2nd edition. New York: Springer, 2009.
2. *Non-linear Dimensionality Reduction.* `en.wikipedia.org/wiki/Nonlinear_dimensionality_reduction`
3. *Hinton J.* Non-linear dimensionality reduction: Lecture 11. 2013. University of Toronto. `www.cs.toronto.edu/~hinton/csc2535/notes/lec11new.pdf`
4. *Deep Learning.* `en.wikipedia.org/wiki/Deep_learning`
5. *LeCun Y., Bengio Y., Hinton G.E.* Deep Learning // Nature. 2015. Vol. 521. Pp. 436-444.
6. *Deep Machine Intelligence and its Applications: Workshop. Skoltech, 4 – 5 June, 2016.* `www.skoltech.ru/en/2016/04/deep-machine-intelligence-and-its-applications-skoltech/`

7. *Autoencoder.* https://en.wikipedia.org/wiki/Autoencoder
8. *Hinton G.E., Salakhutdinov R.R.* Reducing the Dimensionality of Data with Neural Networks // Science. 2006. Vol. 313. Pp. 504-507.
9. *Makhzani A., Shlens J., Jaitly N., Goodfellow I.* Adversarial Autoencoders Venue. 2016. arXiv:1511.05644v2 [cs.LG]. http://arxiv.org/abs/1511.05644, http://research.google.com/pubs/pub44904.html
10. *Toderici G., O'Malley S.M., et al.* Variable Rate Image Compression with Recurrent Neural Networks. 2016. arXiv:1511.06085 [cs.CV]. http://arxiv.org/abs/1511.06085, http://research.google.com/pubs/pub44844.html
11. *theanets.* https://pypi.python.org/pypi/theanets, theanets.readthedocs.io/en/stable/api/generated/theanets.recurrent.Autoencoder.html
12. *Lasagne.* https://github.com/Lasagne, https://github.com/Lasagne/Recipes/tree/master/examples/variational_autoencoder
13. *mlpack.* http://mlpack.org/docs/mlpack-1.0.9/doxygen.php?doc=classmlpack_1_1nn_1_1SparseAutoencoderFunction.html

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Auto-encoders: examples of their using for data dimensionality reduction

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The talk is devoted to a short review of some kinds of auto-encoders with illustrative examples of their usage for data dimensionality reduction.

Keywords: machine learning, neural networks.

UDC 004.4

On Internet of Things Programming models

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Abstract. In this paper, we present the review of existing and proposed programming models for Internet of Things (IoT) applications. The requests by the economy and the development of computer technologies (e.g., cloud-based models) have led to an increase in large-scale projects in the IoT area. The large-scale IoT systems should be able to integrate diverse types of IoT devices, support big data analytics. And, of course, they should be developed and updated at a reasonable cost and within a reasonable time. Due to the complexity, scale and diversity of IoT systems, programming for IoT applications is a great challenge. And this challenge requires programming models and development systems at all stages of development and for all aspects of IoT development. The first target for this review is a set of existing and future educational programs in information and communication technologies at universities, which, obviously, must somehow respond to the demands of the development of IoT systems.

Keywords: internet of things, Smart Cities, streaming, sensor fusion, programming, education.

1. Introduction

The Internet of Things (IoT) world is becoming an important direction for technology development. In general, the IoT promotes a heightened level of awareness about our world. IoT plays a basic role in many other things. For example, IoT is a base for Smart Cities, etc.

IoT ecosystem is currently presented by multiple (sometimes - competing) technologies and platforms. IoT platforms (at least, nowadays) are varied across the vertical and horizontal segments of the markets. Of course, it complicates and delays the development and deployment, makes the support of IoT systems more expensive than it should be, etc. So, IoT standards are highly demanded [1].

In the same time, it is a very competitive area. We cannot expect that a general solution will be agreed upon by all players. Standards proposals in IoT (and M2M) come from formal standards development organizations (e.g., the European Telecommunications Institute - ETSI) or non-formal groups (the Institute of Electrical and Electronics Engineers - IEEE). Standards can target the connectivity for a particular set of devices (e.g., Bluetooth Low Energy) or provide common application interfaces up to developers (e.g., oneM2M) [2]. In this paper, we would like to discuss the common elements of IoT programming models and perform this review from the perspective of educational programs.

The rest of the paper is organized as follows. In Section 2, we discuss programming systems for IoT. In Section 3, we discuss data models, data persistence, and processing.

2. IoT programming models

The choice of programming languages for IoT platforms does not depend on a hardware platform. Also, new hardware platforms makes programming embedded (nowadays - cyber-physical) systems easier. Even more, the diversity in hardware platforms enhances the interest to the platform independent on hardware.

Standards for the IoT could be classified as. downward-facing standards that establish connectivity with devices and upward-facing standards that provide common application interfaces up to end users and application developers.

By our opinion, confirmed by the practical experience and academic papers, the key moment for software development in telecom and related areas (IoT is among them) is time to market indicator [3]. The main question to any software standard is the generalization. Shall the standard follow to the "all or nothing" model and covers all the areas of the life cycle? In software standards, the excessive generalization (unification) could be the biggest source of the problems. Actually, all the standards should make its implementation by the most convenient way for developers. Because only the developers are finally responsible for the putting new services in place.

It is especially true for such areas as IoT or Smart Cities. The services here are not finalized (and it is very probably that they could not be finalized at all). This means that we will constantly try (test) new services and to refuse from the old ones. Naturally, this process needs to be fast and inexpensive. As the next step, it means that the most of IoT application could be described as mashups [4]. Mashups use data from several data sources. On programming level, it should stimulate the interest in scripting languages and to the systems for fast prototyping. In the modern software architecture world, we can mention also micro-services approach [5]. Of course, these directions should have an appropriate reflection in educational programs.

Another direction, which is very close to mashups, actually, is so-called Data-as-a-Service (DaaS) approach [6]. In its technical aspects, DaaS is an information provision and distribution model in which data files (including text, images, sounds, and videos) are made available to customers over a network. The key moment is the separation for data and proceedings. It lets delivery data (e.g., in some open format, like JSON), rather than some API with the predefined model for data processing.

The next significant visible trend is the growing interest to the dynamic languages. And the perfect example here is JavaScript. We can mention the following reasons for JavaScript in IoT applications [7]. At the first hand, there is a big army of web developers. So, the entry level for programming is low. Most of the Internet applications already use JavaScript. JavaScript nowadays covers both server-side and client-side programming. It could be useful to use the same language across the whole project. So, it makes sense to extend the same standard platform to the Internet of Things, communicating to a larger set of devices using the same language.

JavaScript has matured as a language and international standards cover its extensions. JavaScript has a range of already existing libraries, plugins, etc. And what is also important, there is a huge Open Source community behind JavaScript.

Technically, this language has got a great support for event-driven apps. The nature of IoT project is mostly associated with asynchronous communications. And event-driven models are the most suitable solution. In JavaScript it is very easy to implement models, where an application can receive and respond to events, then wait for a callback from each event that notifies us once it is complete. It lets respond to events as they happen, performing many tasks simultaneously as they come in.

Also, the recent development shows more and more direct involvement JavaScript into data processing. Actually, the winning data format (JSON) has its origin in JavaScript.

As a recent example of JavaScript in IoT, we can mention the developments from Samsung. Samsung Electronics recently opened the development of IoT.js, a web-based Internet of Things (IoT) platform that connects lightweight devices. Examples of lightweight devices include microcontrollers or devices with only a few kilobytes of RAM available [8]. The idea is to make all devices interoperable in the IoT space by enabling more devices to be interoperable, from complex and sophisticated devices such as home appliances, mobile devices, and televisions, to lightweight and small devices such as lamps, thermometers, switches and sensors. The IoT.js platform is comprised of a lightweight version of the JavaScript engine, and a lightweight version of node.js. We think that JavaScript for IoT world should be in educational programs. This is very important because until now, this language is often seen as simple web pages scripting. But it's not for a long time already.

By the same reason, any attempt to replace JavaScript with the similar idea of portability could be also interested in IoT programming. In this connection, we should mention Dart programming language from Google [9].

3. Data persistence and processing

It terms of the data processing in IoT applications, we should pay attention in educational programs to the following two moments: sensor fusion and streaming. There are different data mining and data science

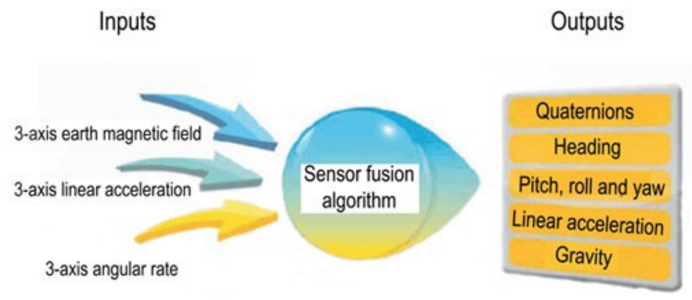

Figure 1. Sensor fusion [12]

approaches which are applicable to IoT. And of course, they should be a subject of the separate courses for statistics, machine learning, etc. For example, in many cases, IoT (Smart City) measurements are time series. Of course, it should be a subject of a separate course among other data-mining techniques [10].

But one moment is important, in our opinion, and should be discussed separately for IoT applications. It is sensor fusion. Sensor fusion is combining of sensory data or data derived from disparate sources such that the resulting information has less uncertainty than would be possible when these sources were used individually [11]. It is illustrated in Figure 1.

here are many ways of fusing sensors into one stream. Each sensor has its own strengths and weaknesses. The idea of sensor fusion is to take readings from each sensor and provide a more useful result which combines the strengths of each. Actually, such a fusion is the main idea for all IoT and Smart City projects, related to some measurements. The next big issue for IoT data processing is streaming. By our opinion, it is a key technology in data acquisition and proceeding for Smart Cities and IoT.

There are many tasks in IoT with the requirements for real-time (or near real-time) processing. In this case, the common use case is associated with some messaging bus. And it is very important to present the software architectures associated with streaming. At the first hand, it is so-called Lambda Architecture [13]. Originally, the Lambda Architecture is an approach to building stream processing applications on top of MapReduce and Storm or similar systems (Figure 2). Currently, we should link it to Spark and Spark streaming too [14].

The main idea behind this schema is the fact that an immutable sequence of source data is captured and fed into a batch system and a stream processing system in parallel. Of course, the negative impact of this decision is the need to implement business logic twice, once in the batch system and once in the stream processing system.

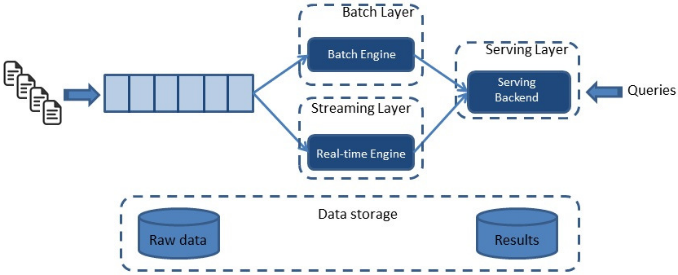

Figure 2. Lambda architecture [15]

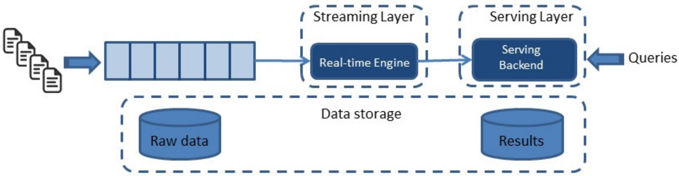

Figure 3. Kappa architecture [15]

The Lambda Architecture targets applications built around complex asynchronous transformations that need to run with low latency. One proposed approach to fixing this is to have a language or framework that abstracts over both the real-time and batch framework [16].

Another solution here is so-called Kappa architecture [17]. The Kappa architecture simplifies the Lambda architecture by removing the batch layer and replacing it with a streaming layer (Figure 3).

With Kappa, everything in the system is a stream. All batch operations become a subset of streaming operations. Data source (raw data) is persisted and views are derived. Of course, a state can always be recomputed where the initial record is never changed. This feature lets us support replay functionality. Computations and results can evolve by replaying the historical data from a stream. With Kappa, only a single analytics engine is required. It means that code is considerably reduced. Also maintenance and upgrades are cheaper.

The hearth for such implementations is a scalable, distributed messaging system with events ordering and at-least-once delivery guarantees. At this moment, it is almost always Kafka system [18].

Apache Kafka is a distributed publish-subscribe messaging system. It is designed to provide high throughput persistent scalable messaging. Kafka allows parallel data loads into Hadoop. Its features include the use of compression to optimize performance and mirroring to improve availability, scalability. Kafka is optimized for multiple-cluster scenarios In general, publish-subscribe architecture is the most suitable approach for mostly asynchronous measurements in IoT.

As per Kafka's semantics, when publishing a message, developers have a notion of the message being "committed" to the log. Once a published message is committed, it will not be lost. Kafka is distributed system, so messages are replicated to partitions. For message commit, at least one replicating broker should be alive.

Kafka guarantees at-least-once delivery by default. It also allows the user to implement at most once delivery by disabling retries on the producer and committing its offset prior to processing a batch of messages. Exactly-once delivery requires co-operation with the destination storage system (it is some sort of two-phase commit).

In connection with Kafka, we should highlight Apache Spark [19]. Apache Spark is an open-source cluster computing framework for big data processing. It has emerged as the next generation big data processing engine, overtaking Hadoop MapReduce. Apache Spark provides a comprehensive, unified framework to manage big data processing requirements with a variety of diverse data sets (text data, graph data, etc) and data sources (batch data and real-time streaming data). Spark enables applications in Hadoop clusters to run up to 100 times faster in memory and 10 times faster even when running on disk. Spark lets developers write applications in Java, Scala, or Python using a built-in set of high-level operators. In addition to MapReduce operations, Apache Spark supports SQL queries, streaming data, graph data processing, and machine learning. Developers can use these capabilities stand-alone or combine them to run in a single data pipeline use case.

Another model is the recently introduced Kafka Streams. Kafka models a stream as a log, that is, a never-ending sequence of key/value pairs. Kafka Streams is a library for building streaming applications, specifically applications that transform input Kafka topics into output Kafka topics (or calls to external services, or updates to databases, or whatever). It lets you do this with concise code in a way that is distributed and fault-tolerant [20].

The above-mentioned models describe the modern view of the building IoT systems from the position of data architecture. A review for some IoT and/or Smart Cities related program (of course, we target technology only) presented in [21], for example.

References

1. *Chen, Yen-Kuang.* "Challenges and opportunities of internet of things." Design Automation Conference (ASP-DAC), 2012 17th Asia and South Pacific. IEEE, 2012.

2. *Namiot, Dmitry, and Manfred Sneps-Sneppe.* "On IoT Programming." International Journal of Open Information Technologies 2.10 (2014): 25-28.
3. *Namiot, Dmitry, and Manfred Sneps-Sneppe.* "On software standards for smart cities: API or DPI." ITU Kaleidoscope Academic Conference: Living in a converged world-Impossible without standards?, Proceedings of the 2014. IEEE, 2014.
4. *Im, Janggwan, Seonghoon Kim, and Daeyoung Kim.* "IoT mashup as a service: cloud-based mashup service for the Internet of things." Services Computing (SCC), 2013 IEEE International Conference on. IEEE, 2013.
5. *Namiot, Dmitry, and Manfred Sneps-Sneppe.* "On Micro-services Architecture." International Journal of Open Information Technologies 2.9 (2014): 24-27.
6. *Bahrami, Mehdi, and Mukesh Singhal.* "The role of cloud computing architecture in big data." Information Granularity, Big Data, and Computational Intelligence. Springer International Publishing, 2015. 275-295.
7. *Raggett, Dave.* "The internet of things: W3C plans for developing standards for open markets of services for the IoT." Ubiquity 2015.October (2015): 3.
8. *Samsung Iot.js* https://news.samsung.com/global/samsung-electronics-opens-development-of-iot-js-an-iot-platform-that-expands-interoperability-to-lightweight-devices Retrieved: May, 2016.
9. *Who uses Dart* https://www.dartlang.org/community/who-uses-dart.html Retrieved: May, 2016.
10. *Aggarwal, Charu C., ed.* Managing and mining sensor data. Springer Science & Business Media, 2013.
11. *Wang, Meisong, et al.* "City Data Fusion: Sensor Data Fusion in the Internet of Things." arXiv preprint arXiv:1506.09118 (2015).
12. *Introduction to sensor fusion.* http://projects.mbientlab.com/introduction-to-sensor-fusion/ Retrieved: May, 2016.
13. *Marz, Nathan, and James Warren.* Big Data: Principles and best practices of scalable realtime data systems. Manning Publications Co., 2015.
14. *Ranjan, Rajiv.* "Streaming big data processing in datacenter clouds." IEEE Cloud Computing 1 (2014): 78-83.
15. *Applying the Kappa architecture in the telco industry.* https://www.oreilly.com/ideas/applying-the-kappa-architecture-in-the-telco-industry Retrieved: May, 2016.
16. *Villari, Massimo, et al.* "Alljoyn lambda: An architecture for the management of smart environments in iot." Smart Computing Workshops (SMARTCOMP Workshops), 2014 International Conference on. IEEE, 2014.
17. *Erb, Benjamin, and Frank Kargl.* "A conceptual model for event-sourced graph computing." Proceedings of the 9th ACM International

Conference on Distributed Event-Based Systems. ACM, 2015.
18. *Garg, Nishant.* Apache Kafka. Packt Publishing Ltd, 2013.
19. *Shanahan, James G., and Laing Dai.* "Large scale distributed data science using apache spark." Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. ACM, 2015.
20. *Kafka Streams.* http://www.confluent.io/blog/introducing-kafka-streams-stream-processing-made-simple Retrieved: May, 2016.
21. *Namiot, Dmitry.* "On Internet of Things and Smart Cities educational courses." International Journal of Open Information Technologies 4.5 (2016): 26-38.

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A Ciclic Queueing System With Priority Customers And T-strategy Of Service

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Abstract. We review the queuing system, the input of which is supplied with the Poisson process of priority customers and N number of the Poisson processes of non-priority customers. Durations of service for both priority and non-priority customers have a distribution functions of $A(x)$ and $B_n(x)$ for applications from priority flow and for customers from n flow $(n = 1 \dots N)$ respectively. By using methods of systems with server vacations and asymptotic analysis in conditions of a large load we have found the asymptotic probability distribution of values of virtual time of waiting for non-priority applications. It is shown that this distribution is exponential.

Keywords: ciclic queueing system with server vacations,priority customers, asymptotic analysis, exponential distribution.

1. Introduction

A ciclic queueing systems with priority customers mathematical models of telecommunication systems, which are pretty common in practice [1].Method of research of such systems is her decomposition and research of system with server vacations. In real systems "vacations" are considered as a temporal suspension of service either for device other applications or for its breakdown or repair [2].

2. Main section

Let's review the queuing system with one service device, the input of which is supplied with the Poisson process of priority information with the intensity of τ and a N number of the Poisson processes of non-priority information with the intensity of λ_n, where $n = 1, N$ [3]. The flows of non-priority customers will be called λ_n-flows, and the flow of priority information will be called τ-flow. Let's assume that the intensity of a τ-flow is substantially lower than the total intensity of λ_n-flows. Applications of each λ_n-flow form a queue with an unlimited number of waiting seats.

Device visits queues in a cyclic order, starting from a first queue and finishing with the N, then the cycle repeats. Duration of a visit is random and has the distribution function of $T_n(x)$. During that time device receives customers of a λ_n-flow for service, duration of which has the distribution function of $B_n(x)$. If there are no customers in the queue when device addresses it or if device has already serviced all applications in the queue, device is still addressing the queue till the end of a visit, duration of which is determined by the distribution function of $T_n(x)$.

τ-flow customers form their queue with an unlimited number of waiting seats as well. If the system receives an application from a τ-flow, device stops the service of a common application and instantly starts servicing priority customers for a time, duration of which is distributed by the function of $A(x)$. Upon finishing the service of a priority customer device resumes the service of non-priority customers. If during a service of a priority customer device receives another priority application device services all priority application and only then returns to the queue of a non-priority customers and resumes servicing them.

Cyclic system research method is it's decomposition and the research of a system with server vacations. Let's review the queuing system with one service device and two queues with an unlimited number of waiting seats. The system receives a Poisson process of priority customers with the intensity of τ (τ-flow) and a Poisson process of non-priority customers with the intensity of λ (λ-flow).

The system functions in cyclic mode, the cycle of which consists of two consecutive intervals. During the first interval customers of a λ-flow are serviced at the device. If there are no customers in the queue at the start of an interval or if device has serviced all customers that were in the queue during that interval, device still remains in this mode, waiting for customers to come. At the end of an interval device goes on a "vacation" during a second interval of the said cycle. Customers of a λ-flow that were received during a vacation are accumulating in the queue and wait till device returns to servicing them [4].

