Analysis of thin shells

USING FGM FOR CYCLIC SHELL STRUCTURES

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This article deals with FGM for cyclic shell structure and the finite element modeling and analysis of functionally graded (FG) shell structures under self-loading. In order to study the influences of important parameters on the responses of FG shell structures, some types of cyclic shells have been considered. The responses obtained for FG shells are based on the analysis; some important results are presented and discussed for thin cyclic shells.
KEY WORDS: Functionally Graded Materials, Cyclic Shells, Epitrochoidal Shells, Thermo-Elastic Property.

Introduction:

Thin shells as structural elements occupy a leadership position in civil, mechanical, architectural, aeronautical and marine engineering, since they give rise to optimum conditions for dynamic behavior, strength and stability. In other words, these structures support applied external forces efficiently by virtue of their geometrical shape. An important aspect in the successful applications of these structures is fact that shells cover large pans. As for many other shape kinds, conical, cylindrical shells and shells of complex geometry are very common structural elements.

The concept of functionally graded materials (FGMs) introduced a class of highly engineered structures tailored to specific properties, resulting of compositional changes in used materials. The necessity to bring into practice new materials appears crucial with, for instance, space vehicles: on the surface side and the skin plates should have very good heat-resistance, on the inside, however, – high mechanical qualities (e.g. toughness) were needed.

The problem was successfully solved in Japan in the mid of 1980s by manufacturing specific composite: metallic matrix and ceramic particles with graded distribution of these particles. That solution is closed with ingenious structural systems in some plants, e.g. bamboo. After 20 years of intensive research and practical applications, the field of FGMs is still in development and a precise definition of that new class of materials is till now not accepted. Modeling of FGMs is recognized as indispensable step in designing at the microstructural level to meet specific requirements of an intended application. Many production technologies were proved to be useful for practical adoption.

Today, production of graded structures can be considered as the next step in composite materials development. Functionally graded materials are a relatively new class of engineered materials in which the composition and/or microstructure varies in one specific direction. They are made by a continuous change in composition and do not possess a specific interface. Therefore, it is generally assumed that such a structure should better resist thermal and mechanical cycling. The application of this concept to metal matrix composites (MMCs) leads to the development of materials/components designed with the purpose of being selectively reinforced only in regions requiring increased modulus, strength and/or wear resistance.

Definition and Modeling of FGM

Functionally graded materials (FGMs) are a class of composite materials that have been taken into consideration in the last three decades because of their special performance compared with conventional materials. FGMs are inhomogeneous materials made from different phases of material constituents, usually ceramic and metal, and their material properties change gradually along a certain direction, usually in the thickness one.

The term FGM was originally presented in the 1984 by a group of scientists in Japan when they conducted research into materials that are resistant to extremely high temperatures for aircrafts and aerospace applications. Due to particular characteristics of functionally materials, these can resist high temperatures in various environments. Different types of gradations laws are available in the literature. In the present work power law Gradation has been considered in order to calculate the material properties of FG structures. FGM Consisting of two constituent materials has been considered.

The top surface is assumed to be rich in Material-2 (ceramic) and bottom surface is assumed to be rich in material-1 (metal).The region between the two surfaces consists of a combination of the two materials with continuously varying mixing ratios of two materials.

$$V_c(z) = \left(\frac{(z+h)}{2h}\right)^m$$
$$V_c(z) = 1 - V_{mat}(z)$$
$$P(z) = P_{mat} + (P_c - P_{mat})V_c(z)$$

where z is the distance from mid-surface and m is the power law index, a positive real number. P_{mat} Is the material property of metal (steel) surface and P_c is material property of ceramic (Al_2O_3) surface. Vc *(z)* And V_{mat} (z) are the volume fractions of the ceramic and metal surface. In the present analysis, the material properties are varying though the thickness from bottom to top. The material properties such as young's modulus, thermal expansion, conductivity and density are varying in thickness direction only and Poisson's ratios is constant throughout the thickness as shown in Fig 1. The material properties used for modeling and analysis of FG structures is presented in the Table 1.