Let's assume that durations of these intervals are random and are determined by distribution functions of $T_1(x)$ and $T_2(x)$ respectively. During the first interval device services customers for a random time with a distribution function of $B(x)$. If a server vacation interrupted the service of a common application then after vacation device resumes the service of this application. When the system receives priority customers of a τ-flow, then regardless of where the device previously was (in a service mode or on a vacation) it starts servicing priority customers for duration of time, that has distribution function of $A(x)$. After a device has serviced all priority customers it either resumes the service of non-priority application or returns to a vacation.

In order to research the waiting time in systems with server vacations we have to find the characteristic function and the probability distribution of value $V(t)$ of an unfinished work on servicing all non-priority customers that were in a system at the time t. Let's denote

$V(t)$ the volume of work on servicing all non-priority customers that were in a system at the time t.

$Y(t)$ — the volume of work on servicing all priority customers that were in a system at time t.

$k(t)$ — device status: 1 —device is available for λ-flow applications , 2 —device is on a vacation, 3 —device is servicing priority customers, having interrupted the interval of being available for a common information (1st device status), 4 —device is servicing priority information, having interrupted a vacation.

$z(t)$ — the remaining time of a vacation or servicing priority information.

Let's review the Markov process $\{V(t), Y(t), k(t), z(t)\}$ and set up a direct system of Kolmogorov differential equations for a following probability distribution:

$$P_k(v,z,t) = P\{V(t) < v, k(t) = k, z(t) < z\}, k = 1,2,$$
$$P_k(v,y,z,t) = P\{V(t) < v, Y(t) < y, k(t) = k, z(t) < z\}, k = 3,4.$$

Let's assume that a system functions in a stationary mode, then:

$$-\left(\lambda+\tau\right)P_1(v,z) + \lambda\int\limits_0^v B(v-x)dP_1(x,z) + \frac{\partial P_2(v,0)}{\partial z}T_1(z)+$$
$$+\frac{\partial P_3(v,0,z)}{\partial y} + \frac{\partial P_1(v,z)}{\partial v} + \frac{\partial P_1(v,z)}{\partial z} - \frac{\partial P_1(v,0)}{\partial z} = 0,$$
$$-\left(\lambda+\tau\right)P_2(v_1,z) + \lambda\int\limits_0^v B(v-x)dP_2(x,z) + \frac{\partial P_1(v,0)}{\partial z}T_2(z)+$$
$$+\frac{\partial P_4(v,0,z)}{\partial y} + \frac{\partial P_2(v,z)}{\partial z} - \frac{\partial P_2(v,0)}{\partial z} = 0,$$
$$-\left(\lambda+\tau\right)P_3(v,y,\ z)+$$
$$+\lambda\int\limits_0^v B(v-x)dP_3(x,y,z) + \tau\int\limits_0^y A(y-x)dP_3(v,x,z)+$$
$$+P_1(v,z)A(y)\tau + \frac{\partial P_3(v,y,z)}{\partial y} - \frac{\partial P_3(v,0,z)}{\partial y} = 0,$$
$$-\left(\lambda+\tau\right)P_4(v,y,\ z)+$$
$$+\lambda\int\limits_0^v B(y-x)dP_4(x,y,z) + \tau\int\limits_0^y A(y-x)dP_4(v,x,z)+$$
$$+P_2(v,z)A(y)\tau + \frac{\partial P_4(v,y,z)}{\partial y} - \frac{\partial P_4(v,0,z)}{\partial y} = 0.$$

Let's introduce functions

$$H_k(u,z)=\int\limits_0^\infty e^{-uv}dP_k(v,z), k=1,2,$$

$$H_k(u,y,z)=\int\limits_0^\infty e^{-uv}dP_k(v,y,z), k=3,4,$$

for which we will rewrite a system of Kolmogorov equations in a following form:

$$\begin{cases} [\lambda\beta(u)-(\lambda+\tau)+u]\,H_1(u,z)-uP_1(0,z)+\\ +\frac{\partial H_2(u,0)}{\partial z}T_1(z)+\frac{\partial H_3(u,0,z)}{\partial y}+\frac{\partial H_1(u,z)}{\partial z}-\frac{\partial H_1(u,0)}{\partial z}=0,\\ [\lambda\beta(u)-(\lambda+\tau)]\,H_2(u,z)+\\ +\frac{\partial H_1(u,0)}{\partial z}T_2(z)+\frac{\partial H_4(u,0,z)}{\partial y}+\frac{\partial H_2(u,z)}{\partial z}-\frac{\partial H_2(u,0)}{\partial z}=0,\\ [\lambda\beta(u)-(\lambda+\tau)]\,H_3(u,y,\ z)+H_1(u,z)A(y)\tau+\\ +\tau\int_0^y A(y-x)dH_3(u,x,z)+\frac{\partial H_3(u,y,z)}{\partial y}-\frac{\partial H_3(u,0,z)}{\partial y}=0,\\ [\lambda\beta(u)-(\lambda+\tau)]\,H_4(u,y,\ z)+H_2(u,z)A(y)\tau+\\ +\tau\int_0^y A(y-x)dH_4(u,x,z)+\frac{\partial H_4(u,y,z)}{\partial y}-\frac{\partial H_4(u,0,z)}{\partial y}=0. \end{cases} \tag{1}$$

Here

$$\beta(u)=\int\limits_0^\infty e^{-uv}dB(v),\int\limits_0^\infty e^{-uv}d\frac{\partial P_1(v,z)}{\partial v}=\beta(u)H_k(u,z),$$

$$\int\limits_0^\infty e^{-uv}d\left(\int\limits_0^v B(v-x)dP_k(x,y,z)\right)=\beta(u)H_k(u,y,z).$$

$P_1(0,z)$ is probability of situation where device stays in the service mode, and there are no customers in the system [5]. Let's make changes in the third and the fourth equations of a system (1)

$H_3(u,y,z)=H_1(u,z)H_3(u,y), H_4(u,y,z)=H_2(u,z)H_4(u,y),$

$$\begin{cases} [\lambda\beta(u)-(\lambda+\tau)]\,H_3(u,y)+A(y)\tau+\\ +\tau\int_0^y H_3(u,y-x)dA(x)+\frac{\partial H_3(u,y)}{\partial y}-\frac{\partial H_3(u,0)}{\partial y}=0,\\ [\lambda\beta(u)-(\lambda+\tau)]\,H_4(u,y)+A(y)\tau+\\ +\int_0^y H_4(u,y-x)dA(x)+\frac{\partial H_4(u,y)}{\partial y}-\frac{\partial H_4(u,0)}{\partial y}=0. \end{cases} \tag{2}$$

Let's take a Laplace-Stieltjes transform of the equations of system (2), by denoting:

$$G_k(u,v)=\int_0^\infty e^{-yv}dH_k(u,y),\alpha(v)=\int_0^\infty e^{-yv}dA(y),$$

Then we have:

$$\begin{cases} [\lambda\beta(u)-(\lambda+\tau)+\tau\alpha(v)+v]\,G_3(u,y)+\alpha(v)\tau-\frac{\partial H_3(u,0)}{\partial y}=0, \\ [\lambda\beta(u)-(\lambda+\tau)+\tau\alpha(v)+v]\,G_4(u,y)+\alpha(v)\tau-\frac{\partial H_4(u,0)}{\partial y}=0. \end{cases}$$

Solution of the last system exists for $v=v(u)$ when

$$[\lambda\beta(u)-(\lambda+\tau)+\tau\alpha(v)+v]=0.$$

Then

$$v(u)=(\lambda+\tau)-\lambda\beta(u)-\tau\alpha(v(u)). \tag{3}$$

We have

$$\frac{\partial H_3(u,0)}{\partial y}=\frac{\partial H_4(u,0)}{\partial y}=\alpha(v)\tau.$$

Now let's make changes in the first and the second equations of a system (1)

$$\frac{\partial H_3(u,0,z)}{\partial y}=\alpha(v)\tau H_1(u,z),\qquad \frac{\partial H_4(u,0,z)}{\partial y}=\alpha(v)\tau H_2(u,z),$$

we'll get

$$[\lambda\beta(u)-(\lambda+\tau)+u]\,H_1(u,z)-uP_1(0,z)+$$
$$+\frac{\partial H_2(u,0)}{\partial z}T_1(z)+\alpha(v)\tau H_1(u,z)+\frac{\partial H_1(u,z)}{\partial z}-\frac{\partial H_1(u,0)}{\partial z}=0,$$
$$[\lambda\beta(u)-(\lambda+\tau)]\,H_2(u,z)+$$
$$+\frac{\partial H_1(u,0)}{\partial z}T_2(z)+\alpha(v)\tau H_2(u,z)+\frac{\partial H_2(u,z)}{\partial z}-\frac{\partial H_2(u,0)}{\partial z}=0.$$

Considering that (3), we can write down this

$$\begin{cases} [-v(u)+u]\,H_1(u,z)-uP_1(0,z)+ \\ +\frac{\partial H_2(u,0)}{\partial z}T_1(z)+\frac{\partial H_1(u,z)}{\partial z}-\frac{\partial H_1(u,0)}{\partial z}=0, \\ -v(u)H_2(u,z)+\frac{\partial H_1(u,0)}{\partial z}T_2(z)+\frac{\partial H_2(u,z)}{\partial z}-\frac{\partial H_2(u,0)}{\partial z}=0. \end{cases} \tag{4}$$

The last system will be solved by using a method of asymptotic analysis in conditions of a large load [6]– [7]. Let's denote S — the bandwidth value of a system with vacations, b — first moment of a random value, which is determined by a function of servicing time of non-priority customer $B(x)$, ε — is a small positive parameter, which in theoretical research is considered to be $\varepsilon \to 0$. Let's make changes in the system (4) of equations .

$$\lambda = (1-\varepsilon)S/b,\ u = \varepsilon w, P_1(0,z) = \varepsilon\pi_1(z,\varepsilon).$$
$$H_k(u,z) = F_k(w,z,\varepsilon), H_k(u,y,z) = F_k(w,y,z,\varepsilon),$$

S is the system's load. It's value will be found below. We'll get

$$\begin{cases} [-v(\varepsilon w)+\varepsilon w]\,F_1(w,z,\varepsilon) - \varepsilon^2 w\pi_1(z,\varepsilon) + \\ +\frac{\partial F_2(w,0,\varepsilon)}{\partial z}T_1(z) + \frac{\partial F_1(w,z,\varepsilon)}{\partial z} - \frac{\partial F_1(w,0,\varepsilon)}{\partial z} = 0, \\ -v(\varepsilon w)F_2(w,z,\varepsilon) + \\ +\frac{\partial F_1(w,0,\varepsilon)}{\partial z}T_2(z) + \frac{\partial F_2(w,z,\varepsilon)}{\partial z} - \frac{\partial F_2(w,0,\varepsilon)}{\partial z} = 0. \end{cases} \tag{5}$$

Theorem 1.The limit value for $\varepsilon \to 0$ $F_k(w) = F_k(w, \infty)$ solutions $F_k(w, z, \varepsilon)$ of the system (5) has the following form

$$F_k(w,z) = R_k(z)\Phi(w),$$

where $S = \frac{T_1}{T_1+T_2}(1-\tau a)$,

$$\begin{cases} R_1 = R_1(\infty) = \frac{T_1}{T_1+T_2}(1-\tau a) = S, \\ R_2 = R_2(\infty) = \frac{T_2}{T_1+T_2}(1-\tau a) = 1 - S - \tau a. \end{cases}$$

Asymptotic characteristic function $\Phi(w) = \frac{S\gamma}{S\gamma + w}$ is a Laplace-Stieltjes transform of an exponentially distributed random function with the parameter of γ type.

$$\gamma = \left(S\frac{b_2}{2b} + \tau\left(\frac{S}{1-\tau a}\right)^2 \frac{a_2}{2} + \frac{\Delta}{1-\tau a} \right)^{-1}.$$

Here

$$\Delta = -\frac{T_1 T_2}{T_1+T_2}R_1R_2 + \frac{R_1R_2}{T_1+T_2}\left\{ T_2\frac{T_1^{(2)}}{2T_1} + T_1\frac{T_2^{(2)}}{2T_2} \right\},$$

b and b_2 - are the starting moments of a first and second orders of time of a service of non-priority customer, a and a_2 — are the starting moments of a first and second orders of time of a service of priority customer. $\int\limits_0^\infty (1 - T_k(x))dx = T_k$ is the average time of staying in the corresponding mode. $T_k^{(2)}$ is the second initial moment of the time of device's staying in mode k. Let's find an asymptotic function of volume of work on servicing all customers

$$H(u) = \frac{(S - \lambda b)\,\gamma}{(S - \lambda b)\,\gamma + u}.$$

3. Conclusions

The found function and an exponential probability

distribution of a value of an unfinished work $V(t)$ let us perform research of a virtual time of waiting in cyclic systems by reviewing models with vacations once again.

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References

1. *Pechinkin A. V., Sokolov I. A.* Queueing system with an unreliable device in discrete time. Journal Inform. and its appl. 5(4), 6 —17, 2005. (in Russian)
2. *Saksonov E. A.* Method of the calculation of probabilities of modes for one-line queueing systems with the server vacation. Juornal Automat. and tele-mech. 1, 101 —106, 1995. (in Russian)
3. *Nazarov A. A., Terpugov A. F.* Queueing theory: educational material. NTL, Tomsk, 2004. (in Russian)
4. *Nazarov A. A., Paul S. V.* Research of queueing system with the server vacation that is controlled by T-strategy // Proc. of the International science conference Theory of probabilies, random processes, mathematical statistics and applications. —P. 202–207, 2015 (in Russian)
5. *Nazarov A. A., Paul S. V.* A Number of Customers in the System with Server Vacations. Communications in Computer and Information Science. Switzerland: Springer —Vol. 601. Distributed Computer and Communication Networks. —P. 334-343, 2016.
6. *Moiseeva E. A., Nazarov A. A.* Research of RQ-system MMP|GI|1 by using method of asymptotic analysis under large load. // TSU's

herald/messenger. Administration, calculating technics and informatics 4(25). —P 83–94, 2013. (in Russian)

7. *Nazarov A. A., Moiseev A. N.* Analysis of an open non-Markovian GI—(GI|∞)K queueing network with high-rate renewal arrival process. Problems of Information Transmission 49(2). DOI: 10.1134/S0032946013020063.

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Transient Change Detection in Mixed Count and Continuous Random Data and the Cyber-Physical Systems Security

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Abstract. The problem of sequential transient change detection is considered in the paper. The original contribution of this paper is twofold : first, a mixed count/continuous statistical model with abrupt changes is considered in the paper; second, a new sequential test for such a mixed count/continuous statistical model is designed and studied. The theoretical findings are applied to the problem of cyber-physical attack detection.

Keywords: Mixed count and continuous random data, Sequential change detection, Minimax criterion, Cyber-physical attacks, Cyber-physical systems.

1. Introduction and motivation

The problem of cyber-physical systems security is of great importance nowadays. A typical distributed cyber-physical system (networked control systems, SCADA, etc.) is composed of several physical and cyber layers. These layers are connected by different computer networks (WAN, LAN, VPN, etc.) The recent studies have established that the cyber-physical systems are vulnerable to cyber-physical attacks, when both, physical and cyber, components are sabotaged by attackers (see for details [1]).

A typical feature of cyber-physical systems is the presence of mixed count and continuous parallel data flows. The count data represent the number of events $N(t)$ occurring during a fixed time interval $(0, t]$ (for example, the number of requests per second). The data with continuous state space describe the physical parameters $\{X_t\}_{t\geq 1}$ like temperature, pressure, position/speed, etc., usually in discrete time $t = 1, 2, \ldots$. The theory and tests for sequential detection are well-developed for the observations with continuous state space and for the observations with discrete states (also for some types of point processes). To the best of our knowledge, the theory of sequential detection is practically not developed for the case of mixed count/continuous statistical models with abrupt changes.

2. Sequential detection of transient changes

Let ν be the number of the first post-change observation. It is assumed that the changepoint ν is unknown and not necessarily random. Let P_k and $\mathbb{E}_k$ denote the probability measure of $\{X_t\}_{t\geq 1}$ and its expectation when

$\nu = k$ and let P_∞ and $\mathbb{E}_\infty$ denote the same when $\nu = \infty$, i.e., there is no change. This means that $X_t \sim \mathsf{P}_0$ for every $t < \nu$ and $X_t \sim \mathsf{P}_1$ for every $t \geq \nu$ under the measure P_ν and $X_t \sim \mathsf{P}_0$ for every $t \geq 1$ under the measure P_∞. A sequential change detection test consists in calculating the stopping time T at which the change-point ν is detected. In the classical abrupt change detection, the post-change period is assumed to be infinitely long. The conventional (Shiryaev-type, Lorden-type and Pollak-type) criteria of optimality involve the minimization of the average detection delay for a given value of false alarms (see details in [2]). For instance, the minimax Lorden-type criterion of optimality based on the minimization of the worst-worst-case average detection delay (ADD) is given by [3] :

$$\inf_{T \in \mathbb{C}_\gamma} \left\{ \mathsf{ESADD}(T) = \sup_{\nu \geq 1} \operatorname{esssup} \mathbb{E}_\nu[(T - \nu + 1)^+ | \mathcal{F}_\nu] \right\} \tag{1}$$

over the class

$$\mathbb{C}_\gamma = \{T : \mathbb{E}_\infty(T) \geq \gamma\}$$

of stopping times T.

Unfortunately, such criteria of optimality (like (1)) are not adequate for the detection of transient changes of duration L (i.e., the changes of short duration) because the detection of changes after their disappearance or with the detection delay greater than a prescribed value L is considered as missed. Moreover, for the safety-critical applications, it is no matter if the true duration of the post-change period is greater than L. The changes should be detected with the delay which satisfies the following condition $T - \nu + 1 \leq L$ due to safety requirements. The penalty function related to the detection delay is quite nonlinear.

The conventional (Shiryaev-type, Lorden-type or Pollak-type) criterion warranties that some large detection delays can be compensated with some short detection delays and, hence, the (worst-worst-case) mean detection delay will be optimal. In safety-critical applications, such philosophy does not work : a detection delay greater than L cannot be compensated with a detection delay shorter than L.

Motivated by safety-critical applications, we use through this paper the criterion of optimality introduced in [4,5], which involves the minimization of the worst-case conditional probability of missed detection (under the assumption that no change occurs during the "preheating" period (i.e. it is assumed that $\nu \geq L$))

$$\inf_{T \in \mathbb{C}_\alpha} \left\{ \overline{\mathbb{P}}_{\mathrm{md}}(T; L) = \sup_{\nu \geq L} \mathbb{P}_\nu (T - \nu + 1 > L | T \geq \nu) \right\} \tag{2}$$

over the class

$$\mathbb{C}_\alpha = \left\{ T : \overline{\mathbb{P}}_{\mathrm{fa}}(T; m) = \sup_{\ell \geq L} \mathbb{P}_0 (\ell \leq T < \ell + m - 1) \leq \alpha \right\},$$

where $\overline{\mathbb{P}}_{\mathrm{md}}$ denotes the worst-case probability of missed detection and $\overline{\mathbb{P}}_{\mathrm{fa}}$ stands for the worst-case probability of false alarm within any time window of length m.

3. Main results

Let us formalize the transient change detection problem considered in this paper as follows. We sequentially observe n_c parallel independent sequences $\{X_{i,t}\}_{t\geq 1}$ of random variables (also independent) with absolutely continuous distributions F_i, $i = 1, \ldots, n_c$. We also sequentially observe n_d parallel sequences $\{N_{i,t}\}_{t\geq 1}$ of independent random variables with discrete distributions P_i, $i = 1, \ldots, n_d$. Therefore, the generative model of the continuous distributions with transient changes is given by :

$$X_{i,t} \sim \begin{cases} F_{i,0} & \text{if} \quad 1 \leq t < \nu \\ F_{i,\theta_{t-\nu+1}} & \text{if} \quad \nu \leq t \leq \nu + L - 1 \end{cases},$$

where $F_{i,\theta}$ is the parameterized cumulative distribution function during the transient change period L and $(\theta_{i,1}, \ldots, \theta_{i,L})$ is the set of known parameters defining the dynamic profile of the transient change. Without loss of generality, it is assumed that the pre-change parameter is $\theta_{i,0} = 0$. Hence, the pre-change cumulative distribution function is denoted by $F_{i,0}$.

Analogously, the generative model of the discrete distributions with transient changes is given by :

$$N_{i,t} \sim \begin{cases} P_{i,0} & \text{if} \quad 1 \leq t < \nu \\ P_{i,\theta_{t-\nu+1}} & \text{if} \quad \nu \leq t \leq \nu + L - 1 \end{cases},$$

where $P_{i,\theta}$ is the parameterized probability mass function of the discrete random variable $N_{i,t}$.

The motivation and rationalities of the Window Limited (WL) CUSUM test as a solution to the transient change detection problem can be found in [4, 5]. Let us first consider that the pre-change density is f_0 and the post-change density is f_θ. Because the conventional CUSUM test can be interpreted as a set of parallel open-ended sequential probability ratio tests (SPRTs), which are activated at each time n with the upper threshold h and the lower threshold $-\infty$ [2] :

$$T_k = \begin{cases} \min\{n \geq k : S_k^n \geq h\} \\ \infty \;\text{ if no such } n \text{ exists} \end{cases}, \; S_k^n = \sum_{t=k}^{n} \log \frac{f_\theta(X_t)}{f_0(X_t)},$$

where $k = 1, 2, \ldots$, the stopping time T_{CS} is defined as

$$T_{\mathrm{CS}} = \inf\{T_k \mid k = 1, 2, \ldots\}. \tag{3}$$

The rationality of the WL CUSUM test is due to the fact that any detection with a delay greater than L is considered as missed. Hence, the WL CUSUM test uses at each moment only L last observations. To get a more general stopping time, we consider now the following definition of the truncated SPRT with the upper variable threshold $h_1, \ldots, h_L$ and the lower threshold $-\infty$

$$T_k = \begin{cases} \min\{k \leq n \leq k+L-1 : S_k^n \geq h_{n-k+1}\} \\ \infty \text{ if no such } n \text{ exists} \end{cases},$$

$$S_k^n = \sum_{t=k}^{n} \log \frac{f_{\theta_{i-k+1}}(X_t)}{f_0(X_t)}, \; k = 1, 2, \ldots$$

Putting together the above mentioned equations, we get the stopping time of the Variable Threshold Window Limited CUmulative SUM (VTWL CUSUM) test :

$$T_{\text{VTWL}} = \inf\left\{n \geq L : \max_{1 \leq k \leq L} \left[S_{n-k+1}^n - h_k\right] \geq 0\right\},$$

$$S_{n-k+1}^n = \sum_{t=n-k+1}^{n} \log \frac{f_{\theta_{k-n+t}}(X_t)}{f_0(X_t)}.$$

Let us consider the time instant n. Considering the parallel truncated SPRTs $T_1, T_2, \ldots, T_n$, we get a set of stopping times. All these tests, consequently activated at each time m, $1 \leq m \leq n$, accumulate the statistics in the "direct‘" time. Say, the test activated at time m calculates the log-likelihood ratios (LLR) $S_m^m, S_m^{m+1}, S_m^{m+2}, \ldots$. From a practical point of view, it is more convenient to consider the on-line detection algorithm in the "inverse" time, i.e., to re-write equations (3) and (4) for a sliding window $[n-L+1; n]$. In the other words, the procedure of observation is stopped and a transient change is declared at the first time instant n when $S_{n-k+1}^n \geq h_k$ for some k such that $1 \leq k \leq L$. These LLRs S_{n-k+1}^n are calculated by using the replicas of the profile : $(\theta_1, \ldots, \theta_L)$, $(\theta_1, \ldots, \theta_{L-1}), \ldots$ (θ_1) in the sliding window $[n-L+1; n]$.

The originality of this paper with respect to previous publications is the parallel processing several mixed continuous-discrete data flows. The following results will be presented at the conference and in the extended version of the paper.

1. The design of the VTWL CUSUM test for the mixed count/continuous statistical model. In this case the LLR is given by

$$S_{n-k+1}^n = \sum_{t=n-k+1}^{n} \sum_{i=1}^{n_c} \log \frac{f_{i,\theta_{k-n+t}}(X_{i,t})}{f_{i,0}(X_{i,t})} + \sum_{t=n-k+1}^{n} \sum_{i=1}^{n_d} \log \frac{P_{i,\theta_{k-n+t}}(N_{i,t})}{P_{i,0}(N_{i,t})}.$$

The main problem with respect to the previously published results is the co-existence of parallel continuous and discrete data flows. Hence, it is necessary to study the LLR as a function of continuous state space data, represented by $\{X_t\}_{t\geq 1}$ and discrete (countable) data, represented by the number of events $\{N_t\}_{t\geq 1}$ per sampling period and to define its statistical properties.

2. The calculation of the error probabilities, i.e., the worst-case probability of missed detection $\overline{\mathbb{P}}_{\text{fa}}$ and the worst case probability of false alarm $\overline{\mathbb{P}}_{\text{fa}}$ for the proposed VTWL CUSUM test.
3. The optimization of the VTWL CUSUM test with respect to criterion (2).

References

1. V. L. Do, L. Fillatre, I. Nikiforov, and P. Willett "Security of SCADA systems against cyber-physical attacks", *IEEE Aerospace & Electronics Systems Magazine*, (in print), 2016
2. Tartakovsky, A., Nikiforov, I. and Basseville, M. (2014). *Sequential Analysis : Hypothesis Testing and Changepoint Detection*, CRC Press, Taylor & Francis Group.
3. Lorden, G. (1971). Procedures for reacting to a change in distribution. *Annals of Mathematical Statistics*, 42(6): pp. 1897–1908, Dec.
4. B. K. Guépié, L. Fillatre, and I. Nikiforov, "Sequential detection of transient changes," *Sequential Analysis*, vol. 31, no. 4, pp. 528–547, 2012.
5. B. K. Guepie, L. Fillatre, and I. Nikiforov, "Detecting an abrupt change of finite duration," in *Signals, Systems and Computers (ASILOMAR), 2012 Conference Record of the Forty Sixth Asilomar Conference on.* IEEE, 2012, pp. 1930–1934.
6. V. L. Do, L. Fillatre, and I. Nikiforov, "Sequential monitoring of SCADA systems against cyber/physical attacks," in *9th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS 2015)*, vol. 48, Paris, France, September 2015, pp. 746–753.
7. V. L. Do, L. Fillatre, and I. Nikiforov, "Sequential detection of transient changes in stochastic-dynamical systems," *Journal de la Société Française de Statistique (JSFdS)*, vol. 156, no. 4, pp. 60–97, 2015.

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Анализ развития современных услуг связи и их влияние на перераспределение трафика

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Аннотация. . В работе рассмотрены результаты исследования современных услуг связи, рассмотрены тенденции развития новых услуг, влияющие на перераспределение абонентского трафика. Определены основные проблемы, связанные с этим процессом. Статья содержит: анализ тенденций развития новых услуг, таких как OTT услуги доступа к видеоконтенту, VoIP, социальные сети, услуги дополненной реальности, анализ и прогноз изменения числа пользователей и трафика новых услуг, анализ проблем перераспределения трафика, приводится модель перераспределения трафика, построенная на основе прогнозов изменения численности пользователей и соотношения цена/качество услуг. Приведены результаты анализа развития технологий связи, тенденций развития абонентских устройств и клиентских приложений, влияния развития услуг на перераспределение абонентского трафика. Была выбрана модель абонентского трафика с учетом развития услуг. Сформирован предварительный прогноз изменения трафика.

Ключевые слова: телекоммуникации, сеть, перераспределение трафика, данные, услуги, связь, провайдер, пользователь, наложенные сервисы.

1. Введение

Одной из наиболее динамично развивающихся отраслей современности является связь. Телекоммуникации, информационные и интернет технологии — сфера деятельности, которая подвержена кардинальным, стремительным и непрерывным изменениям. На начальном этапе развития телекоммуникационный рынок был полностью монополизирован во всех странах. Деятельность в этой области, осуществляла, как правило, единая государственная компания. Эволюция инфокоммуникационных технологий привела к расширению возможностей по реализации традиционных, а также к появлению новых услуг связи. Повышение уровня развития технической базы создает предпосылки к развитию новых способов предоставления услуг связи, по этой причине происходит перераспределение абонентского трафика.

2. Основная часть

В настоящее время, говоря о технологиях связи, принято выделять несколько поколений. К первому поколения относят аналоговые системы, действующие в пределах одной страны, в рамках национальных границ, что приводило к несовместимости таких сетей. Второе поколение составляют системы, охватывающие отдельные регионы земного шара. Данный период характеризуется повсеместным распространением систем второго поколения, расширением спектра предоставляемых услуг, передача данных, наиболее распространенным стандартом, имеющим небывалый успех, стал стандарт GSM. Огромный спрос в мире на услуги подвижной связи и успехи в развитии технологии радиопередачи, обеспечившие надёжную связь с высокими скоростями, привели к созданию систем третьего поколения. Процесс глобализации мобильной связи призван завершить системы третьего поколения. Началом работ над четвертым поколением можно считать семинар по развитию сети радиодоступа, который прошел в Канаде в ноябре 2004 года. Именно там были сформированы основные направления развития сетей четвертого поколения: снижение стоимости передачи бита информации, расширение спектра предоставляемых услуг, упрощенная архитектура и открытые интерфейсы. Опережающее развитие телекоммуникаций является необходимым условием для создания инфраструктуры бизнеса и важным фактором подъема национальной экономики, роста деловой и интеллектуальной активности общества, укрепления авторитета страны в международном сообществе. Этапы развития отрасли связи можно представить в виде трех платформ. Первая строится на базе универсальных серверов и терминалов, на которых работало огромное количество приложений и пользователей; основание второй образуют традиционные персональные устройства, сеть Интернет, архитектура клиент-сервер и сотни тысяч приложений; последняя – третья платформа опирается на стремительно растущее количество непрерывно подключенных к сети Интернет персональных устройств, постоянное широкое использование социальных сетей, а так же на развитую облачную инфраструктуру, которая применяется для решения комплексных задач. Платформы эволюции рынка информационных технологий представлены на Рис. 1.

Повышение уровня развития технической базы и расширение возможностей сети и развитие рынка телекоммуникационных технологий приводит к увеличению возможностей по реализации услуг связи и, как следствие ведет за собой увеличение числа провайдеров, способных предоставить необходимые услуги связи посредством наложенных (OTT) сервисов. При создании OTT сервисов провайдер услуги организует серверную часть, сопрягая ее сетью связи, а также разрабатывает клиентские приложения, реализующие доступ пользователя к услуге[1]. Операторы связи не могут как-либо ограничивать доступ своих абонентов к услугам-заменителям, это значит, что в подобных условиях деятельность операторов на рынке оказания услуг связи начинает сужаться до роли, при которой они обеспечивают лишь доступ

Рис. 1. Платформы эволюции рынка ИТ

абонентов к необходимым услугам. Иными словами сеть оператора превращается в «трубу» для передачи трафика, при этом сеть будет не в состоянии справляться с растущими объемами трафика, в то время как оператор связи не будет обладать необходимыми для развития сети финансовыми возможностями. По данным исследований, только в 2014 г. российские операторы мобильной связи не досчитались не менее 15-20 млрд рублей или около 2% своих доходов из-за мобильных интернет-мессенджеров и VoIP-сервисов (Рис. 2)[2].

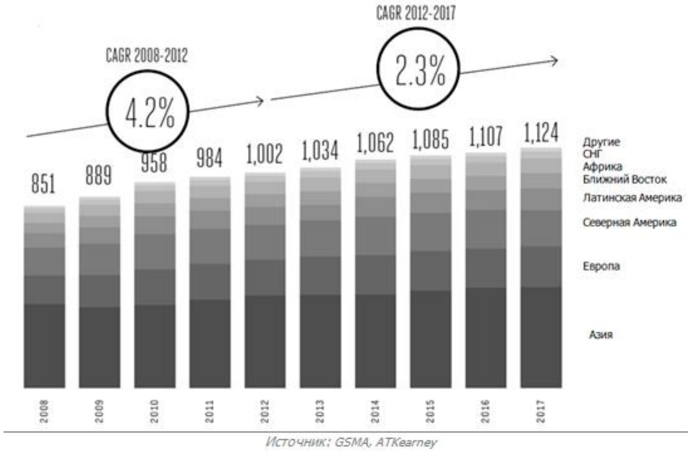

Рис. 2. Выручка мобильных операторов

В сетях операторов стал преобладать трафик приложений типа Skype, Viber, с различными видами видео- и аудиоинформации[3]. Преобладавший ранее трафик услуг телефонной связи и отдельно услуг по передачи данных трансформировался к разноклассовому трафику (видео, аудио, данные), передаваемому на базе единых технологий, и чувствительному к задержкам и потерям. Учитывая эти факторы, а также повсеместный переход на единые технологии оказания услуг связи на базе сетей с коммутацией пакетов, приведут к определенной потере операторами контроля над пользователями в части выбора той или иной услуги. Единственным действенным методом остается возможность управления качеством услуг, предоставляемых на платформе оператора связи, или при взаимодействии с ней. Развитие технологий и услуг связи неизбежно приводит к изменению свойств абонентского трафика. Это изменение определяется рядом факторов, таких как: изменение базовых принципов реализации услуг связи во внедряемых технологиях; изменение набора услуг связи; изменение спроса пользователей на услуги связи; изменением проникновения технологий (числа пользователей); изменение стоимости услуг связи; изменение способов учета стоимости услуг.

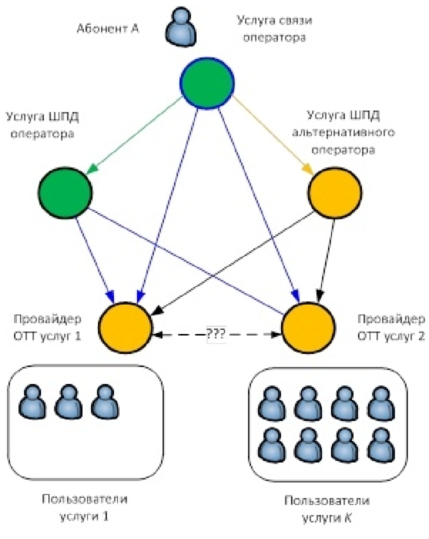

Рис. 3. Выручка мобильных операторов

Для планирования развития той или иной технологии связи, решения задач проектирования и эксплуатации сетей связи необходимо использование данных об абонентском трафике, описывающих основные, значимые для конкретной задачи характеристики[4,5]. Таким образом, для организации взаимовыгодного взаимодействия всех участников телекоммуникационной системы необходимо определить множество условий и правил их взаимодействия. Для этого необходимо иметь модель, описывающую изменение абонентского трафика в данных условиях. При построении модели полагается, что основные факторы, влияющие на перераспределение абонентского трафика: численность абонентов OTT сервисов (определяет доступность услуги), соотношение цена качество (определяет субъективные предпочтения абонента). На Рис. 3 приведено схематичное представление модели распределения абонентского трафика.

Приняв данную модель получен прогноз изменения соотношения долей трафика базовых услуг оператора связи и OTT сервисов (Рис. 4)

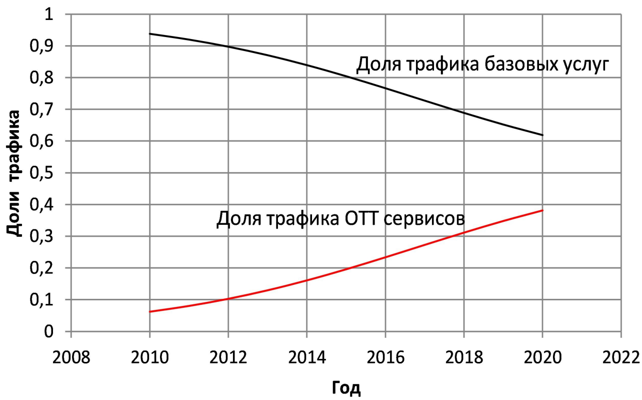

Рис. 4. Выручка мобильных операторов

Из приведенных на Рис.4 результатов прогнозирования перераспределения трафика можно заметить, что в настоящее время, доля трафика OTT сервисов составляет примерно 18% от общего голосового трафика, а до 2020 г. возрастает до 40

3. Заключение

Результаты исследований показали, что в настоящее время происходит интенсивное развитие наложенных (OTT) сервисов, которое сопровождается интенсивным ростом числа их пользователей и абонентского трафика. Рынок услуг таких сервисов является на сегодняшний день одним из самых динамично развивающихся. Результатом работы является модель перераспределения трафика услуг связи, учитывающая их доступность и соотношение цена-качество, а так же прогнозы перераспределения трафика ряда популярных услуг связи. Существует очевидная задача урегулирования отношений между операторами связи и провайдерами услуг связи, с целью соблюдения интересов обоих сторон и интересов пользователей, а так же обеспечения дальнейшего развития услуг связи и качества их предоставления.

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Литература

1. *Кучерявый А. Е., Парамонов А. И.* Миграция речевого трафика в современных сетях связи // Электросвязь. – 2007. - № 12. – С. 20-22.
2. Analytics Данные официальной финансовой отчетности российских операторов связи [Электронный ресурс] / URL: *https : //www.strategyanalytics.com*
3. Оптимальные коммуникации [Электронный ресурс] / URL: $http : //www.oc.ru/media/el_15_paramonov.html$
4. *Парамонов А. И.* Разработка и исследование комплекса моделей трафика для сетей связи общего пользования // Диссертационная работа
5. *Кучерявый А. Е., Нестеренко В. Д., Парамонов А. И., Ревелова З. Б.* Анализ трафика пользователей Интернета в ТфОП // Электросвязь. - 2004.

UDC 621.395

Analysis Of Modern Communication Services And Their Impact On The Redistribution Of Traffic

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The results of the analysis of the traffic changes related to the rapid development of Internet services. The tendencies of development of new services and providers, affect the redistribution of subscriber traffic. The main problems associated with this process. The work includes: analysis of trends in the development of new services, -Analysis and forecast changes in the number of users and traffic of new services, the analysis of the problems of redistribution of subscriber traffic.

Keywords: telecommunications, net, traffic redistribution, data, services, communication, provider, user, imposed services.

УДК 004.057.4

Методология исследования и управления трафиком мультисервисных радиосистем обмена информацией

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Аннотация. Настоящая статья описывает общую методологию исследования ресурсных и временных параметров конвергентных телекоммуникационных сетей радиодоступа. Рассматриваются основные задачи декомпозиции, анализа и синтеза, модернизации и обеспечения качества обслуживания в конвергентных (мультисервисных) системах обмена информацией, использующих каналы радиодоступа.

Ключевые слова: сети радиодоступа; мультисервисные системы обмена информацией.

1. Введение

Объект исследования: коммуникационные системы (КС) с радиоканалами, относящиеся к классу конвергентных (мультисервисных) систем обмена информацией (МСОИ).

Предмет исследования: декомпозиция, моделирование, расчет, оценка, анализ, выбор параметров, синтез, обеспечение качества обслуживания в КС класса МСОИ с радиоканалами доступа.

Актуальность: постоянно растет объем передаваемых данных, появляются новые услуги, увеличиваются объемы мультимедиа, растут требования к скорости передачи.
Необходим выбор параметров КС на всех этапах жизненного цикла:

— проектирования;
— развертывания сети;
— стадия модернизации;
— режим функционирования (реальный масштаб времени).

КС класса МСОИ являются сложными гетерогенными системами, состоящими из множества элементов нескольких типов. Наряду с опорной сетью (ядром сети) использование систем радиодоступа в них становится базовым и самым массовым.
Общемировые данные о сетях 4 поколения (4G LTE и LTE-Advanced) постоянно обновляются и по состоянию на 2015 год:

— сетей 4G – более 500;
— абонентов 1 млрд.

Прогноз на 2020 год:

— переход к сетям 5 поколения 5G (m2m, IoT, огромная скорость);
— абонентов 4G и 5G будет 5 млрд;
— устройств 50 млрд.

В таких сетях необходимо:

— выделять и распределять ресурсы;
— обеспечивать качество обслуживания;
— своевременно выявлять слабые места сети;
— вовремя изменять параметры (модернизировать сеть).

Для решения задач необходима общая методология: постановка задач, модели элементов, систем, кластеров покрытий и сети в целом, методики решения этих задач.
Основным критерием, который определяет качественные параметры сети, является ее производительность.

2. Постановка задач исследования

Существует множество различных моделей и методов, которые ориентированы на отдельные подсистемы, кластеры, режимы, этапы жизненного цикла КС.
Выбор моделей, методов, разработка методик определяются ограничениями, требуемой детализацией и точностью моделирования элементов сети, систем и кластеров покрытий КС. Общий перечень задач методологии исследования КС [3] следующий:

— выбор структуры, топологии КС (размещение базовых станций, БС);
— расчет, оценка, выбор параметров БС и кластеров;
— расчет рабочей и пиковой производительности;
— балансировка нагрузки на каналы, БС, узлы сети.

Исходными данными при решении задач исследования являются следующие ограничения:

— количество пользователей и сервисных услуг;
— отсутствие последействия (марковский трафик);
— задана структура, топология сети, предполагаемая нагрузка;
— пропускная способность и количество каналов связи ограничено;
— ограничены размеры пакетов запросов, длины сообщений, транзакций по типам услуг (данные, голос, видео и др., например, m2m, IoT).