Table.1. Thermo-elastic properties for metallic (Steel) and ceramic (Al2O3) phases

Material	Thermal expansion coefficient $\alpha(^{o}c^{-1})$	Poisons' ratio μ	Young's modulus E (GPa)	Density(kg/m^3)	Conductivity(k) (W/m^{0}K)
AL_2O_3	6.9×10^{-6}	0.25	390	3.89×10^3	25
Steel	14×10^{-6}	0.25	210	7.85×10^3	40

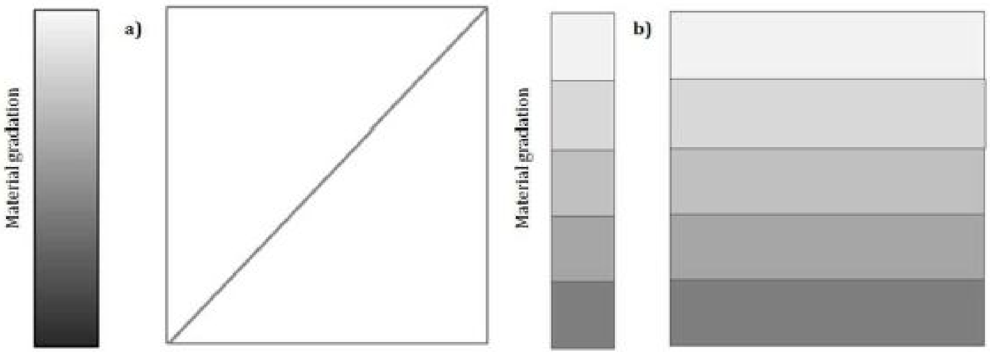

Figure 1

There are two types of graded structures which can be prepared in case of FGCM, continuous structure as shown in figure 1 (a) and stepwise structure shown in figure 1(b). In the case of continuous graded structure, the change in composition and microstructure occurs continuously with position. On the other hand, in the case of step wise, microstructure feature changes in step wise manner, giving rise to a multilayered structure with interface existing between discrete layers.

Many researchers proposed several kinds of processing methods for FGCM [1]. Powder metallurgy is one of the most important methods of producing FGMs. An example of a typical fabrication process by the powder metallurgy is schematically illustrated in flow chart (figure 2). At first, material A and material B are weighed and mixed. Each of mixed-powder is mixed uniformly by a V-shape mill/ball mill. Next step is stepwise stacking of premixed powder according to a predesign spatial distribution of the composition in respective die.

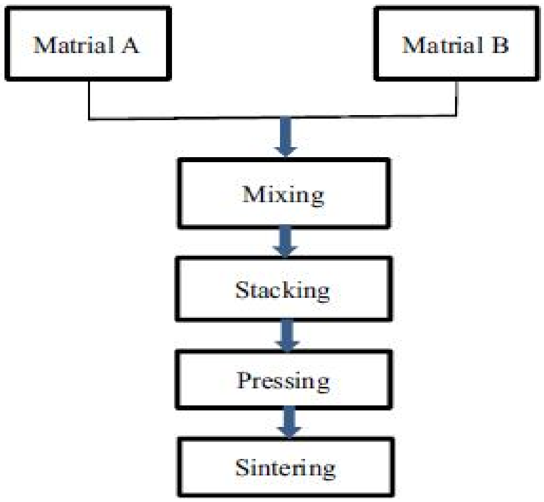

Figure 2

Last step is sintering high quality materials in short periods by charging the intervals between powder particles with electrical energy and high sintering pressure. The FGM fabricated by this method should have the stepwise structure, and it is difficult to produce the FGMs with continuous gradients. The continuous graded structure can be created by a centrifugal force [2].

Cyclic Shell:

The investigation of the class of cyclic surfaces started from the research of tubular surfaces of constant diameter having straight or curvilinear axes. Further, many subclasses and types of cyclic surfaces were discovered and examined [3]. Some cyclic surfaces have been named in honor of the geometricians presented these surfaces for the applications. For example, one may mention Joachimsthal's surface, Dupin's cyclides, or the surface of Virich. It should be noted that surfaces of revolution are the cyclic surfaces with straight axis but they are singled out into a special class of Surfaces of Revolution, which is why these surfaces will not be presented in this review.

Terminology and Classification of Cyclic Surfaces:

The cyclic surface is formed by movement of a circle of variable or constant radius under any law in a three dimensional space [4]. The equation of a cyclic surface in the vector form may be written as

$$r = r\,(u, v) = \rho\,(u) + R\,(u)\,e\,(u, v),$$

where $r(u, v)$ is the radius-vector of a cyclic surface; ρ(u) is the radius-vector of the directrix, i.e. of the line of the centers of generating circles; $R(u)$ is the law of change of radius of circular generatrices.