Для охвата всего жизненного цикла КС необходима целостная методология оценки и выбора параметров КС, проектирования, разработки, модернизации сети, а также – распределения и управления ресурсами КС в рабочем режиме.
Рассматриваются и моделируются следующие уровни модели OSI/ISO:

— канальный уровень и

— два сетевых подуровня:
  – подуровень управления и
  – подуровень коммутации.

Канальный уровень, подуровни управления и коммутации оказывают наибольшее влияние на качество обслуживания трафика КС.

3. Методология решения задач исследования

Предлагается следующая методология исследования (декомпозиции, анализа синтеза, модернизации и) сети ТКС:

— декомпозиция сети на элементы, системы, кластеры покрытий;
— расчет, оценка, анализ и выбор параметров сети;
— сбор исходных данных;
— методики расчета, оценки и выбора параметров сети, кластеров покрытий и систем.

Виды декомпозиции ТКС:

— функциональная (по видам услуг);
— структурно-топологическая.

Декомпозиция сети выполняется по:

— видам услуг;
— элементам системы;
— кластерам покрытий.

Традиционная классификация информационных услуг (triple play) – это: данные, голос и видео. В последнее время так же к услугам относят получение координат (quad play). Возможны и другие услуги.
Кроме этого, предлагался протокол DSCP типов обслуживания сервисных потоков (услуг), в частности: BE (best effort), RT (real time), nRT (non-real time), UGS (unsolicited grant service) и приоритезация от 0 до 7. Однако DSCP в большинстве реализаций практически не используется. Предлагается более простая классификация информационных потоков:

— квазисинхронная информация (голос, аудио: это жесткий (hard) трафик, который задается частотой следования коротких пакетов и малым джиттером, причем последовательность пакетов не должна нарушаться);
— асинхронные данные (передача видео, контента, файловый обмен: гибкий (resilient) трафик, задается средней скоростью передачи, пакеты длинные, очередность их следования может нарушаться).

В проведенных исследованиях даны необходимые определения элементов КС. Проанализированы альтернативные модели КС, выбраны модели, адекватные исследуемым процессам [1] [2] [3]:

1. элементов КС: узлов $Y(\mu)$ (активных элементов) и каналов $K(c)$ (пассивных элементов), где μ – интенсивность обслуживания узлов, c – пропускная способность каналов;

2. трафика: жесткий (hard) – голос, гибкий (resilient) – данные; модели: on/off; экспоненциальная и фрактальная; δ-функция и учет оверхеда, скорости v следования голосовых пакетов ($v = b * f$=длина*частоту) и/или пакетов данных;
3. производительности:
   (a) в переходных режимах (случайный доступ, регистрация абонентов) – гиперэкспоненциальная модель;
   (b) в рабочем режиме – модели производительности: $G/G/1/N$ $(n = \mu/\lambda + 1)$, $G/G/m/N$ $(n = m(\mu/\lambda + 1))$;
   (c) в рабочем режиме – разностные модели нагрузки: $\Delta\mu = \mu - \lambda$ – для узлов и $\Delta c = c - \lambda$ – для каналов.
4. балансировки нагрузки – по методу оврагов (шаг по оврагу);
5. управления – метод брокера, фильтрация Калмана, модель Беллмана (функция цены, минимизация суммарной задержки).

Методология.
Разработана методология [3], состоящая из методик, моделей и сценариев развертывания систем, кластеров и КС в целом:

1. Методика сбора и анализа исходных данных;
2. Метод декомпозиции КС на узлы, каналы, кластеры, услуги;
3. Модели элементов КС (узлов и каналов, кластеров);
4. Модели трафика (данные, голос/аудио);
5. Методика расчета и оценки производительности элементов КС (узлов, каналов, кластеров);
6. Методика сценариев развертывания КС и проникновения на рынок;
7. Балансировка нагрузки (с целевыми функциями по минимизации энергопотребления (энергосбережения) или максимизации пропускной способности, производительности);
8. Модель рабочего режима функционирования КС (БС, кластеров);
9. Модель переходного режима (например, при включении БС);
10. Модель управления гибридной сетью (построение самоуправляющейся КС, которая увязана с бизнес-целями и реализует политики по нескольким контурам с минимальным вмешательством человека – предложена компанией Ericsson).

Модели КС:

1. Передачи данных (уровень анализа, выбора параметров, проектирования, модернизации КС).
2. Уровня управления (режим рабочего функционирования КС).
3. Модели управления ресурсами КС, как динамическими системами массового обслуживания (СМО) и сетями очередей (СеМО)

Решена задача построения совместного управления состояниями сети и наблюдениями.
Модельные ограничения этой задачи:

— СМО или СеМО – марковская;

— исходные данные: полные или неполные;
— входные потоки – последовательность моментов прихода требований.

Конкурирующие модели управления ресурсами КС:

— Фильтры Калмана;
— Метод Брокера [4](использует немарковскую предисторию процесса);
— Функция Беллмана [5] (функция цены или задержки).

Предлагается использовать метод управления ресурсами КС по Беллману, который позволяет учитывать, как производительность сети (максимизировать количество пользователей), так и минимизировать энергопотребление (как основные эксплуатационные затраты).

4. Заключение

1. Предложены адекватные модели, методы, методики (сценарии), необходимые для проектирования, создания, выбора параметров, модернизации, управления трафиком КС, т.е. для охвата полного жизненного цикла КС [1] [2] [3].
2. Основными параметрами КС, которые необходимо учитывать и управлять ими, являются следующие [6]:
   (a) Учет временных задержек и джиттера при обмене данными или при передаче информации;
   (b) Максимизация производительности КС (количества пользователей по услугам всех типов) в пересчете на БС и/ или кластер;
   (c) Минимизация энергопотребления.

Литература

1. *Широков В. Л.* Модели оценки производительности многофункциональных систем обмена трафиком на примере беспроводных сетей доступа Wi-Fi, Wireless MAN и WiMAX // Эл.журнал BC/NW, 2004, № 2 (5), раздел 6, статья 1. URL: http://network-journal.mpei.ac.ru.
2. *Широков В. Л.* Разработка моделей и методов для оценки и выбора параметров мультисервисных систем обмена информацией // Диссертация на соискание ученой степени к.т.н. — М.: МЭИ (ТУ). — 2006. 225 с.
3. *Широков В. Л.* Методология создания беспроводных мультисервисных сетей класса WiMAX. Части 1-3 // Технологии и средства связи. №1, 2010, стр.32-33; №3, 2010, стр.24-25; №5, 2010, стр.37-36.
4. *Широков В. Л.* Использование метода брокера при оценке и резервировании пропускной способности каналов // Эл.журнал

BC/NW 2012, №2 (21), раздел 8, статья 1. URL:http://network-journal.mpei.ac.ru.
5. *Солодянников Ю. В.* Управление и наблюдение для динамических СеМО. Части I-II // Автоматика и телемеханика. 2014. Т. 75. No.3, стр. 14-45; No.5, стр. 91-114.
6. *Широков В. Л.* Влияние временных задержек на производительность мультисервисных коммуникационных сетей // Эл.журнал BC/NW 2008, №2 (13), раздел 9, статья 1. URL:http://network-journal.mpei.ac.ru.

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Methodology of Research and Control Parameters of Convergent Radio Access Network

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The paper describes the general methodology for the research of the resource and time parameters of the convergence (multiservice) telecommunication radio access networks. Looks at the main issues of decomposition, analysis and synthesis, modernization, and maintenance of quality of service in the convergent information sharing systems using a radio access channels.

Keywords: convergent network; radio access network.

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On telecom services evaluation

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Abstract. Communication specialists around the world are facing the same problem: shifting from circuit switching to packet switching. Network paradigm shift means the transition from Signal System Seven (SS7) signaling to Internet Protocol (IP). SS7 is a set of protocol standards that controls signaling for the public switched telephone network, allowing mobile carriers around the world to pass information across their networks. The transition to IP assumes that the IP protocol will be the only means of communication between the transport layer and applications. Telecom services evaluation due to the transition from Time-division multiplexing network to IP are considered and some lessons for Russian telecommunications are named.

Keywords: circuit switching, packet switching, ISDN, SS7, intelligent network, softswitch, SIP, AS-SIP.

1. Introduction

Communication specialists around the world are facing the same problem: shifting from circuit switching to packet switching, from ISDN signaling and AIN (Advanced Intelligent Network) architecture to all-IP world. For example, most of the broadcasters now recognize that IP networks are more flexible, cheaper to upgrade and extremely reliable when configured correctly [1]. For operators, all-IP world lets bypass security problems in SS7 [2], etc. In the same time, this process has own issues, especially, on the global level [3].

The article is devoted to the discussion of the telecommunications development strategy. We will provide examples to illustrate the difficulties that complicate the transition from CS to PS to web-oriented services.In Section 2, we discuss the orientation towards AIN. In Section 3, we discuss the transition from TDM to IP. Sections 4 and 5 consider MFSS services.In Section 6, some lessons for Russian telecommunications are named.

2. On the orientation towards AIN

The AIN architecture was developed by Bell Labs in the 1970s. The basic AIN design includes (Figure 1):

- STP (Signaling Transfer Point)

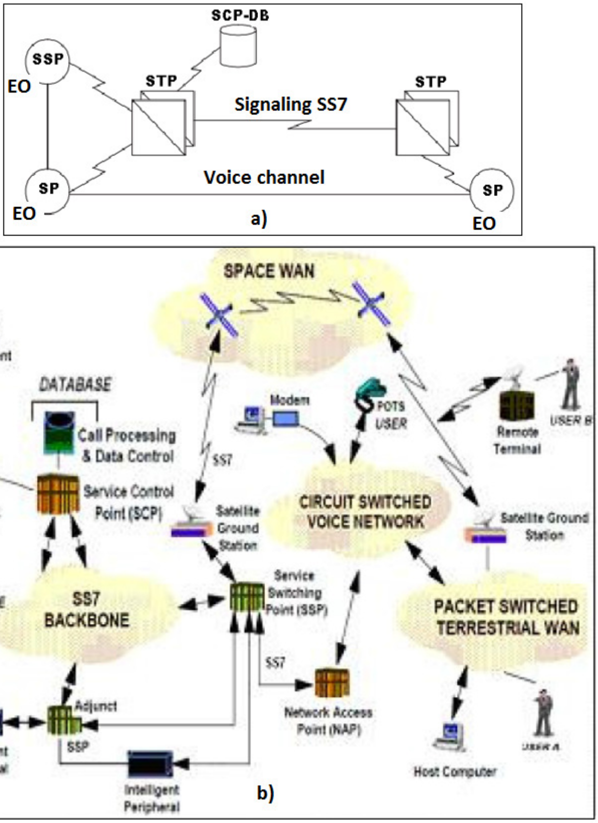

Figure 1. a) AIN basic design. b) An implementation of AIN Service Architecture

- SSP (Service Switching Point)
- SCP-DB (Service Control Point with Database)
- each End Office (EO) contains Signaling Point (SP).

The AIN provides integrated "one stop" end-user services, such as voice, data, video, email, images, office applications, and 800 services. SS7 is a means by which elements of telephone networks exchange information. Information is conveyed in the form of messages. SS7 defines the procedures for the setup, ongoing management, and clearing of a call between users. The key points of AIN are the following: Service Control Point and Database of services, as well as TCAP (Transaction Capabilities

Application Part) - a main protocol in the SS7 protocol stack, providing access to databases.

Intelligent Peripheral also plays an important role: its functions include tone generation, voice recognition, speech and data compression, dialing recognition, and much more, including tactical and strategic services for personnel identification. The Adjunct provides the same operation as the SCP but is configured for one or fewer services for a single switch. The Network Access Point (NAP) is a switch that has no AIN functions. It is connected off a SSP and interfaces to trunks with SS7 messages. It will route the call to its attached SSP or AIN services based on the called and calling number received. Channel switching network subscribers, as well as packet switching network subscribers, can be AIN users. Point out the attention to the Service Creation Environment (SCE) as a standardized means for service software development.

3. MFSS - the transition from TDM to IP

The Information System Network paradigm shift means the transition from SS7 signaling to IP protocol. It is assumed that the IP protocol will be the only means of communication between the transport layer and applications.

Note that SIP protocol was designed to solve a small but important set of issues and to allow interoperability with a broad spectrum of existing and future IP telephony protocols. SIP is regularly deployed alongside SOAP, HTTP, XML, VXML, WSDL, UDDI, SDP, RTP and a variety of other protocols.But SIP, as a signaling protocol, does not have the ability to break into ongoing calls. The support for Multi-Level Precedence and Preemption (MLPP) can be used instead. For this reason, particularly, Assured Services SIP protocol was invented. Understanding the differences between AS-SIP and standard SIP is not a trivial task.RFC 5638 (Simple SIP) specifies the support of only 11 RFCs necessary to create a SIP appliance with presence, instant messaging, audio and video communications. The UCRrequires support for nearly 200 RFCs. It is the substantially large number of requirements for the end instrument that make AS-SIP different from the typical SIP stack.

The most important step for network modernization is the replacing of channel switching electronic Multifunctional switches (MFS) by packet switching routers under AS-SIP signaling. The transition phase is based on Multifunctional SoftSwiches (MFSS) developed by CISCO.

The MFSS (Figure 2) will be interfacing between the TDM and IP backbone network and will have much more complex circuit-switched based interfaces along with simple packet-switched based IP interfaces. The ISDN network uses ISDN User Part (ISUP) signaling protocol for the session/call control. So, MFSS will also need to provide ISUP-SIP interworking function (IWF). It is expected that TDM switching portion of the MFSS will be retired as soon as all users/systems migrate to IP.

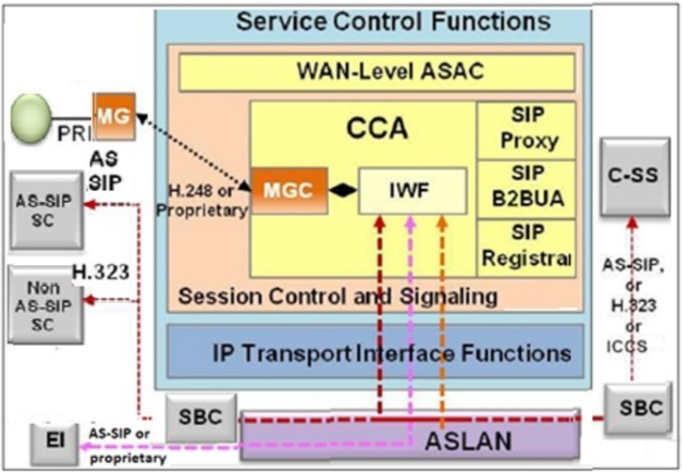

Figure 2. Reference model for Multifunction SoftSwitch.

The MFSS provides all required PSTN/ISDN interface functions, including ISUP, CCS7/SS7, and CAS signaling and media conversion. A signaling gateway (SG) deals with all signaling protocols such as ISUP, CCS7/SS7, and CAS. The MFSS also operates as a media gateway (MG) between TDM circuit-switching and IP packet-switching under the control of the media gateway controller (MGC) while communications control protocol like H.248 is used between MG and MGC. Besides, there are EI (End Instrument) and AEI (Assured Services End Instrument), following AS-SIP, as well as PIE (Proprietary Internet Protocol Voice End Instrument). H.323 is the leading protocol for video conferencing. With its superior handling of video and conference control, no other protocol comes close to matching the capabilities of H.323 for video.

4. MFSS services

Take an attention to the Service Control Function (Figure 3). The SCF cooperates with 19 servers by a lot of protocols. Besides, the UC Extensible Messaging and Presence Protocol (XMPP) supports the full potential of Instant Messaging (IM), Chat, and Presence.

It is worth to pay attention to TCAP (Transaction Capabilities Application Part) Application Server. The TCAP enables the deployment of advanced intelligent network services by supporting non-circuit related information exchange between signaling points using the Signaling Connection Control Part (SCCP) connectionless service in SS7 networks of

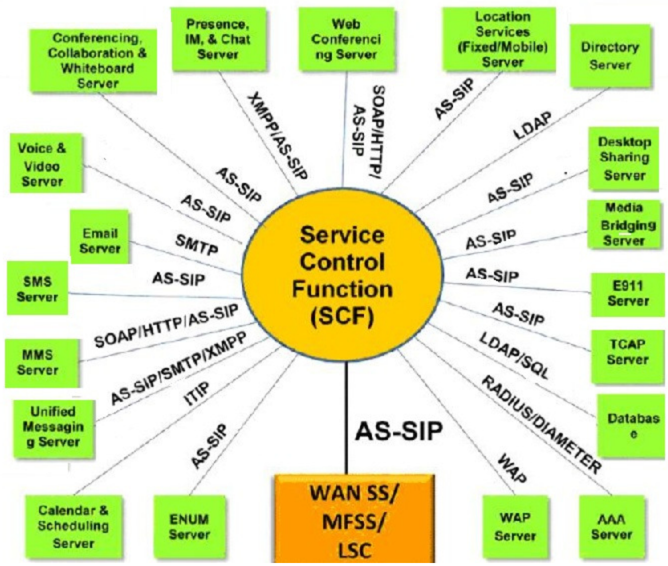

Figure 3. Service Control Function and its 19 servers

the TDM network. A service switching point (SSP) uses TCAP to query a signaling point control point(SCP) to determine the routing number(s) associated with a dialed 800, 888, or 900 numbers. The SCPuses TCAP to return a response containing the routing number(s) (or an error or reject component) back to the SSP. Calling card calls are also validated using TCAP query and response messages.

5. IETF moves towards MLPP services

The Multilevel Precedence and Preemption (MLPP) service is the key feature of AS-SIP. It allows properly validated users to place priority calls. If necessary, users can preempt lower priority phone calls. Precedence designates the priority level that is associated with a call. Preemption designates the process of terminating lower precedence calls that are currently using the target device, so a call of higher precedence can be extended to or through the device. This capability assures high-ranking personnel of communication to critical organizations and personnel during network stress situations, such as a national emergency or degraded network situations.

RFC 4542 [4] describes six precedence classes, in descending order:

1. Executive Override (or Flash Override): used by the Commander in Chief, Secretary of Defense, and Joint Chiefs of Staff, commanders

of combatant commands when declaring the existence of a state of war. Commanders of combatant commands when declaring Defense Condition Flash Override cannot be preempted.

2. Flash Override: the same users. Commanders of combatant commands when declaring Defense Condition One or Defense Emergency and other national authorities the President may authorize. Flash Override cannot be preempted in the DSN.
3. Flash: reserved generally for telephone calls pertaining to command and control of military forces essential to defense and retaliation, critical intelligence essential to national survival, conduct of diplomatic negotiations critical to the arresting or limiting of hostilities, dissemination of critical civil alert information essential to national survival, continuity of federal government functions essential to national survival, fulfillment of critical internal security functions essential to national survival, or catastrophic events of national or international significance.
4. Immediate: reserved generally for telephone calls pertaining to situations that gravely affect the security of national and allied forces, reconstitution of forces in a post-attack period, intelligence essential to national security, conduct of diplomatic negotiations to reduce or limit the threat of war, implementation of federal government actions essential to national survival, situations that gravely affect the internal security of the nation, Civil Defense actions, disasters or events of extensive seriousness having an immediate and detrimental effect on the welfare of the population, or vital information having an immediate effect on aircraft, spacecraft, or missile operations.
5. Priority: reserved generally for telephone calls requiring expeditious action by called parties and/or furnishing essential information for the conduct of government operations.
6. Routine: designation applied to those official government communications that require rapid transmission by telephonic means but do not require preferential handling.

MLPP is intended to deliver a higher probability of call completion to the more important calls. The rule, in MLPP, is that more important calls override less important calls when congestion occurs within a network. More than one call might properly be preempted if more trunks or bandwidth is necessary for this higher precedence call.A video call (perhaps of 384 KBPS, or 6 trunks) competing with several lower- precedence voice calls is a good example of this situation.

6. On the future service capabilities

Service capabilities are the integration of voice, video, and/or data services delivered ubiquitously across a secure and highly available network infrastructure, independent of technology, to provide increased mission effectiveness to the war fighter and business communities.

For example, Voice Features and Capabilities are the following:

- Call Forwarding: on Busy Line, Don't Answer, Selective Call Forwarding
- Multi-Level Precedence and Preemption (MLPP): Interactions With Call Forwarding , at a Busy Station, No Reply at Called Station
- Precedence Call Waiting: Busy With Higher Precedence Call, Busy With Equal Precedence Call, Busy With Lower Precedence Call, No Answer,Line Active With a Lower Precedence Call, Call Waiting for Single Call Appearance VoIP Phones
- Call Transfer: at Different Precedence Levels, at Same Precedence Levels
- Call Hold
- Three-Way Calling

The capabilities described in the previous section are provided through a collection of services, where a service is defined as 'a mechanism to enable access to a set of one or more capabilities'.

7. Some lessons for Russian telecommunications

On emergency services. Development of the system 112 is a complex project of national importance. The project covers all aspects of the life of Russian society. In the course of its realization are exposed many shortcomings of the country's economy, accumulated over a quarter-century of capitalism development. To illustrate the diversity of network requirements NG9-1-1, we present the scheme of NG9-1-1 activities for single US state. According to official documents [5], the future network of emergency services has to be packet-switched networks. Particularly it noted that NG9-1-1 network must support multimedia and practically cover all aspects of social life, as shown in Fig. 4. It is worth to consider the transition to the AS-SIP protocol.

On critical infrastructure protection. Data network is of the highest importance for the sustainable city.In order to increase its reliability, we offer the new architecture of the data network.It should not focus on client-server architecture (data centers and terminals), but on a fully distributed architecture, in which each object is acting as a terminal and a server at the same time and where critical objects interact with each other using DDS publish-subscribe model to specify the QoS. This applies particularly to sensor networks, mobile units, the operational headquarters on the site of disaster and others. Messaging zones can overlap with global data networks.

Circuit switching versus packet switching. Conduct a systematic study and compare the features of circuit switching and packet switching technology. A policy of import substitution has discussed now in Russia. If we really go for the construction of communications networks on its own, it should return to the state of knowledge achieved before - some 20 years ago, and to develop them further. In this case, the system SS7, as well as intelligent network are reasonable to consider as a reference point. In Russia, the gap from the more advanced countries is the great one,

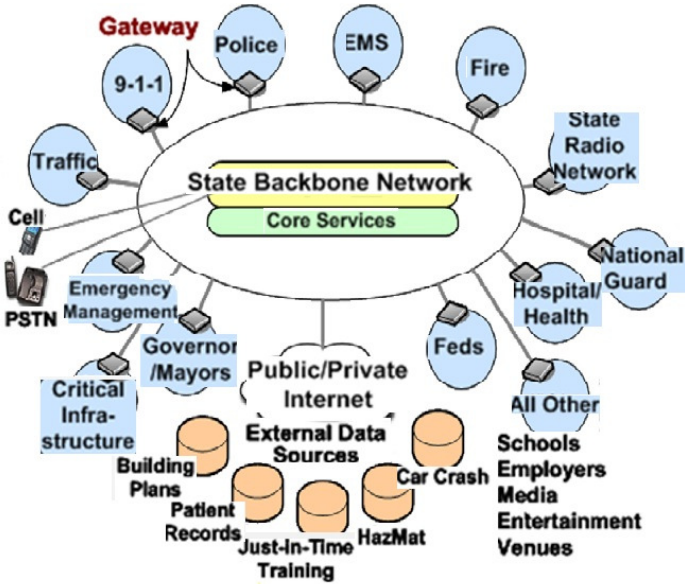

Figure 4. NG9-1-1 activities for a single U.S. state [5]

especially on the packet switching technique, which requires a high level microelectronics. Therefore, channel switching is presently preferred

References

1. *Lin Y., Pang A. C.* Wireless and mobile All-IP networks. — John Wiley & Sons, 2005.
2. *Sneps-Sneppe M.* "On telecom services and the DISN evolution." International Journal of Open Information Technologies 3.8 (2015): 40-47
3. *Sneps-Sneppe M.* "Circuit Switching versus Packet Switching." International Journal of Open Information Technologies 3.4 (2015): 27-37.
4. *Baker F.* Implementing an Emergency Telecommunications Service (ETS) for Real-Time Services in the Internet Protocol Suite, RFC 4542, May 2006.
5. Model State 911 Plan, National Association of State 911 Administrators, DOT HS 811 36. 9 February 2013.

UDC 004.4

Performance Modeling of Finite-Source Congnitive Radio Networks Using Simulation

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1. Introduction

Cognitive radio has emerged as a promising technology to realize dynamic spectrum access and increase the efficiency of a largely under utilized spectrum. In a cognitive radio network (CRN), a cognitive or secondary users (SUs) are allowed to use the spectrum by primary users (PUs) as long as the PUs do not use it. This operation is called opportunistic spectrum access [1]. To avoid interference to PUs, SUs must intelligently release the unlicensed spectrum if a licensed user appears [3].

In this paper we introduce a finite-source queueing model with two (non independent) frequency channels. According to the CRN modelling the users are divided into two types: the Primary Users (PUs) have got a licensed frequency,which does not suffer from overloading feature. The Secondary Users (SUs) have got a frequency band too, but suffers from overloading. A newly arriving SU request can use the band of PUs (which is not licensed for SUs) if the band of SUs is engaged, in the cognitive way: the non-licensed frequency must be released by the SU when a PU request appears. In our environment the band of the PUs is modelled by a queue where the requests has preemprive priority over the SUs requests. The band of the SUs is described by a retrial queue: if the band is free when the request arrives then it is transmitted. Otherwise, the request goes to the orbit if both bands are busy. We assume that the radio transmission is not reliable, it will fail with a probability p for both channels. If happens then the request retransmission process starts immediately [3].
Hence, it should be noted that the novelty of this work is that we create a new model to analyze the effect of distribution of inter-event time on the mean and variance of the response time of the PUs and SUs. In several combinations of the distribution of the involved random variables and using simulation we compare the effect of their distribution on the first and second moments of the response times illustrating in different figures.

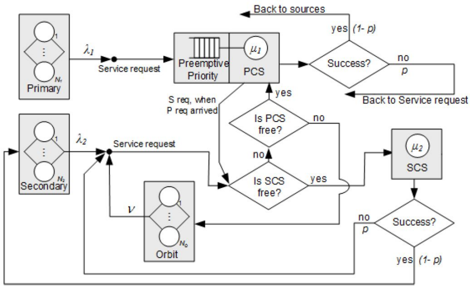

Figure 1. A priority and a retrial queue with components

2. System Model

A Fig.1 illustrates a finite source queueing system which is used to model the considered cognitive radio network. The queueing system contains two interconnected, not independent sub-systems. The first part is for the requests of the PUs. The number of sources is denoted by N_1. In order to analyse the effect of the distribution, these sources generate high priority requests with hypo-exponentially and hyper-exponentially distributed inter-request times with parameter λ_1. The generated requests are sent to a single server unit (Primary Channel Service - PCS) with preemptive priority queue. The service times are supposed to be also hypo-exponentially and hyper-exponentially distributed with parameter μ_1. The second part is for the requests of the SUs. There are N_2 sources,the inter-request times and service times of the single server unit (Secondary Channel Service - SCS) are assumed to be hypo-exponentially and hyper-exponentially distributed random variables with parameters λ_2 and μ_2 respectively.

A generated high priority packet goes to the primary service unit. If the unit is idle, the service of the packet begins immediately. If the server is busy with a high priority request, the packet joins the preemptive priority queue. When the unit is engaged with a request from SUs, the service is interrupted and the interrupted low priority task is sent back to the SCS. Depending on the state of secondary channel the interrupted job is directed to either the server or the orbit. The transmission through the radio channel may produce errors, which can be discovered after the service. In the model this case has a probability p, and the failed packet is sent back to the appropriate service unit. When the submission, is successful (probability *1-p*), the requests goes back to the source.
In case of requests from SUs. If the SCS is idle, the service starts, if the SCS is busy, the packet looks for the PCS. In case of an idle PCS, the service of the low priority packet begins at the high priority channel

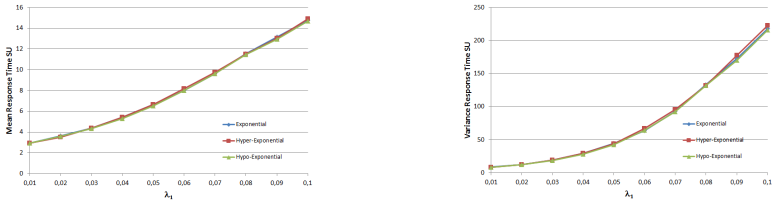

Figure 2. The effect of inter-request time distribution of the PUs on the variance and mean response time of SUs vs λ_1

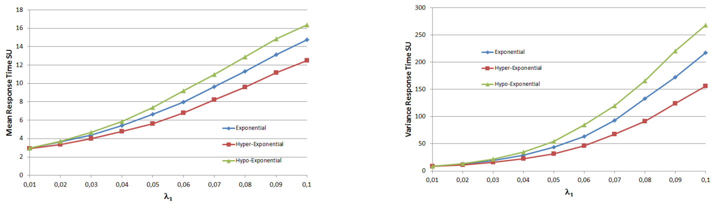

Figure 3. The effect of service time distribution of the PUs on the variance and mean response time of SUs vs λ_1

(PCS). If the PCS is busy the packet goes to the orbit. From the orbit it retries to be served after an exponentially distributed time with parameter ν. The same transmission failure with the same probability can occur as in the PCS segment.
The Figure 1 shows the functionality of the system.

In order to analyse the effect of the distribution of the inter-event times, we used simulation and standard methods to calculate the mean and variance of the response time of the PUs and SUs. the following figures illustrates that effect [2][4].

The figure 2 shows that the distribution of the inter-arrival time of the primary packets has no effect on the mean and variance of response time of the secondary users knowing that the inter-request time of the SUs and the service time of both servers units are exponentially distributed. The other operation mode is where the service time of the primary server unit is hyper-exponentially, hypo-exponentially distributed and the inter-arrival time of PUs and SUs, the service time of the secondary server are exponentially distributed. The figure 3 shows that the value of the mean response time and variance is greater when the service time is hypo-exponentially distributed.

Table 1

Numerical values of model parameters

No.	N_1	N_2	λ_1	λ_2	μ_1	μ_2	ν	p
Fig.2	10	50	x - axis	0.03	1	1	20	0.1
Fig.3	10	50	x - axis	0.03	1	1	20	0.1

The numerical values of parameters are collected in Table 1.

3. Conclusions

In this paper a finite-source retrial queueing model was proposed with two bands servicing primary and secondary users in a cognitive radio network. Primary users have preemptive priority over the secondary ones in servicing at primary channel. At the secondary channel an orbit is installed for the secondary packets finding the server busy upon arrival. The simulation was used to carry numerical calculations illustrating the effect of the distribution of the inter-events times on the first and second moments of the response time.

Acknowledgments

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References

1. *Wong E. W., Foch C. H., Adachi F.,* Analysis of cognitive radio spectrum decision for cognitive radio networks.IEEE Journal on Selected Areas of Communication 2011. —Vol.29, —P. 757–769.
2. *Law A. M., Kelton W. D.* Simulation Modeling and Analysis. — Second Edition, 1991.
3. *Sztrik J., Bérczes T., Almási B., Kuki A., Wang J.* Performance Modeling of Finite-Source Cognitive Radio Networks. Acta Cybernetica 2016. —Vol.22, no. 3. —P. 617–631.
4. *White A. J., Schmidt J. W., Bennett G. K.* Analysis of Queueing Systems. 1975.

UDC 004.4

Dynamic multi-criteria virtual machine allocation in cloud data centers

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Abstract. The problem of dynamic virtual machine (VM) placement in cloud data centers is addressed. The aim of dynamic VM placement is to provision hosted applications in real-time and to keep the parameters of the data center within limits. The key aspects of this problem are monitoring conditions under which it is necessary to take action on VM migration and switching on / off physical servers, selection of virtual machines for migration and selection of destination nodes. In this paper we propose a method of dynamic multi-criteria VM placement in data centers. It includes the algorithm of prediction of parameters of such criteria functions as temperature dissipation, power consumption, and uniform resource utilization of physical servers. Furthermore, the multi-criteria algorithm of selection of destination nodes makes a prediction of the state of the server after VM migration. Efficiency of the proposed method is assessed through simulation using random and real-world traces.

Keywords: optimal server placement, cloud computing, resource management, virtual machine allocation; multi-objective optimization.

1. Introduction

The problem of optimal resource allocation appeared relatively long ago. At the beginning it was the problem of allocation of physical resources [1]. But more recently, the need for dynamic resource allocation has raised, due to the increased demand for data center services. One of the solutions to meet this demand is to increase data center space and to connect to additional power sources. It is a very expensive way. Therefore, the problem is addressed with the help of virtualization technology that allows to create multiple virtual machines (VMs) on a single physical server, and move them from one server to another without shutting down (live migration). This allowed without a significant increase in the data center area and power consumption to meet the demand and to provide qualitatively new services called cloud computing. But the process of increasing demand has not stopped. More and more people are using cloud services and more data is stored in data centers. Therefore, the problem of effective use of the physical and virtual data center resources is still urgent.

The focus of this work is on multi-criteria virtual machine placement strategies that can be applied in a virtualized data center by a Cloud provider. The criteria includes CPU temperature, power consumption, and uniform resource utilization.

The problem of dynamic virtual machine placement can be divided into several stages:

1. Monitor the conditions under which the server is assumed to be overloaded or underutilized. Then, start the process of migration of virtual machines from this server.
2. Select appropriate VM(s) for migration.
3. Select the physical server (destination host) on which the VM will be moved.

The main focus of our work is on the method of determining the problem server and forecasting its parameters in the near future. Server parameters such as CPU temperature and utilization are monitored. The forecast is based on the usage of Group Method of Data Handling (GMDH). At each iteration, during the calculation of the function coefficients, the proposed algorithms takes into account the dispersion of the experimental data. Experimental data on the CPU utilization and temperature, obtained using the monitoring software show that variation is significant. It was shown that the accuracy of prediction in determining the step-ahead value is increased by at least 10%.

The remainder of the paper is organized as follows. In Section 2 we discuss the related work. In Section 3 we introduce the model of the data center. In Section 4 we describe the proposed method of determining the problem host and forecasting its parameters. Evaluation and analysis of the experimental results described in Section 4. Section 5 is a conclusion.

2. Related Work

This problem was addressed in a series of articles. Among them are works of Beloglazov A. and Buyya R. [2] and Xu J. and Fortes J. [3]. In [2] studied the algorithms of VM allocation based on the single criterion of energy efficiency. In [3] a multi-criteria approach to the problem of VM allocation is proposed. Also authors used least squares method to search parameters of the linear model to predict the behavior of the servers in the step-ahead observation. However, our experimental data have shown significant dispersions. Therefore, to improve the accuracy of the forecast we propose to use the GMDH-method, which is based on the algorithm of least squares method with measurements of different accuracy. A more accurate server temperature and utilization forecast as well as taking into account multiple criteria for the selection of the destination host will reduce the number of unnecessary migrations and balance the state of the data center as a whole.

3. System Model

In this work the target system is an IaaS environment, represented by a large-scale data center consisting of N heterogeneous physical nodes.

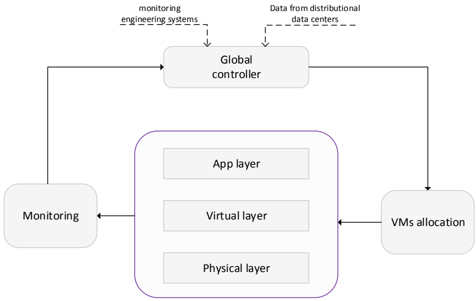

Figure 1. System architecture

In the data center management architecture the function of the dynamic distribution of virtual machines (VMs) assigned to global controller, which receives the necessary information from the monitoring system. The monitoring system saves system information, such as resource utilization, power consumption and server temperature in a central repository. Global controller analyzes this information and in accordance with it places virtual machines (see Fig. 1).

Dynamic allocation of virtual machines in data centers requires from global controller answering the following questions:

1. When it is necessary to distribute VMs?
2. Which VMs are optimal for migration?
3. Where to move VMs?

To solve these problems global controller continuously analyzes data received from multiple sensors. If working parameters of the model are out of a threshold, global controller initiates the process of virtual machines migration.

System indicators are checked sequentially, in accordance with criteria importance (temperature, resource wastage, energy efficiency) (see Fig. 2). The monitoring process is carried out continuously, even in cases of performing actions on the movement of VMs. The system administrator may change the priority of criteria, but due to their contradictory, the step of verification should remain consistent.

4. The Problem Host Determination

At the first stage the main difficulty lies in how to evaluate the proximity of server state to some critical thresholds. They can be static [3] or dynamic [2]. Different prediction methods can be incorporated, such as Markov chains [4], moving average [5], linear least-squares approach [3]

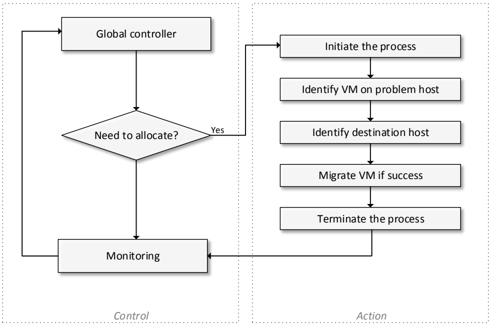

Figure 2. Controller's control flow

and Kalman filters [6]. A sliding-window detection is applied, in which the analysis of the time-varying data such as CPU temperature and resource utilization is performed over the values covered by a window of finite length [3]. In [2] the authors propose a multisize sliding window approach that auto adjust to current conditions. The sliding window method assumes the selection of data for a certain period of time (the size of the window), averaging parameters and comparing them with previous results. Information from the sensors is stored as a profile, which is a typical behavior of the object. Exceeding values of parameters beyond a certain threshold indicates the need to move the VM. The size of the window influences the aggressiveness of observation and is defined by the system administrator, but its minimum size must be longer than VM migration time as it is obvious that the moving process introduces an error in the observations. Furthermore, it is assumed that in the window of a certain size, we observe a stationary process.

Comparison of the average deviation may be incorrect for the trend (ie monotonically rises or falls) strategies. In addition, it is desirable to predict the step-ahead values of the parameters in order to avoid unnecessary VM migrations in such situations as exceeding the critical threshold due to short-term events, such as local load surge. For this reason the second level threshold is proposed, which is used to check if the step-ahead value of parameters is beyond the second threshold. If it is true, the global controller initiates the migration process. We conducted a study on different kinds of computers. To emulate a variety of data center loads we used workload generators SysBench and LookBusy for Linux. We investigated such CPU parameters as temperature and utilization, depending on the dynamically changing load. As shown in Tables 1 and 2, the scatter in the experimental data in case of high temperature is significant and differs by more than 15%.

The results indicate that the use of the classical algorithm of least squares method to find the parameters of the linear model may not provide enough accurate predictive assessment of server parameters.

To improve the accuracy of the forecast we propose to use combinatorial group method of data handling (GMDH) proposed by Ivakhnenko [7]. This method is characterized by high speed of data processing, as well as the possibility of obtaining results in a statistically insignificant sample, when the number of unknown factors at a fixed time may be greater than the number of such moments. It is based on full or reduced sorting-out of gradually complicated models and evaluation of them by external criterion on separate part of data sample. To build models we used the Kolmogorov-Gabor polynomial, which is as follows:

$$Y(x_1, ..., x_n) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_i x_j + \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=j}^{n} a_{ijk} x_i x_j x_k + ...$$

As a model selection criterion has been used minimum regularity criterion.

To determine the parameters a_i in models used the least squares method. Since, as discussed above, the scatter of experimental data is significant, it is proposed to use a method of least squares algorithm with measurements of different accuracy. The use of this algorithm for determining the tempera-ture and the CPU utilization at the next step improves prediction accuracy not less than 10%, which is essential for the dynamic allocation of virtual machines.

5. Virtual machine selection

What virtual machines to move depends on the nature of the event.

Table 1

CPU temperature scatter

CPU temperature (°C)	31-35	36-40	41-45	46-50	51-55
Scatter (%)	4	6	8	10	16

Table 2

CPU utilization scatter

CPU utilization (%)	0-20	21-40	41-60	61-80	81-100
Scatter (%)	10	14	21	26	28

1. Overheating – VM(s) with the maximum CPU usage have to be migrated, as having the direct impact on the CPU temperature of the physical server. Since the migration process causes extra strain on physical resources, the VM with minimum amount of RAM should be chosen to make the migration faster.

2. The lack of resources – the total average utilization is calculated on the affected physical server and VM are selected from the above-average performance. As in the previous case, all VM on the server should be sorted in order of increasing size of memory.

3. Low energy efficiency – in this case, the physical server has low resource utilization. All VMs from this server should be moved and the server itself should be switched off.

6. Destination host selection

The next challenge in the dynamic VM allocation process is the determi-nation of the destination host. It should take into account all the criteria and make the prediction of server state after migrating VM on it. In general, the following recommendations are relevant:

— in case of overheating - to balance the temperature in the data center the destination node should be the "coolest"server. With the use of the temperature model derived from the data monitoring system, the global controller is able to predict the temperature of the server after placing VMs on it;
— lack of resources - preference is given to the server with the largest amount of available resources;
— energy efficiency - in terms of energy savings, preference is given to loaded servers with sufficient resources. Servers with low load have to be released and switched off.