Epitrochoidal Surface:

The M point located on a plane of a circle with the a radius, which rolls without sliding on other motionless circle with b radius, forms an epitrochoidal line [5]. The planes of these two circles constitute a constant corner γ. The distance from a point of M to the center of a mobile circle is equal to $\mu\, a$ ($\mu = 1, or\ \mu < 1, or\ \mu > 1$).

changing parameter γ from 0 to 2π, it is possible to receive a family of epitrochoidal curves which will form epitrochoidal surface (Figure 3) [5]. Surface *Ф* envelops a system of spherical surfaces and touches with them along the circles. The theorem of Joachimsthal's proves that the family of circles of epitrochoidal surfaces is lines of curvature; hence, a surface *Ф* is a special case of canal surface of Joachimsthal.

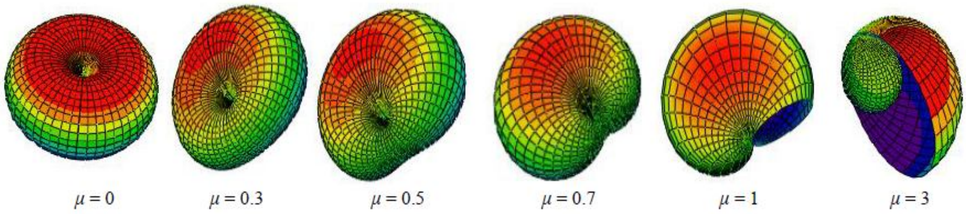

Figure 3

An epitrochoidal surface maybe defined by the parametrical equations:

$$x = x(\alpha, v) = 2R(\alpha)\cos^2 v \cos\alpha, \quad y = y(\alpha, v) = 2R(\alpha)\cos^2 v \sin\alpha, \quad z = z(\alpha, v) = R(\alpha)\sin 2v,$$

where $R(\alpha) = a(1 + \mu \cos\alpha)$ is the radius of a generating circle, α is a corner between an axis *Ox* and a plane of a circular generatrix. At this way of the task, one recognizes that a surface generates by rotation of a mobile circle with radius a about its tangent in the point of a contact with a motionless circle with radius $b = a$. Generating circles of the surface lie in a plane of one pencil. The beginning of coordinates is placed in a double conic point of the surface.

Additional information can be taken in [6].

Modeling of FG Structures using ANSYS:

The material properties of the FGM change throughout the thickness; the numerical model has been divided into various layers in order to make the changes in properties. Each layer has the finite portion of the thickness and treated like isotropic material.

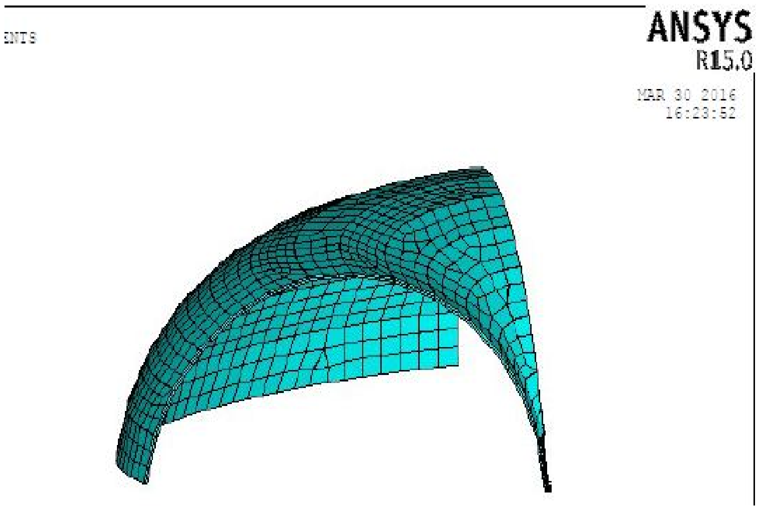

Fig.4. Functionally Graded epitrochoidal shell

Material properties have been calculated at the mid-plane of each of this layer by using the power law gradation. The thickness of FG shell structures have been discretized through the thickness into two layers in order to model FG shell structures. The finite element (FE) modelling has been carried out using ANSYS. A layered shell element (SHELL181) has been used for modelling of FG shell structures. The element has six degrees of freedom at each node. Translations are in the nodal *x, y*, and z directions and rotations about the nodal *x, y*, and *z*-axis. In this paper finite element modelling of functionally graded cyclic shell (Epitrochoidal shells) structures have been done.