Results for various server selection criteria can differ from each other. For this reason it is proposed to use the convolution of multiple criteria into a single objective function having the following form [8]:

$$\min_{T, u_{CPU}, u_{RAM}} f = \alpha_1 f_t(T) + \alpha_2 f_{resource}(u_{CPU}, u_{RAM}) + \alpha_3 f_{power}(u_{CPU}),$$

where f_t–temperature criterion

$$f_t(T) = 1 - 1/(1 + e^{(T - T_s)}),$$

where T–current CPU temperature;
T_s–safe CPU temperature.
$f_{resource}$–unused resources criterion

$$f_{resource}(u_{CPU}, u_{RAM}) = 1 - u_{CPU} u_{RAM},$$

where u_{CPU}–CPU utilization;

u_{RAM}–main memory utilization.
f_{power}–power consumption criterion

$$f_{power}(u_{CPU}) = \begin{cases} p_0 + (p_1 - p_0)u_{CPU}, & u_{CPU} > 0 \\ 0, & u_{CPU} = 0 \end{cases},$$

where p_0–power consumption by an underutilized server;
p_1–power consumption by a fully utilized server;
α_1, α_2, α_3–criteria weights, which values the administrator sets, based on his current tasks. All three criteria functions are normalized in [0,1] range. Optimization is performed under the constraints on the parameters of physical servers. Also, before the migration process is started, the state of the destination host should be taken into account to ensure its stability in the near future. For prediction of the server's step-ahead parameters on this stage we also propose to use GMDH-method.

7. Experimental results

The effectiveness of multi-criteria algorithm is assessed through simulation. We used simulation platform CloudSim [9], which allows to simulate various scenarios for cloud data center and incorporate various resource allocation algorithms. The simulation was performed for the data center, consisting of 100 homogeneous physical servers with one processor core, output of 2,000 million instructions per second (MIPS) and 16 GB of RAM. Underloaded server consumed 175 watts, and at 100% CPU usage – 250 watts. Each VM required processor capacity of 250, 500, 750 or 1,000 MIPS, and 128 GB of RAM. The number of VMs is 250, that fill the capacity of the data center. The change in CPU usage was modelled in accordance with a uniform distribution, reflecting the change in application workloads on running virtual machines. Also, we used real-world PlanetLab workload traces, which are provided with CloudSim.

The proposed multi-criteria algorithm for selecting the destination host was compared with used in practice bin packing heuristics – First Fit Decreasing (FFD) and Best Fit Decreasing (BFD).Also, the comparison was done with an exhaustive search method by a single criteria. It was taken as an 100% result.

These results demonstrate the effectiveness of the proposed algorithm by 10-15% compared with algorithms BFD and FFD (see Fig. 3).

8. Conclusion

We proposed a method of dynamic computing resources allocation of the data center, which includes three stages: determination of the conditions for moving VM(s), virtual machine selection for migration and determination of the destination host.

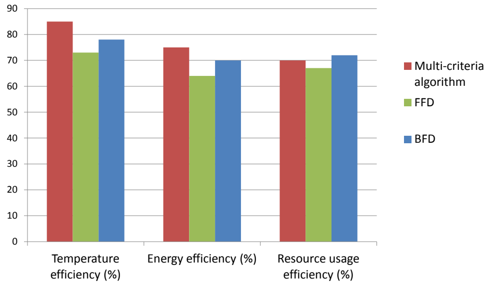

Figure 3. Performance comparisons of algorithms

At the first stage it is important to determine the right moment to move VM(s) from the problem server. A method of predicting the state of the host in the step-ahead moment based on the group method of data handling which allows to increase the forecast accuracy by not less than 10%.

While choosing the destination host, it should be taken into account such criteria as server temperature, uniform resource utilization and energy efficiency.

The proposed multi-criteria destination host selection algorithm allows to avoid unnecessary VM migrations and to take into account criteria relating to the state of the data center, as well as meeting SLAs.

References

1. *Tutova N. V* The Techniques of Optimal Resource Allocation in the Internet Data Centers. // T–Comm, 2008, No 6, P. 44–46.[in Russian].
2. *Beloglazov A., Buyya R.* Optimal Online Deterministic Algorithms and Adaptive Heuristics for Energy and Performance Efficient Dynamic Consolidation of Virtual Machines in Cloud Data Centers // Concurrency and Computation: Practice and Experience (CCPE), Volume 24, Issue 13, Pages: 1397-1420, John Wiley & Sons, Ltd, USA, 2012.
3. *Xu J. and Fortes J.* A Multi-objective Approach to Virtual Machine Management in Datacenters. // Proceedings of the 8th International Conference on Autonomic Computing, ICAC 2011, Karlsruhe, Germany, June 14-18, 2011. P. 225-234.
4. *Beloglazov A., Buyya R.* Managing Overloaded Hosts for Dynamic Consolidation of Virtual Machines in Cloud Data Centers Under Quality of Service Constraints // IEEE Transactions on Parallel and Distributed Systems (TPDS), Volume 24, Issue 7, Pages: 1366-1379, IEEE CS Press, USA, 2013.

5. *Zhang Q., Zhani M. F., Zhang S., Zhu Q., Boutaba R., Hellerstein J. L.* Dynamic Energy-Aware Capacity Provisioning for Cloud Computing Environments // Proceedings of the 9th international conference on Autonomic computing, 2012.
6. *Wu Y., Yuan Y., Yang G., Zheng W.* Load Prediction Using Hybrid Model for Computational Grid // Proceedings of 8th IEEE/ACM International Conference on Grid Computing, 2007.
7. *Ivahnenko A. G., Stepashko V. S.* Error controlled modeling. Kiev, 1985, 216 p. [in Russian].
8. *Vorozhtsov A. S., Tutova N. V., Tutov A. V.* Optimal Cloud Servers Placement in Data Centers // T-Comm, No6, 2015. P. 4–8. [in Russian].
9. *CloudSim toolkit*, URL:http://www.cloudbus.org/cloudsim/ (14.04.16)

УДК 004.9

Разработка алгоритма формирования графических образов документов для интегрированного в СДОУ модуля сканирования

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Аннотация. Вследствие активного ввода в эксплуатацию каналов связи между подразделениями крупных территориально-распределенных организаций, а также массового внедрения систем документального обеспечения управления (СДОУ), все актуальнее становится задача перехода от бумажного документооборота к безбумажному. Внедряемая в крупных государственных организациях система межорганизационного электронного документооборота (МЭДО) определила единый формат передаваемых документов. Каждый передаваемый документ состоит из карточки документа в формате XML (паспорт документа), содержащей все основные реквизиты, и графического образа подписанного документа. Поскольку на сегодняшний день еще не сформулированы единые требования к формату подписанного электронной цифровой подписью (ЭЦП) документа для передачи по системе МЭДО, основным способом формирования графического образа является сканирование бумажной версии документа. Если документ отправляется нескольким адресатам, для каждой организации должен быть сформирован отдельный образ с соответствующим титулом. В общем случае количество адресатов может достигать нескольких десятков организаций, что приводит к большим временным и техническим издержкам на формирование графических образов. В данной работе рассматривается алгоритм формирования графических образов документов для интегрированного в СДОУ модуля сканирования. Предложенный алгоритм позволяет упростить и ускорить процесс формирования графических образов документов, направляемых нескольким адресатам по системе межорганизационного взаимодействия.

Ключевые слова: информационные системы, организационное управление, межорганизационный электронный документооборот.

1. Введение

В условиях внедрения каналов связи между подразделениями крупных территориально-распределенных организаций (выделенные линии, VPN) [1], а также широкого распространения систем документального обеспечения управления (СДОУ), являющихся частным случаем систем электронного документооборота (СЭД) [2], все большее число предприятий переходит на безбумажный документооборот [3].

Внедряемая в крупных государственных организациях система межорганизационного электронного документооборота (МЭДО) определила единый формат передаваемых документов. В работе [4] была спроектирована общая структура программного комплекса, который позволяет реализовать обмен документами между действующими СЭД организаций. В работе [5] представлена реализация модуля отправки документов по системе межорганизационного документооборота, интегрированного в программный комплекс взаимодействия между организационными СДОУ. Для отправки документа по системе межорганизационного взаимодействия необходимо сформировать в СДОУ его электронную карточку и прикрепить к ней отсканированный графический образ документа. Если документ отправляется нескольким адресатам, для каждой организации должен быть сформирован отдельный образ с соответствующим титулом, что приводит к большим временным и аппаратным издержкам. В данной работе рассматривается алгоритм, позволяющий упростить и ускорить процесс формирования графических образов документов, направляемых нескольким адресатам по системе межорганизационного взаимодействия.

2. Постановка задачи

Поскольку на сегодняшний день еще не сформулированы единые требования к формату подписанного электронной цифровой подписью (ЭЦП) документа для передачи по системе МЭДО, основным способом формирования графического образа является сканирование бумажной версии документа. Сформированный графический образ сохраняется в базе данных в формате PDF, прикрепляется к карточке документа в СДОУ и в дальнейшем может быть отправлен по системе межорганизационного взаимодействия. Если документ отправляется нескольким адресатам, необходимо сформировать для каждой организации отдельный образ документа с титульным листом, содержащим реквизиты данного адресата. Также требуется предусмотреть возможность формирования визового экземпляра документа для последующего отображения в СДОУ организации. Таким образом, процесс формирования графических образов документов состоит из следующих этапов:

— сканирование бумажной версии документа;
— формирование образов документов для всех адресатов и визового экземпляра;
— сохранение образов документов в БД СДОУ организации.

Разработанный алгоритм предназначен для использования в модуле сканирования, интегрированного в СДОУ организации.

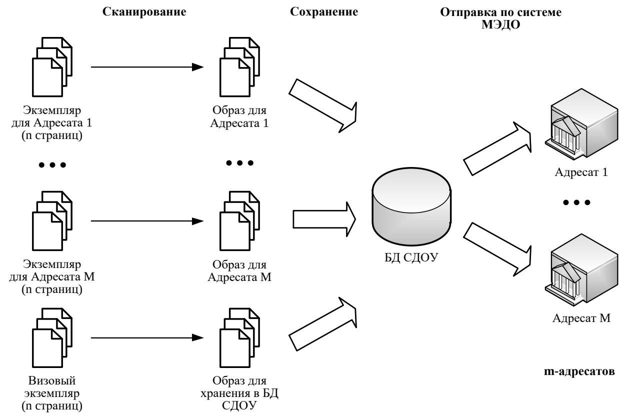

Рис. 1. Формирование графических образов документов средствами типовой СДОУ

3. Алгоритм формирования графических образов документов

В общем случае у одного документа может быть несколько адресатов. Также бывает необходимо хранить в СДОУ визовый экземпляр документа. Формирование графических образов документов средствами большинства СДОУ осуществляется по схеме, приведенной на рис. 1.

Согласно схеме, для каждого адресата необходимо отсканировать все страницы адресованного ему бумажного экземпляра документа. Полученный образ сохраняется в базе данных СДОУ в формате PDF и в дальнейшем доступен для просмотра и отправки по системе МЭДО. Таким образом, для документа с количеством страниц n, направляемого в m организаций необходимо создать m+1 образов (включая визовый экземпляр), отсканировав стандартными средствами (m+1)*n страниц. Для ускорения процесса сканирования и снижения нагрузки на оборудование предлагается схема, представленная на рис. 2.

Для упрощения приведен случай формирования образа документа для двух адресатов. Алгоритм разработан исходя из того, что экземпляры документов для разных адресатов отличаются лишь титульным листом, на котором указаны реквизиты адресатов. Визовый экземпляр содержит на титуле все необходимые визы. Таким образом, если у документа n страниц и m адресатов, необходимо сначала отсканировать все титульные листы для всех адресатов (m-страниц) и визовый лист,

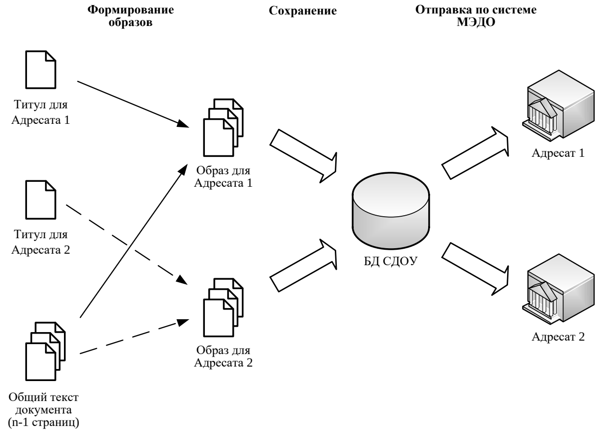

Рис. 2. Алгоритм формирования графических образов документов для интегрированного в СДОУ модуля сканирования

а затем остальную часть документа, общую для всех адресатов (общий текст документа) (n-1 страниц). Всего при использовании данной схемы необходимо отсканировать m+n страниц. Формирование образов для просмотра и отправки осуществляется программными средствами модуля сканирования. Для титульных страниц и визового листа пользователь устанавливает соответствующие отметки с помощью интерфейса модуля. Затем модуль автоматически формирует образ документа для каждого адресата, соединяя соответствующий титульный лист с общим текстом документа. После сохранения в БД СДОУ сформированные графические образы доступны для просмотра из карточки документа и последующей отправки по системе межорганизационного взаимодействия всем адресатам. В целях уменьшения объема дискового пространства, необходимого для хранения образов документов, можно сохранять в БД СДОУ лишь титулы и общий текст документа по отдельности, а образы для просмотра и отправки формировать динамически при вызове соответствующих функций системы. Оптимизация использования дискового пространства особенно актуальна в случаях, если организация работает с большим количеством документов объемом порядка нескольких десятков или сотен страниц, отправляемых многим адресатам. Разработанный алгоритм может быть реализован в модуле сканирования для формирования образов документов средствами СДОУ организации при отправке документов нескольким адресатам по системе межорганизационного взаимодействия.

4. Заключение

В результате работы спроектирован алгоритм формирования образов документов для интегрированного в СДОУ модуля сканирования. Алгоритм позволяет упростить и ускорить процесс формирования графических образов документов, направляемых нескольким адресатам по системе межорганизационного взаимодействия. Использование представленного метода позволяет также уменьшить объем дискового пространства, необходимого для хранения образов документов в БД СДОУ.

Литература

1. *Таненбаум Э. С., Ван Стеен М.* Распределенные системы. Принципы и парадигмы. С-Пб.: Питер, 2003. – 877 с.
2. *Баканова Н. Б., Цапаева Ю. А., Гурвиц А. Л., Сурпин В. П.* Создание корпоративной системы сбора и анализа информации на основе Web-технологий. Международный семинар «Распределенные компьютерные телекоммуникационные сети». М.: Техносфера, 2005.
3. *Волчков Д. В., Сурпин В. П.* Разработка механизма синхронизации данных в распределенных системах документационного обеспечения управления // Управление развитием крупномасштабных систем (MLSD'2011): Материалы Пятой международной конференции (3-5 октября 2011 г., Москва, Россия). Том II.М.: Учреждение Российской академии наук Институт проблем управления им. В.А. Трапезникова РАН, 2011. – с.213-216.
4. *Волчков Д. В.* Разработка структуры программного комплекса обмена документами между корпоративными СДОУ // Distributed Computer and Communication Networks: Control, Computation, Communications (DCCN-2013), Moscow: JSC «TECHNOSPHERA», 2013. – 464p. – с.329-331.
5. *Волчков Д. В., Баканова Н. Б.* Разработка модуля отправки документов для программного комплекса взаимодействия между организационными СДОУ // Управление развитием крупномасштабных систем MLSD'2015: Материалы Восьмой международной конференции, 29 сент. – 1 окт. 2015 г., Москва: в 2 т. / Ин-т проблем упр. им. В.А. Трапезникова Рос. акад. наук; под общ. ред. С.Н.Васильева, А.Д.Цвиркуна. – Т. 2. Секции 2–12. – М.: ИПУ РАН, 2015. – С. 225-228.

UDC 004.9

Development of an algorithm for forming graphic images of the documents for scanning module integrated into DMS

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Due to the active commissioning of the communication channels between departments of large geographically distributed organizations and mass introduction of document management system (DMS), all the more urgent becomes the problem of the transition from a paper-based flow of documents to an paperless one. Introduced in large public institutions of Interorganizational System of Electronic Document Flow (ISEDF) has determined a uniform format of transmitted documents. Each transmitted document consists of a document card in XML format (document passport), containing all the basic details, and a graphic image of signed document. As for today we have not yet formulated unified requirements to the format of the signed digital signature (EDS) document for transmission over ISEDF, the basic method of forming the graphic image is a scan of a paper version of the document. If the document is sent to multiple recipients, for each organization separate image with the corresponding title should be formed. In general, the number of recipients may reach several tens of organizations. It leading to considerable time and technical costs for forming graphic images. In this paper, we consider the algorithm for forming graphic images of the documents for the scan module integrated into DMS. The proposed algorithm allows to simplify and accelerate the process of formation of graphic images of documents to be sent to multiple recipients over an interorganizational system.

Keywords: information systems, organizational management, interorganizational electronic document flow.

УДК 519.248:62

О подходах к задаче расстановки базовых станций для внутренних систем позиционирования

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Аннотация. Рассматривается задача расстановки базовых станций с целью улучшения точности работы системы локации в помещении. Помещение моделируется в виде неориентированного графа, узлы которого соответствуют возможным положениям отслеживаемого объекта.

Ключевые слова: внутренние системы позиционирования, базовая станция, метрическая размерность графа.

1. Введение

В настоящее время широкое распространение получают системы определения местоположения мобильных объектов внутри помещений. Большинство из этих систем основано на использовании беспроводных сетей стандартов WiFi, ZigBee, Bluetooth или UWB. Такие системы состоят из нескольких стационарных точек доступа и мобильного устройства, местоположение которого необходимо определить.

Для определения местоположения объекта используются разные методы и алгоритмы. При этом точность локации зависит от многих факторов, один из которых – число и расположение стационарных точек доступа. Задача расстановки точек доступа рассматривалась рядом авторов (см., например, [1]). В большинстве таких моделей помещение разбивается на конечное число зон, следовательно местоположение отслеживаемого объекта определяется с точностью до зоны. При этом задача определения конфигурации точек доступа часто формулируется либо в форме задачи математического программирования, либо в форме комбинаторной задачи об оптимальном взвешенном покрытии множества системой его подмножеств. В ряде статей рассматривается вероятностная модель определения погрешности локации мобильного объекта на основе изменчивости измерений уровня сигнала.

В настоящей работе предпринята попытка рассмотреть два подхода (детерминированный и вероятностный) для помещения, моделируемого в виде неориентированного связного графа [5].

2. Детерминированный подход

В качестве модели помещения рассматривается неориентированный связный граф $G = (V,\ E,\ \omega)$, где V – множество вершин, которым соответствуют места возможного расположения объекта; E – множество ребер, описывающее возможные пути перемещения объекта; ω – весовая функция:

$$\omega\colon\ E \to \mathbb{N}.$$

Ряд исследователей, например [3], связывает задачу расстановки точек доступа в помещении с задачей из теории графов, известной под названием метрическая размерность [4].

Обозначим через $d(u, v)$ расстояние между двумя вершинами u и v графа (минимальная длина связывающей их цепи). Подмножество вершин $W \subset V$ называется разрешающим, если для каждой пары вершин $u, v \in V$ найдется вершина $w \in W$, для которой $d(w, u) \neq d(w, v)$. Метрическая размерность $\beta(G)$ графа G – это минимальное число вершин в разрешающем множестве. Разрешающее множество минимальной мощности называется базисом метрик графа.

Известно, что задача определения метрической размерности графа в общем случае является NP-полной. Для некоторых частных случаев получены ее решения в явном виде [2]. Множеству точек доступа можно сопоставить разрешающее множество вершин графа, при этом минимально необходимое число точек доступа соответствует метрической размерности графа.

3. Вероятностный подход

Рассмотрим математическую модель, учитывающую случайные ошибки при определении расстояний.

Пусть $V = \{v_1, ..., v_n\}$. Зафиксируем индекс $i \in \{1, \ldots, n\}$ и рассмотрим двумерную случайную величину $(\xi,\ \eta_i)$, где ξ принимает значения из множества $\{1, \ldots, n\}$, η_i принимает целые значения, которые будем интерпретировать как результат определения расстояния между вершиной v_i, соответствующей точке доступа, и вершиной v_ξ, соответствующей местоположению мобильного объекта.

Пусть задано распределение

$$P(\xi = j) = p_j, \qquad j = 1, \ldots, n,$$

$$P(\eta_i = s \mid \xi = j) = q_{ij}(s), \qquad s \in \mathbb{Z},$$

$$\sum_{s \in \mathbb{Z}} q_{ij}(s) = 1, \quad i = 1, \ldots, n, \quad j = 1, \ldots, n.$$

Пусть $D \subset \{1, \ldots, n\}$ – множество, состоящее из номеров вершин, соответствующих потенциально возможным точкам доступа. Обозначим $E(d(v_\xi,\ u) \mid \overline{\eta}_D)$ – условное математическое ожидание расстояния между вершиной v_ξ графа и вершиной u относительно случайного вектора $\overline{\eta}_D = \{\eta_i,\ i \in D\}$.

Пусть система локации после регистрации вектора измерений расстояний $\overline{s}_D \in \mathbb{Z}^{|D|}$ в качестве искомой возвращает вершину u с минимальным значением $E(d(v_\xi,\ u) \mid \overline{s}_D)$. Обозначим

$$\Delta(\overline{\eta}_D) = \min_{u \in V}\{E(d(v_\xi,\ u) \mid \overline{\eta}_D)\}.$$

Математическое ожидание погрешности локации мобильного объекта рассчитывается стандартным образом:

$$E(\Delta(\overline{\eta}_D)) = \sum_{\overline{s}_D \in \mathbb{Z}^{|D|}} \Delta(\overline{s}_D) P(\overline{\eta}_D = \overline{s}_D).$$

Обозначим

$$\beta'(\varepsilon) = \min\{|D| \ :\ D \subset \{1, \ldots, n\},\ E(\Delta(\overline{\eta}_D)) \leq \varepsilon\}.$$

Значение $\beta'(\varepsilon)$ существует при $\varepsilon \geq \varepsilon^* = E(\Delta(\overline{\eta}_V))$.

В результате можно поставить следующие оптимизационные задачи:

Задача 1. При заданном значении $\varepsilon \geq \varepsilon^*$ определить $\beta'(\varepsilon)$ и соответствующее множество вершин D.

Задача 2. При заданном m $(1 \leq m \leq n)$ найти множество вершин D, для которого $|D| = m$ и $E(\Delta(\overline{\eta}_D))$ примет минимальное значение.

Решение поставленных задач позволяет найти оптимальную расстановку точек доступа системы локации с учетом погрешности определения местоположения объекта.

4. Заключение

Рассмотрено два подхода к задаче расположения базовых станций во внутренних системах позиционирования. Детеминированный подход основан известном из теории графов понятии метрической размерности. Основная проблема состоит в том, что не учитываются ошибки измерений, в результате чего получаются нетипичные с практической точки зрения конфигурации. Вероятностный подход учитывает погрешности при измерениях. Согласно данному подходу выбирается такая расстановка базовых станций, при которой гарантируется, что математическое ожидание погрешности локации не превысит некоторого заданного значения, либо для заданного числа базовых станций находится конфигурация с минимальной погрешностью локации.

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Литература

1. *Battiti R., Brunato M., Delai A.* Optimal wireless access point placement for location-dependent services, Technical Report DIT-03?052, 2003.
2. *Klein D. J., Yi E.* A comparison on metric dimension of graphs, line graphs, and line graphs of the subdivision graphs //European Journal of Pure and Applied Mathematics. 2012. V. 5. N 3. pp. 302–316.
3. *Simon R F., George A.* On the Metric Dimension of few Network Sheets // International Conference on Robotics, Automation, Control and Embedded Systems – RACE 2015. 18–20 February 2015, Hindustan University, Chennai, India.
4. *Slater P. J.* Leaves of trees // Proc. 6th Southeastern Conference on Combinatorics, Graph Theory, and Computing (Florida Atlantic Univ., Boca Raton, Fla., 1975), Congressus Numerantium 14, Winnipeg: Utilitas Math., pp. 549–559.
5. *Werner M.* Indoor Location-Based Services: Prerequisites and Foundations. Springer, 2014.

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The problem of the base station placement for indoor positioning systems

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We consider the problem of optimal placement of base stations under random error measurements in order to improve the accuracy of the locating system indoors. A door is modeled as undirected graph whose nodes correspond to the possible location of tracking object.

Keywords: indoor positioning systems, base station, metric dimension of graph.

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Применение методов инфракрасной микроспектроскопии для сбора и анализа данных в сетях связи общего пользования и молекулярных наносетях

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Аннотация. В данной статье рассматриваются вопросы, касающиеся новых методов сбора данных, а именно, возможности применения инфракрасной микроспектроскопии, а также приборов, реализующих данную функцию - портативных инфракрасных микроспектрометров для сбора информации о свойствах и состоянии вещества. Также освещены вопросы организации каналов связи для коммуникации подобных приборов с человеком (через терминальное оборудование - смартфон, планшет, ноутбук и т.д.), и другими функциональными узлами сетей связи общего пользования – удалёнными облачными серверами, обеспечивающими хранение и обновление базы данных образцов, поддерживающими возможности удалённого анализа полученных данных. Тематика данной статьи находится на стыке нескольких областей знаний – молекулярных наносетей, материаловедения, физики, химии и т.д. ИК-микроспектрометр рассматривается автором как элемент сети связи общего пользования.

Ключевые слова: молекулярные наносети, медицинские сети, сбор данных в молекулярных наносетях, инфракрасная спектроскопия, сети связи общего пользования, облачные сервисы.

1. Введение

Ни для кого не секрет, что в двадцать первом веке самым важным и значимым ресурсом является информация. Возможности человека по передаче, приёму, обмену, хранению и распределению информации в первую очередь определяются текущим уровнем развития технологической базы. К сожалению, долгое время этот уровень не позволял эффективно извлекать необходимую информацию из объектов окружающего мира, передавать её на требуемые расстояния с обеспечением заданного уровня качества предоставления услуг. Благодаря деятельности многих поколений исследователей человечество на данный момент значительно расширило спектр возможностей в данной области.

2. Основная часть

Первоначально человеку было достаточно остроты собственного зрения, слуха и обоняния, чувствительности тактильных ощущений для получения информации из окружающего мира, поэтому основные усилия были направлены на совершенствование способов передачи информации. Так, например, А.С. Попов больше ста лет назад совершил научный прорыв, открыв для нас передачу информации в радиочастотном спектре. Однако в настоящее время в этой области мы подошли к определённому пределу, который связан с ограниченностью ресурсов радиочастотного спектра, и вынуждены изыскивать альтернативные способы передачи информации. Например, в современной научной литературе описывается возможность использования наносетей для передачи информации [1]. Под наносетью в данном случае понимается телекоммуникационная сеть, в которой передача информации происходит между узлами данной сети, и может осуществляться как классическими способами - с использованием радиочастотного спектра, так и путём перемещения вещества. Часто такие сети ещё называют молекулярными наносетями.

С другой стороны, значительного прогресса достигли методы получения информации из объектов окружающего мира. Сейчас на помощь человеку приходят всевозможные технические устройства и приборы. Так, например, широкое распространение в современной науке и технике получили методы инфракрасной спектроскопии [2]. Это особый раздел спектроскопии, изучающий взаимодействие инфракрасного излучения с различными веществами. Любой инфракрасный микроспектрометр работает по следующему принципу: сенсор излучает инфракрасное излучение, которое проходит через исследуемое вещество, после чего происходит возбуждение колебательных движений молекул или их отдельных фрагментов вещества. Одновременно с этим наблюдается ослабление интенсивности светового излучения, которое проходит через образец. Поглощение света происходит не во всём спектре падающего инфракрасного излучения, а лишь в тех длинах волн, энергия которых соответствует энергиям возбуждения колебаний в изучаемых молекулах. Частоты, при которых наблюдается максимальное поглощение инфракрасного излучения, могут свидетельствовать о наличии в молекулах образца вещества тех или иных функциональных групп и других фрагментов, что широко используется в различных областях для установления структуры соединений. Этот подход возможен благодаря большому количеству накопленной экспериментальной информации - благодаря труду исследователей в настоящее время составлены специальные таблицы, связывающие частоты поглощения инфракрасного спектра с наличием в образце определённых молекулярных фрагментов. Созданы также базы, хранящие в себе данные об инфракрасном спектре некоторых классов соединений. Это позволяют автоматически сравнивать спектр неизвестного анализируемого вещества с уже известными спектрами и идентифицировать это вещество в кратчайшие сроки. Скорость выполнения анализа и сравнения позволяет использовать данный метод в областях, где ключевую роль играет время проведения анализа - в медицине, судебной экспертизе и т.д.. До недавнего времени применение подобных методов исследований было связанно с использование дорогостоящего высокотехнологичного стационарного оборудования, имеющего значительные массогабаритные характеристики. Поэтому проведение подобного анализа с целью получения информации о составе и свойствах вещества могло проходить только в лабораторных условиях высококвалифицированными специалистами, что ограничивало количество потенциальных пользователей узким кругом профессионалов. Между тем совсем недавно учёным и разработчикам совместно удалось создать прототипы носимых персональных инфракрасных микроспектрометров. Примером может послужить микроспектрометр SCiO, недавно представленный компанией Consumer Physics [3].

Рис. 1. Внешний вид инфракрасного микроспектрометра SCiO.

Компания-разработчик позиционирует своё устройство как персональный носимый инфракрасный сканнер, который должен взаимодействовать с программным обеспечением, установленным на смартфоне, планшете или компьютере пользователя. Небольшой карманный ИК-спектрометр позволит буквально за одну минуту определить состав и качество того или иного продукта питания, лекарственного препарата и т.д., стоит лишь поднести прибор к исследуемому объекту, произвести сканирование и сравнить полученный результат с образцами инфракрасного спектра из облачной базы данных.

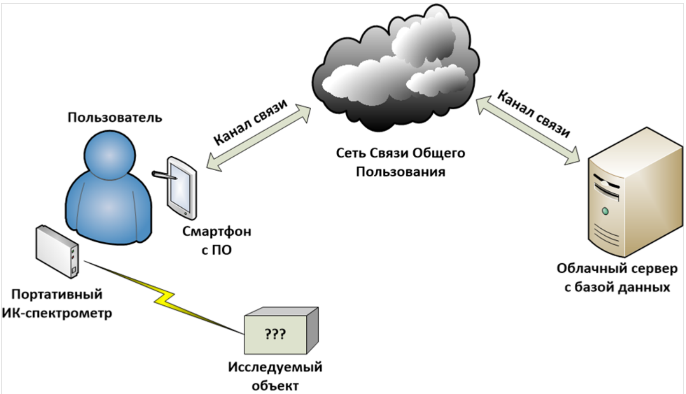

Рис. 2. Использование ИК-спектрометра и его взаимодействие с облачной базой данных.

Для того чтобы верно оценить последствия появления подобных приборов на рынке, потенциальные возможности его применения и особенности взаимодействия приборов с терминальным оборудованием пользователя и ССОП, рассмотрим рабочие характеристики устройства, предполагая также, что они будут являться типовыми для всех инфракрасных спектрометров данного класса.

Для работы программного обеспечения устройства потребуется терминальное оборудование с поддержкой операционных систем iOS 8 и выше или Android 4.3 и выше с поддержкой Bluetooth Low Energy.

В первую очередь стоит отметить, что подобный прибор позволит рядовому пользователю осуществлять взаимодействие с таким типом сетей, как молекулярные наносети. Если ранее использование таких сетей было ограничено лабораторными условиями и предполагало наличие сложного дорогостоящего оборудования, а также его использование высококвалифицированными профессионалами, то теперь доступ к получению информации посредством таких сетей будет открыт для каждого человека. Станет возможным внедрение и использование молекулярных наносетей в коммерческих целях, в целях обеспечения безопасности людей, и в других случаях, когда это необходимо или нет возможности организации классических сетей связи. Также стоит отметить, что ранее передача информации в молекулярные наносетях ограничивалась ограниченным набором веществ – положительные ионы кальция, феромоны, жгутиковые бактерии [4]. Это было связано с тем, что существовали достаточно простые устройства, которые могли реагировать на появление подобных носителей информации и извещать пользователя. Сейчас же благодаря появлению персонального

Таблица 1
Характеристики инфракрасного микроспектрометра SCiO.

№	Характеристика	Значение
1	Спектр излучения сенсора	Инфракрасный
2	Тип сенсора	Инфракрасный спектрометр
3	Время измерения	1.5 сек
4	Дистанция измерения	5-15 мм
5	Тип соединения	Беспроводное
6	Питание	От внутренней батареи
7	Зарядка	Микро USB
8	Время полного заряда батареи	3 часа
9	Габариты	54х36,4х15.4 мм
10	Вес	35 грамм
11	Рабочая температура	5-35 градусов по Цельсию

носимого спектрометра появилась возможность использовать значительно большее количество различных носителей и источников информации. Например, появляется возможность маркировать специальными составами, безвредными для человека, продукты питания, лекарственные препараты и т.д., а покупатели, в свою очередь, получают доступ к лёгкому способу проверки качества и соответствия товара или медицинского препарата путём сканирования нанесенного состава или самого объекта исследования инфракрасным микроспектрометром. Этот метод может послужить дополнительной защитой от контрафактных товаров, а также способом оповещения покупателя или работника медицинского учреждения о таких критериях, как страна-производитель товара или препарата, состав и т.д.

Во вторых, стоит рассматривать подобный прибор как источник дополнительной нагрузки на сети связи общего пользования. Дополнительные потоки трафика, который будут генерироваться в результате работы устройства, должны будут передаваться для обработки и сравнения с эталонными данными с соблюдением требований к параметрам качества обслуживания [5]. Потребность хранения и накопления эталонных данные об инфракрасном спектре различных веществ и соединений подразумевает использование специальных облачных сервисов, которые являются составной частью сетей связи общего пользования. Хранение всей информации на терминальном оборудовании пользователя не представляется возможным в связи с большими объёмами информации в базе данных об инфракрасном спектре веществ и соединений, потребности её обновления и дополнения.

В третьих, внедрение подобного прибора позволит интенсифицировать работы по развитию персональных сетей и нательных сетей, разработке протоколов для взаимодействия пользователя с подобными приборами, M2M и D2D коммуникаций. Как видно из таблицы, разработчиками для взаимодействия с терминальным оборудованием пользователя выбрана технология Bluetooth Low Energy. Эта технология обеспечивает экономию энергии инфракрасного микроспектрометра, и также с успехом может использоваться для реализации построения самоорганизующихся сетей и взаимодействия в рамках концепции Интернета Вещей. При наличии соответствующего программного обеспечения возможно реализовать сервисы, которые будут способны формировать временные сетевые информационные кластеры для обмена информацией между пользователями микроспектрометров, например, если они находятся в помещении магазина и обмениваются информацией о представленных товарах с целью составления рейтингов, рекомендации того или иного товара.

3. Заключение

В данной статье рассмотрены вопросы, касающиеся новых методов сбора и анализа данных об окружающем мире, а именно – принцип работы и методы применения инфракрасных микроспектрометров как составной части сетей связи общего пользования. Приведён пример существующей реализации микроспектрометра и его рабочие характеристики. Отмечено возможное влияние данного устройства на развитие молекулярных наносетей, на увеличение потоков трафика в сетях связи общего пользования, на интенсивность развития персональных и нательных сетей, медицинских сетей, протоколов Интернета Вещей, M2M и D2D взаимодействия. Данные устройства, которые без сомнения в скором времени войдут в широкое употребление, и станут незаменимым инструментом сбора и анализа данных.

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Литература

1. *Кучерявый А.Е., Прокопьев А.Е., Кучерявый Е.А.* Самоорганизующиеся сети. — СПб.: Любавич, 2011.
2. *Бёккер Ю.* Спектроскопия = Spektroskopie / Пер. с нем. Л. Н. Казанцевой, под ред. А. А. Пупышева, М. В. Поляковой. — М.: Техносфера, 2009.

3. Near Infra-Red Spectroscopy at the palm of your hand. [Электронный ресурс] // Consumer Physics Inc. : [сайт]. [2016]. URL: https://www.consumerphysics.com (дата обращения: 09.05.2016).
4. *Пирмагомедов Р.Я., Кучерявый Е.А., Глушаков Р.И., Киричек Р.В., Кучерявый А.Е.* Живые организмы в киберпространстве – проект "Биодрайвер" — Электросвязь. — 2016. —№ 1.
5. *A. Koucheryavy, A.Prokopiev.* Ubiquitous Sensor Networks Traffic Models for Telemetry Applications, The 11th International Conference on Next Generation Wired/Wireless Networking NEW2AN 2011. — Springer LNCS 6869. August 2011 Saint-Petersburg.

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Application of methods of infrared microspectroscopy for collection and data analysis in communication networks public and molecular nanonetworks

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In this article the questions concerning new methods of data collection, namely, possibilities of application of infrared microspectroscopy, and also instruments realizing this function - portable infrared microspectrometers for information collection about properties and a status of substance are considered. Questions of the organization of communication links for communication of similar instruments with the person (through terminal equipment - the smartphone, a pad, a notebook, etc.), and other functional assemblies of communication networks public – the remote cloud servers providing storage and up-dating of the database of samples, supporting possibilities of the remote analysis of data retrieveds are taken also up. The subject of this article is on a joint of several knowledge domains – molecular nanonetworks, materials science, physics, chemistry, etc. Infrared microspectrometer is considered by the author as an element of a communication network public.

Keywords: molecular nanonetworks, medical network, data collection on molecular nanonetworks, infrared spectroscopy, communication networks public, cloud services.

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On strong polynomial bounds of rate of convergence for regenerative processes

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Abstract. We give strong bounds for the rate of convergence of the regenerative process distribution to the stationary distribution in the total variation metric. These bounds are obtained by using coupling method. We propose this method for obtaining such bounds for the queueing regenerative processes.

Keywords: regenerative process, queuing theory, rate of convergence, coupling method.

1. Introduction.

We study the rate of convergence of distribution of regenerative process to the stationary distribution in the total variation metric.

Many queueing processes are regenerative, and establishing bounds for the rate of their convergence is a very important problem for the practical applications of the queueing theory. Recall the definition of regenerative process.

Definition. *The process $(X_t,\, t \geqslant 0)$ on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$, with a measurable state space $(\mathcal{X}, \mathcal{F}(\mathcal{X}))$ is regenerative, if there exists an increasing sequence $\{\theta_n\}$ $(n \in \mathbb{Z}_+)$ of Markov moments with respect to the filtration $\mathcal{F}_{t\leqslant 0}$ such that the sequence*

$$\{\Theta_n\} = \left\{X_{t+\theta_{n-1}} - X_{\theta_{n-1}}, \theta_n - \theta_{n-1}, t \in [\theta_{n-1}, \theta_n)\right\}, \quad n \in \mathbb{N}$$

consists of independent identically distributed (i.i.d.) random elements on $(\Omega, \mathcal{F}, \mathbf{P})$. If $\theta_0 \neq 0$, then the process $(X_t,\, t \geqslant 0)$ is called delayed.

Denote $\zeta_n \stackrel{\text{def}}{=\!=} \theta_n - \theta_{n-1}$, and let $F(s) = \mathbf{P}\{\zeta_n \leqslant s\}$ be the distribution function of regeneration period; we suppose that the distribution F is not lattice.

The first great results about the convergence rate of regenerative processes was obtained in [1], namely.

If the condition

$$\mathbf{E}\,\zeta^K = M_K < \infty, \;\; K > 1 \tag{1}$$

is satisfied, then for every $A \in \mathcal{F}(\mathcal{X})$ and all $\kappa < K - 1$ there exists $C(\kappa)$ such that

$$\left|\mathcal{P}_t^{X_0}(A) - \mathcal{P}(A)\right| < C(\kappa) t^{-\kappa}, \tag{2}$$

where $\mathcal{P}_t^{X_0}(A) = \mathbf{P}\{X_t \in A\}$ *and* $\mathcal{P}$ *is the stationary distribution of* X_t, but bounds of the constants $C(\kappa)$ are unknown.

Later this results was repeated by probabilistic approaches, namely, by *modified* coupling method: shift-coupling and distributional shift-coupling [5, 6], ε-coupling [3, 4]. Emphasize that the application of the coupling method is possible only for the Markov processes. However, in queuing theory, usually the regenerative processes are not Markov. Therefore, the state space of considered regenerative process must be extended so that the regenerative process with this state space would become Markov.

So, for the use of the coupling method for the arbitrary regenerative process X_t we must to extend the state space $\mathcal{X}$ of this process of by such a way that the process X_t with this extended state space $\overline{\mathcal{X}}$ is Markov. For markovization of non-Markov regenerative process we can include to the state X_t, $t \in [\theta_{n-1}, \theta_n)$ full history of the process on the time interval $[\theta_{n-1}, t]$: the process $\overline{X}_t \stackrel{\text{def}}{=\!=} \{X_s, s \in [\theta_{n-1}, t] | t < \theta_n\}$ is Markov and regenerative with the extended state space $\overline{\mathcal{X}}$. Thus, in the sequel we suppose that the regenerative process X_t is Markov.