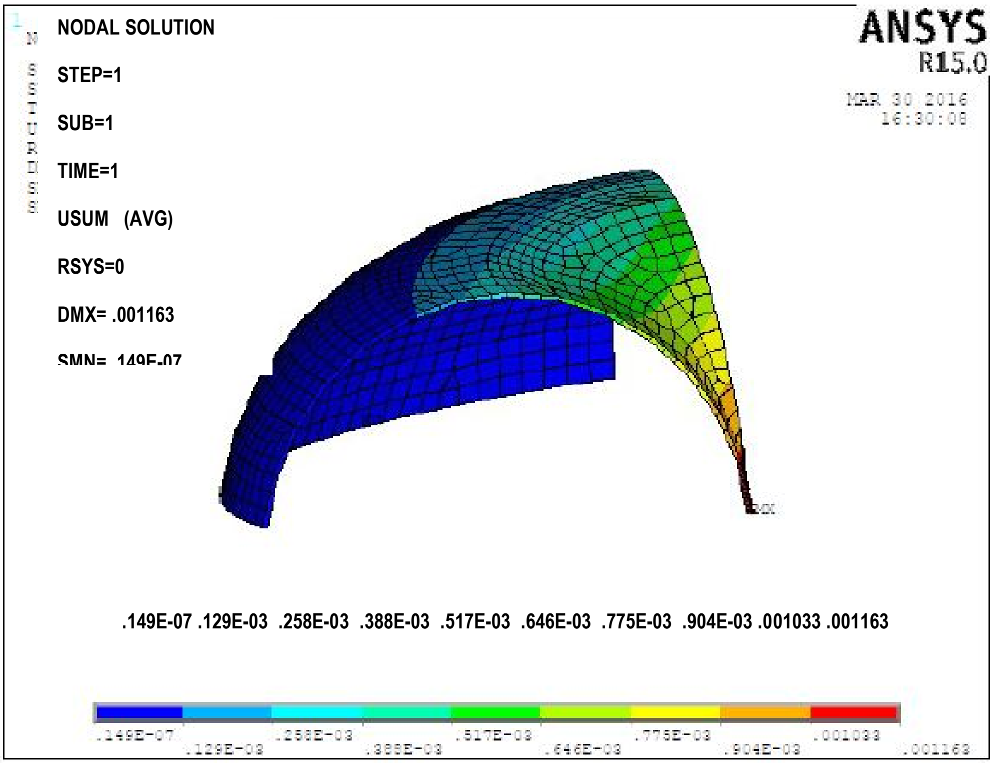

Fig. 5. Displacement variation for FG epitrochoidal shell

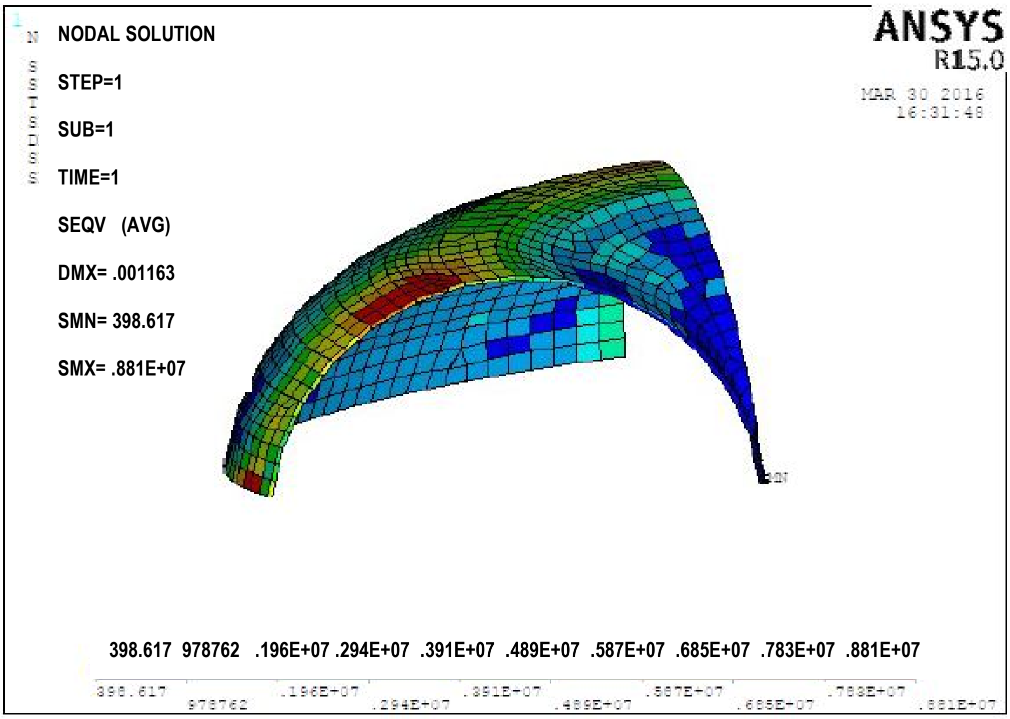

Fig. 6. Von Mises Stress variation for FG epitrochoidal shell

Results and Discussions:

Based on the FE analysis as discussed, FG epitrochoidal shells have been analyzed using ANSYS. Results obtained of different analysis (such as static, and thermal) have been presented in the following subsections.

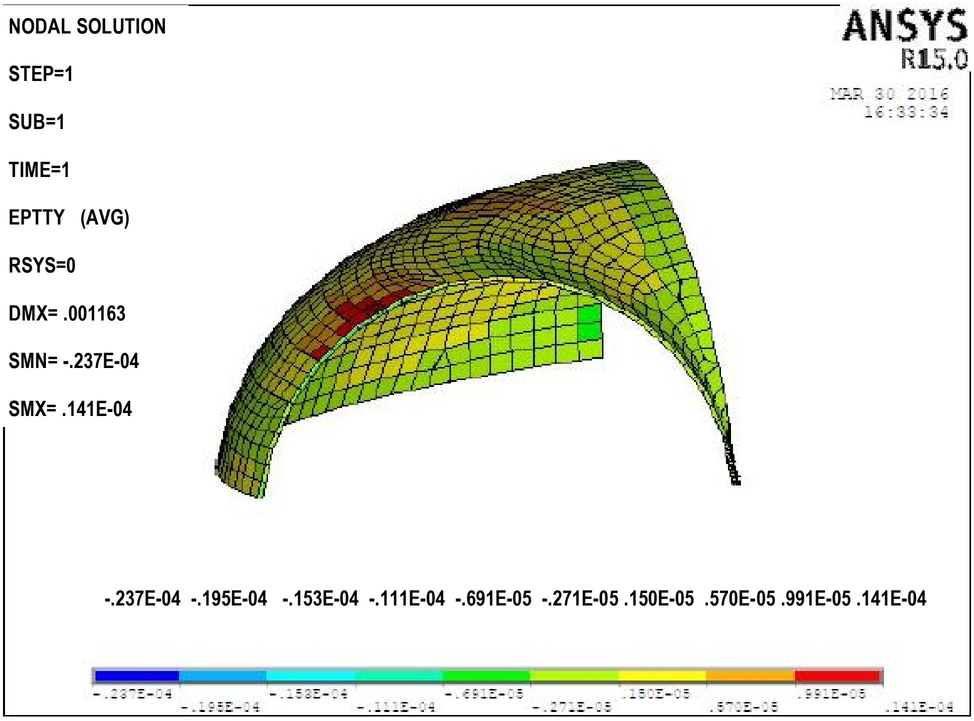

Fig.7. Strain (Y-axis) variation for FG epitrochoidal shell

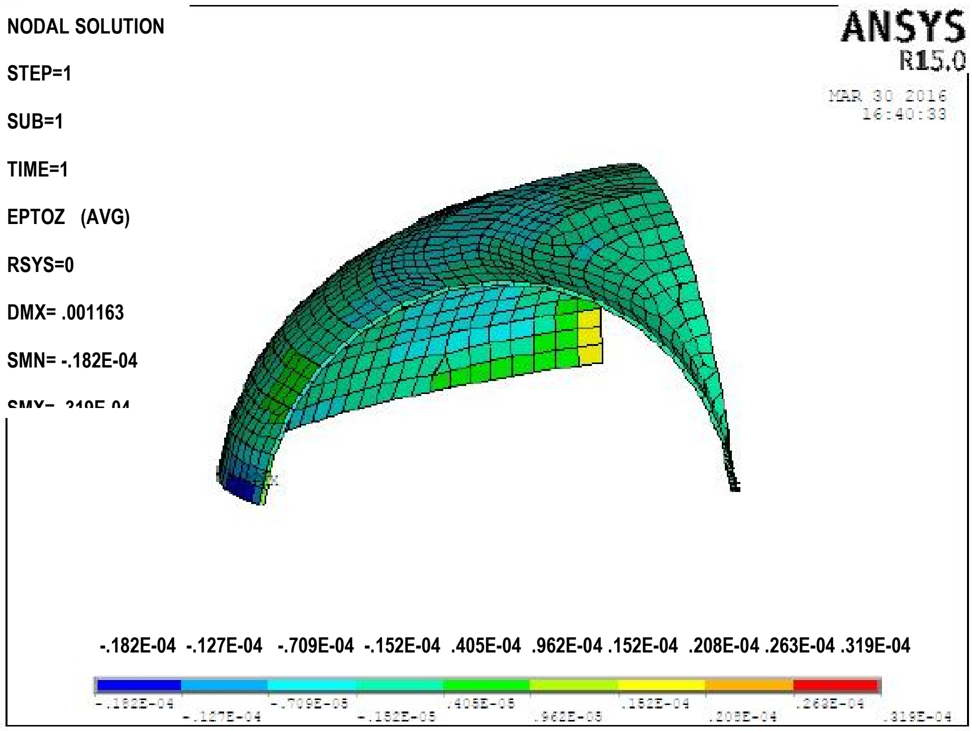

Fig.8. Strain (Z-axis) variation for FG epitrochoidal shell

FE analysis of FG epitrochoidal shell structure under self-weight and thermal loading

An epitrochoidal shell structure with fixed supports is depicted in Fig 4. After validation, it has been analyzed under mechanical as well as thermal loading. Finite element (FE) analyses have been done using our model. Thickness of the shell (*h=1.0 cm*) including ($Al_2O_3 = 5mm, Steel = 5mm$). The mechanical and thermal material properties used in the present study have been listed in the Table 1. Fig. 5 shows displacement variation for FG epitrochoidal shell under self-weight and thermal loading. The overall displacement varies from $0.149 \cdot 10^{-7} m\ to\ 0.001163\ m$.

Fig. 6 shows the Von Mises Stress variation for FG epitrochoidal shell under self-weight and thermal loading. The stress varies from $398.617\ Pa.\ \ to\ \ 0.881 \cdot 10^{7} Pa$.

Fig. 7 shows Y- component of the strain variation for FG epitrochoidal shell under self-weight and thermal loading. The strain varies from $-0.237 \cdot 10^{-4}$ to $0.141 \cdot 10^{-4}$. Fig. 8 shows Z- component of the Strain variation for FG epitrochoidal shell under self-weight and thermal loading. The strain varies from $-0.182 \cdot 10^{-4}$ to $0.319 \cdot 10^{-4}$.

Conclusions

A layered epitrochoidal shell element (SHELL181) has been used for the modeling and analysis of functionally graded (FG) composite shell structures. Important results obtained from the present study have been presented here, that shown the strengthening role of functionally graded materials for shell structure. Static analysis of the FG epitrochoidal shell under self-weight and thermal loading have been done and validated with the published results.

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ПРИМЕНЕНИЕ ФУНКЦИОНАЛЬНО-ГРАДИЕНТНЫХ МАТЕРИАЛОВ В КОНСТРУИРОВАНИИ ЦИКЛИЧЕСКИХ ОБОЛОЧЕК

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Статья посвящена функционально-градиентным материалам (ФГМ) в конструировании и конечно-элементном моделировании циклических оболочек под действием собственного веса. Для исследования влияния функционально-градиентных материалов на прочность оболочек, рассмотрены некоторые виды циклических оболочек. Получены важные результаты, которые представлены и прокомментированы в статье.

КЛЮЧЕВЫЕ СЛОВА: функционально-градиентные материалы, циклические оболочки, эпитрохоидальные оболочки, термо-упругий материал.