The use of *modified* coupling method is explained by the fact that the applying of the "direct" coupling method for not discrete processes in continuous time is impossible. Indeed, for using of coupling method, it is necessary to estimate the distribution of random variable

$$\tau\left(X_0^{(1)}, X_0^{(2)}\right) \stackrel{\text{def}}{=\!=} \inf\left\{t > 0 :\, X_t^{(1)} = X_t^{(2)}\right\},$$

where $X_t^{(1)}$ and $X_t^{(2)}$ are two versions of Markov process X_t with different initial states. Then for any increasing positive function $\varphi(s)$ we have for all $A \in \mathcal{F}(\mathcal{X})$

$$\left|\mathcal{P}_t^{X_0^{(1)}}(A) - \mathcal{P}_t^{X_0^{(2)}}(A)\right| \leqslant \mathbf{P}\left\{\tau\left(X_0^{(1)}, X_0^{(2)}\right) > t\right\} =$$
$$= \mathbf{P}\left\{\varphi\left(\tau\left(X_0^{(1)}, X_0^{(2)}\right)\right) > \varphi(t)\right\} \leqslant (\varphi(t))^{-1}\mathbf{E}\,\varphi\left(\tau\left(X_0^{(1)}, X_0^{(2)}\right)\right)$$

by the coupling inequality.

Now by integration of this inequality with respect to the measure $\mathcal{P}$ we have

$$\left|\mathcal{P}_t^{X_0^{(1)}}(A) - \mathcal{P}(A)\right| \leqslant (\varphi(t))^{-1} \int\limits_{\mathcal{X}} \varphi\left(\tau\left(X_0^{(1)}, X_0^{(2)}\right)\right) \mathrm{d}\mathcal{P}\left(X_0^{(2)}\right) =$$
$$= (\varphi(t))^{-1} C\left(X_0^{(1)}\right), \quad (3)$$

and

$$\left\|\mathcal{P}_t^{X_0} - \mathcal{P}\right\|_{TV} < (\varphi(t))^{-1} 2C\left(X_0^{(1)}\right). \quad (4)$$

But in general case $\mathbf{P}\left\{\tau(X_0^{(1)}, X_0^{(2)}) < \infty\right\} < 1$, and the "direct" use of coupling method is impossible.

We will present a new method for obtaining polynomial bounds with the rigorous bounds for $C(\kappa)$ in (2).

2. Idea.

Our method applies the idea of [2]. This idea is based on the construction (in a special probability space) of the paired stochastic process $\mathcal{Z}_t = \left(Z_t^{(1)}, Z_t^{(2)}\right)$ such that:

1. For all $t \geqslant 0$ the random variables $X_t^{(i)}$ and $Z_t^{(i)}$ have the same distribution, $i = 1, 2$.
2. $\mathbf{E}\,\tau\left(Z_0^{(1)}, Z_0^{(2)}\right) < \infty$ where $\tau(Z_0) \stackrel{\text{def}}{=\!=} \inf\left\{t \leqslant 0 :\, Z_t^{(1)} = Z_t^{(2)}\right\}$.
3. $Z_t^{(1)} = Z_t^{(2)}$ for all $t \geqslant \tau\left(Z_0^{(1)}, Z_0^{(2)}\right)$.

The paired stochastic process $\mathcal{Z}_t = \left(Z_t^{(1)}, Z_t^{(2)}\right)$ satisfying the conditions 1–3 is called *successful coupling.*

For all $a \in \mathcal{F}(\mathcal{X})$ we can use the coupling inequality in the form:

$$\left|\mathcal{P}_t^{X_0^{(1)}}(A) - \mathcal{P}_t^{X_0^{(2)}}(A)\right| = \left|\mathbf{P}\left\{X_t^{(1)} \in A\right\} - \mathbf{P}\left\{X_t^{(2)} \in A\right\}\right| = \\ = \left|\mathbf{P}\left\{Z_t^{(1)} \in A\right\} - \mathbf{P}\left\{Z_t^{(2)} \in A\right\}\right| \leqslant \mathbf{P}\left\{\tau\left(Z_0^{(1)}, Z_0^{(2)}\right) \geqslant t\right\} \leqslant \\ \leqslant \frac{\mathbf{E}\,\varphi\left(\tau\left(Z_0^{(1)}, Z_0^{(2)}\right)\right)}{\varphi(t)} \leqslant \frac{C\left(Z_0^{(1)}, Z_0^{(2)}\right)}{\varphi(t)} \qquad (5)$$

for some constant $C\left(Z_0^{(1)}, Z_0^{(2)}\right)$. As $Z_0^{(i)} = X_0^{(i)}$, the right-hand side of the inequality (5) depends only on $X_0^{(i)}$.

Then, if $\mathcal{P}^{X_0} \Rightarrow \mathcal{P}$ for all initial states, we can use integration of (5) with respect to the measure $\mathcal{P}$ as in (3)–(4).

This applying of *successful coupling* has some difficulties: this is a need to integrate $\mathbf{E}\,\varphi\left(\widetilde{\tau}\left(Z_0^{(1)}, Z_0^{(2)}\right)\right)$ or $C\left(Z_0^{(1)}, Z_0^{(2)}\right)$ with respect to the stationary measure $\mathcal{P}$.

Therefore we propose to apply the schema of *successful coupling for the process X_t with the initial state X_0, and its stationary version*, namely.

We will construct a successful coupling $\mathcal{Z}_t = \left(Z_t, \widetilde{Z}_t\right)$ for the process X_t and its stationary version $\widetilde{X}_t$, and we will estimate the random variable

$\widetilde{\tau}(X_0) = \widetilde{\tau}(Z_0) \stackrel{\text{def}}{=} \inf\left\{t > 0 : Z_t = \widetilde{Z}_t\right\}$. Then

$$\left\|\mathcal{P}_t^{X_0}(A) - \mathcal{P}(A)\right\|_{TV} \leqslant 2\mathbf{P}\left\{\widetilde{\tau}(X_0) > t\right\}.$$

In the sequel, we suppose that

$$\int\limits_{s:\,\exists f(s)} f(s)\,\mathrm{d}\,s > 0. \tag{6}$$

3. Implementation of idea.

We will divide our reasoning into several steps.

Step 1. Consider some renewal processes R_t with the distribution of renewal time $F(s)$. R_t is a counting process: $R_t \stackrel{\text{def}}{=} \sum\limits_{i=0}^{\infty} \mathbf{1}(S_n < t)$ where $S_n \stackrel{\text{def}}{=} \sum\limits_{n=0}^{n} \zeta_i$, and ζ_i, $i \in Z_+$ are mutually independent; $\mathbf{E}\,\zeta_i < \infty$,$i \in Z_+$; $\mathbf{P}\left\{\zeta_i \leqslant s\right\} = F(s)$, $i \in N$.

Then $X_t \stackrel{\text{def}}{=} (t - S_{R_t})$ – backward renewal time of process R_t – is a Markov process.

It is well known that

$$\lim_{t\to\infty} \mathbf{P}\left\{X_t \geqslant s\right\} = \mu^{-1}\int\limits_{s}^{\infty}(1 - F(u))\,\mathrm{d}\,u \stackrel{\text{def}}{=} 1 - \widetilde{F}(s),$$

where $\mu \stackrel{\text{def}}{=} \mathbf{E}\,\zeta_1$.

Our first step is the estimation of the rate of convergence

$$\mathbf{P}\left\{X_t \leqslant s\right\} \to \widetilde{F}(s).$$

For the initial state $X_0 = a$ we suppose

$$\mathbf{P}\left\{\zeta_0 \leqslant s\right\} = F_a(s) \stackrel{\text{def}}{=} \frac{F(s+a) - F(a)}{1 - F(a)}.$$

We will construct a *successful coupling* $\mathcal{Z}_t = \left(Z_t, \widetilde{Z}_t\right)$ for the process X_t with an initial state $X_0 = a$ and its stationary version $\widetilde{X}_t$.

Denote $\varphi(s) \stackrel{\text{def}}{=} \mathbf{1}(\exists\, f(s)) \times \Big(f(s) \wedge \widetilde{F}'(s)\Big)$; the condition (6) implies

$$\int\limits_0^\infty \varphi(s)\,\mathrm{d}\,s \stackrel{\text{def}}{=} \varkappa > 0.$$

Denote $\Phi(s) \stackrel{\text{def}}{=} \int\limits_0^s \varphi(u)\,\mathrm{d}\,u$, $\Psi(s) \stackrel{\text{def}}{=} F(s) - \Phi(s)$, $\widetilde{\Psi}(s) \stackrel{\text{def}}{=} \widetilde{F}(s) - \Phi(s)$.

Put for $a \in [0,1)$, $b \in [0,\varkappa)$, $c \in [\varkappa, 1)$

$$\Xi(a,b,c) \stackrel{\text{def}}{=} \mathbf{1}(a < \varkappa)\Phi^{-1}(\varkappa b) + \mathbf{1}(a \geqslant \varkappa)\Psi^{-1}((1-\varkappa)c);$$

$$\widetilde{\Xi}(a,b,c) \stackrel{\text{def}}{=} \mathbf{1}(a < \varkappa)\Phi^{-1}(\varkappa b) + \mathbf{1}(a \geqslant \varkappa)\widetilde{\Psi}^{-1}((1-\varkappa)c).$$

We will construct the pair $\mathcal{Z}_t$ by induction.

Let $\mathcal{U}_{i,j}$ be i.i.d. uniform random variables on $[0,1)$ defined on a some probability space $(\Omega, \mathcal{F}, \mathbf{P})$.

Basis of induction. Put

$$t_0 \stackrel{\text{def}}{=} F_a^{-1}(\mathcal{U}_{0,0}), \;\; \widetilde{t}_0 \stackrel{\text{def}}{=} \widetilde{F}^{-1}(\mathcal{U}_{0,1}), \;\; \widetilde{Z}_0 \stackrel{\text{def}}{=} F_{t_0}^{-1}(U_{0,2});$$

and we put $Z_t \stackrel{\text{def}}{=} t + a$ for $t \in [0, \theta_0)$ where $\theta_0 \stackrel{\text{def}}{=} t_0 \wedge \widetilde{t}_0$.

Inductive step. Suppose that we have constructed the process $\mathcal{Z}_t$ for $t \in [0, \theta_k)$, $\theta_k = t_i \wedge \widetilde{t}_j$. There are three alternatives.

1. $\theta_k = t_i = \widetilde{t}_j$. In this case we put

$$Z_{\theta_k} = \widetilde{Z}_{\theta_k} = 0, \;\; t_{i+1} = \widetilde{t}_{j+1} = \theta_{k+1} = F^{-1}(U_{0,i+1});$$

and $Z_t = \widetilde{Z}_t \stackrel{\text{def}}{=} t - \theta_k$ for $t \in [\theta_k, \theta_{k+1})$.

2. $\theta_k = \widetilde{t}_j < t_i$. In this case we put

$$\widetilde{Z}_{\theta_k} = 0, \;\; Z_{\theta_k} = Z_{\theta_k - 0}, \;\; \widetilde{t}_{j+1} \stackrel{\text{def}}{=} \widetilde{t}_j + F^{-1}(U_{1,j+1});$$

and $\widetilde{Z}_t \stackrel{\text{def}}{=} t - \theta_k$, $Z_t \stackrel{\text{def}}{=} t - \theta_k + Z_{\theta_k}$ for $t \in [\theta_k, \theta_{k+1})$ where $\theta_{k+1} \stackrel{\text{def}}{=} t_i \wedge \widetilde{t}_{j+1}$.

3. $\theta_k = t_i < \widetilde{t}_j$. In this case we put

$$t_{i+1} \stackrel{\text{def}}{=} t_i + \Xi(\mathcal{U}_{2,i+1}, \mathcal{U}_{3,i+1}\mathcal{U}_{4,i+1}); \;\; \widetilde{t}_j \stackrel{\text{def}}{=} t_i + \widetilde{A},$$

where $\widetilde{A} = \widetilde{\Xi}(\mathcal{U}_{2,i+1}, \mathcal{U}_{3,i+1}, \mathcal{U}_{5,i+1})$; and

$$Z_t \stackrel{\text{def}}{=} t - \theta_k, \;\; \widetilde{Z}_t \stackrel{\text{def}}{=} t - \theta_k + F_{\widetilde{A}}^{-1}(\mathcal{U}_{6,i+1})$$

for $t \in [\theta_k, \theta_{k+1})$ where $\theta_{k+1} \stackrel{\mathrm{def}}{=} t_i \wedge \widetilde{t}_{j+1}$.

It can to check that the constructed process $\mathcal{Z}_t = (Z_t, \widetilde{Z}_t)$ satisfies the conditions 1–3 of definition of the successful coupling, and

$$\mathbf{P}\left\{Z_{t_{i+1}} = \widetilde{Z}_{t_{i+1}} | Z_{t_i} \neq \widetilde{Z}_{t_i}\right\} = \varkappa > 0;$$

we will give the full proof of this inequality in the following publications.

Step 2. Now we can give a strong bounds for the constant $C(\kappa)$ in (2) if the conditions (1) and (6) are satisfied, and for all $\kappa \leqslant k$ we have

$$\widetilde{\tau}(Z_0) \leqslant t_0 + \sum_{i=1}^{\nu}(t_i - t_{i-1}) = t_0 + \sum_{i=1}^{\nu-1} \bar{\zeta}_i + \hat{\zeta}_\nu$$

where
$\mathbf{P}\{\nu > i\} = (1 - \varkappa)^i$, $\bar{\zeta}_\nu \stackrel{\mathrm{def}}{=} \zeta\Big|_{\left\{Z_{t_\nu} \neq \widetilde{Z}_{t_\nu}\right\}}$, and $\hat{\zeta}_\nu \stackrel{\mathrm{def}}{=} \zeta\Big|_{\left\{Z_{t_\nu} = \widetilde{Z}_{t_\nu} \middle| Z_{t_{\nu-1}} \neq \widetilde{Z}_{t_{\nu-1}}\right\}}$.

By Jensen's inequality we have:

$$\begin{aligned}
&\mathbf{E}_{X_0}(\widetilde{\tau}(X_0))^\kappa \leqslant \\
&\leqslant \sum_{n=1}^{\infty}(n+1)^{\kappa-1}\varkappa(1-\varkappa)^{n-1}\left(\mathbf{E}_{X_0}(t_0)^\kappa + (n-1)\mathbf{E}(\bar{\zeta}_i)^\kappa + \mathbf{E}(\hat{\zeta}_n)^\kappa\right) \leqslant \\
&\qquad \leqslant \varkappa \int_0^\infty s^\kappa \,\mathrm{d}\, F_{X_0}(s) \sum_{n=1}^{\infty}(n+1)^{\kappa-1}(1-\varkappa)^{n-1} + \\
&\qquad\quad + \varkappa \mathbf{E}\,\zeta^\kappa \sum_{n=2}^{\infty}(n+1)^{\kappa-1}(n-1)(1-\varkappa)^{n-2} + \\
&\qquad\qquad + \mathbf{E}\,\zeta^\kappa \sum_{n=1}^{\infty}(n+1)^{\kappa-1}(1-\varkappa)^{n-1} \stackrel{\mathrm{def}}{=} C(X_0, \kappa).
\end{aligned} \tag{7}$$

Step 3. Now let X_t be a regenerative process with an extended state space $\mathcal{X}$, and on this state space it is Markov. The regeneration times t_0, t_1,... of X_t form an embedded process $Y_t \stackrel{\mathrm{def}}{=} t - \max\{t_i < t\}$ (the backward renewal time of the embedded renewal process) with the distribution of the renewal time $\mathbf{P}\{\zeta \leqslant s\} = F(s)$; the length of i-th regeneration period is $t_i - t_{i-1} = \zeta_i \stackrel{\mathcal{D}}{=} \zeta$.

The random element $\mathfrak{X}_i = \{X_t, t \in [t_{i-1}, t_i)\}$ depends on the random variable $\zeta_i = t_i - t_{i-1}$; for $A \in \mathcal{B}(\mathcal{X})$ denote

$$\mathcal{G}_a(s, A) \stackrel{\text{def}}{=} \mathbf{P}\left\{X_{t_{i-1}+s} \in A | \zeta_i = a\right\}, \quad s \in [0, a).$$

$\mathcal{G}_a(s, A)$ specify a conditional distribution of X_t on the time period $[t_{i-1}, t_i)$ given $\{t_i - t_{i-1} = a\}$.

Therefore if we know all regeneration times of X_t, then we know the (conditional) distribution of process X_t in every time after the first regeneration time t_0: this distribution is determined by the conditional distribution $\mathcal{G}_{\zeta_i}(s, A)$ of the random elements $\mathfrak{X}_i$.

Then we can repeat the construction of the *Step 1* for an embedded renewal process Y_t of regenerative process X_t and for embedded process $\widetilde{Y}_t$ of the stationary version $\widetilde{X}_t$ of the process X_t. We can complete the pair $\mathcal{Z}_t = \left(Z_t, \widetilde{Z}_t\right)$, $Z_t \stackrel{\mathcal{D}}{=} Y_t$, $\widetilde{Z}_t \stackrel{\mathcal{D}}{=} \widetilde{Y}_t$ by random elements $\mathfrak{X}_i$ on such a way that completed regenerative processes W_t and $\widetilde{W}_t$ have a distribution $\mathcal{P}_t^{X_0}$ and $\mathcal{P}$ correspondingly.

Hence, $\widetilde{\tau}(Z_0) \stackrel{\text{def}}{=} \inf\left\{t :\, Z_t = \widetilde{Z}_t\right\} \geqslant \inf\left\{t :\, W_t = \widetilde{W}_t\right\}$, and we can estimate a bounds for the convergence rate for regenerative Markov process X_t similar to (7).

Now we can return to the original state space of the regenerative (non-Markov) process X_t. The bounds of the convergence rate will remain the same, i.e. we can apply (7) to non-Markov regenerative processes.

4. Applying to the queuing theory.

In the queuing theory the distribution of the regenerative process period is often unknown. But often the regeneration period can be split into two parts, usually this is a busy period and idle period. And as a rule the idle period has a known non-discrete distribution. If the bounds of moments of a busy period are also known, then we can use a modified *Step 1*. This modification is a construction of a successful coupling for alternating renewal process X_t and its stationary version.

Let X_t be an alternating renewal process having two states, 1 and 2, say. The time of the stay of the process in a state i has the distribution function $F_i(s) = \mathbf{P}\left\{\zeta^{(i)} \leqslant s\right\}$, and periods of stay of the process in states 1 and 2 alternate. Again, we consider the markovization of this process, namely.

We complement the state of the process by the time, during which the process located continually in this state: if $X_t = (n, x)$, then at the time $t - x$ it got into the state n; denote $n(X_t) = n$, $x(X_t) = x$. The process X_t has a state space $\{1, 2\} \times \mathbb{R}_+$. Denote $c(n) \stackrel{\text{def}}{=} \mathbf{1}(n = 2) + 2 \times \mathbf{1}(n = 1)$.

If $X_0 = (i, a)$, then the process X_t changes it's first component $n(X_t)$ at the times $\zeta^{(a,i)} = t_{0,i},\ t_{0,c(i)},\ t_{1,i},\ t_{1,c(i)},\ldots;\ \zeta_j^{(i)} = t_{j,c(i)} - t_{j,i} \stackrel{\mathcal{D}}{=} \zeta^{(i)}$; $\zeta_j^{(c(i))} = t_{j,i} - t_{j-1,c(i)} \stackrel{\mathcal{D}}{=} \zeta^{(c(i))}$; $\mathbf{P}\left\{\zeta^{(a,i)} \leqslant s\right\} = \dfrac{F_i(a+s) - F_i(s)}{1 - F_i(s)}$;.

Suppose that the distribution function $F_1(s)$ satisfies the condition (6), $\mathbf{E}(\zeta^{(i)})^K < \infty$, and random variables $\zeta^{(a,i)}$, $\zeta_j^{(1)}$, $\zeta_j^{(2)}$ are mutually independent.

Now we can construct the successful coupling of the process X_t and its stationary version $\mathcal{Z}_t = \left(Z_t, \widetilde{Z}_t\right)$ so that

$$\mathbf{P}\left\{Z_{t_{j,2}} = \widetilde{Z}_{t_{j,2}} | Z_{t_{j-1,2}} \neq \widetilde{Z}_{t_{j-1,2}}\right\} \geqslant \int\limits_{\left\{\exists\, F_1'(s)\right\}} F_1'(s) \wedge \frac{1 - F_1(s)}{\mathbf{E}\,\zeta^{(1)}}\,\mathrm{d}\,s > 0.$$

Then, by calculations similar to (7), we can obtain a rigorous bound for the rate of convergence for marginal distributions of process X_t.

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References

1. *Borovkov A. A.* Stochastic processes in queueing theory. — Nauka 1972, Springer-Verlag 1976.
2. *Griffeath D.* A maximal coupling for Markov chains // Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete 1975, — Volume 31, Issue 2, pp 95–106.
3. *Kalashnikov V. V.*, Uniform Estimation of the Convergence Rate in a Renewal Theorem for the Case of Discrete Time // Theory Probab. Appl., — 22(2), pp 390–394.
4. *Kalashnikov V. V.*, Topics on Regenerative Processes, — CRC Press, 1994.
5. *Thorisson H.* Coupling, Stationarity, and Regeneration. — Springer, 2000.
6. *Thorisson H.*, On Maximal and Distributional Coupling // Ann. Probab. — Volume 14, Number 3 (1986), 873–876.

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