ИННОВАЦИОННЫЕ ПРОГРАММЫ ИНЖЕНЕРНЫХ ИССЛЕДОВАНИЙ

SHAKEDOWN ANALYSIS OF THE TRUSS AND COMPARING WITH THE FUNDAMENTAL THEOROMS OF ALASTIC-PLASTIC ANALYSIS IMPLEMENTED IN A HOME-PAKEGE AND ANSYS

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Research results in shakedown analysis of space steel structures are presented in this paper. The theory of shakedown and conventional methods of analysis are discussed, and the example of a shakedown of a truss-column is provided to illustrate the concepts of the nonlinear analysis and shakedown for space trusses [1; 4].

Key words: Shakedown Analysis, Limit Analysis, Plasticity, Geometrical and Material Nonlinearities, Truss Structures.

Behavior of steel subjected to uniaxial states of stress. The space trusses which are analyzed in this paper are steel structures. Their behavior is therefore strongly influenced by the properties of steel. Of particular importance is the state of stress at a material point which results from a given state of strain at the same point. The mathematical formulation of this relationship depends on the state of stress. The relationship for a uniaxial state of stress is considerably simpler than the relationship for a three-dimensional state of stress. Figure 1 shows the stress σ as a function of the strain ε for a uniaxial state of stress in a mild steel specimen. The stress-strain diagram is subdivided into three zones.

Elastic behavior. In the elastic zone the stress σ is proportional to the strain ε. The proportionality constant is called the modulus of elasticity E. Let the stress be increased from null to ε so that the material reaches state (ε, σ) in the elastic zone. If the stress is then removed (reduced to null), the strain is also reduced to null. Elastic strains are thus fully reversible [1; 5].

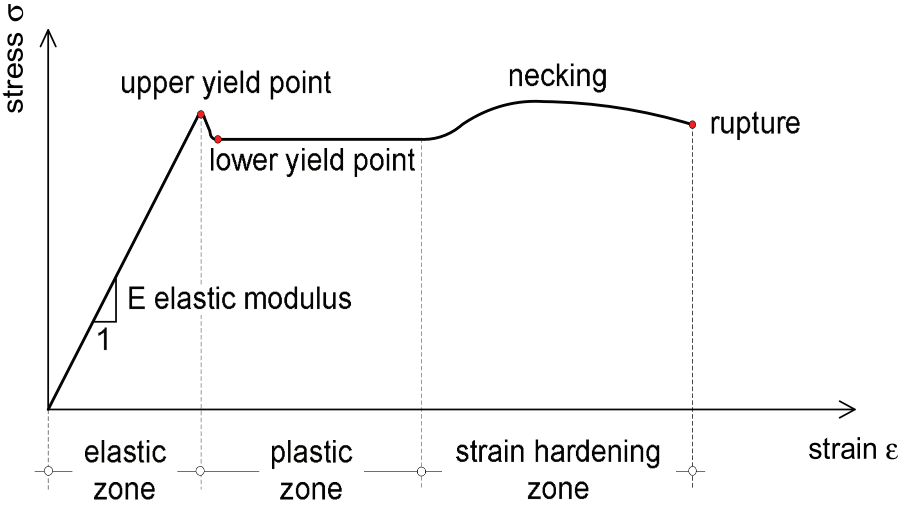


Fig. 1. Stress-strain diagram for mild steel

Plastic behavior. Consider a steel specimen with the load history shown in figure 2. Let the stress be increased from null until the material reaches the upper yield point. The state of the specimen then enters the plastic zone. If the strain is increased further to point B, the total strain equals the sum of an elastic strain component and a plastic strain component. The elastic strain at point B equals the elastic strain at the entry point A to the plastic zone. The plastic strain component is the difference of the total applied strain at point B and the elastic strain component [2].

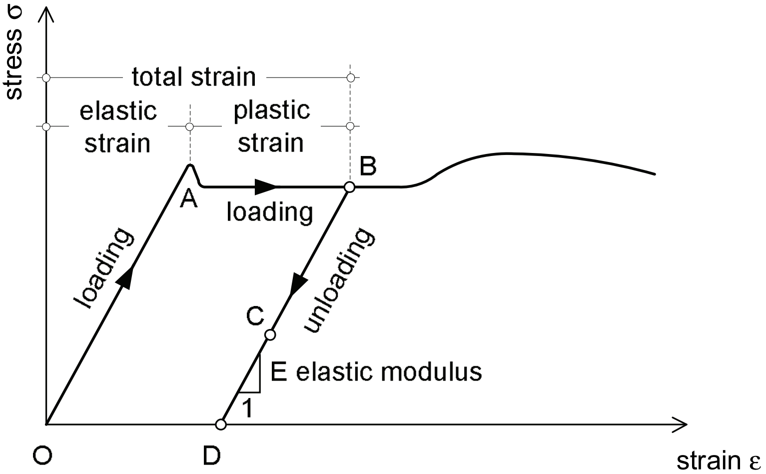


Fig. 2. Load history of a steel specimen

Unloading. The specimen is unloaded at point B by reducing the applied total strain by Δε until point C is reached. Since the plastic strain component is not reversible, the elastic strain component is reduced by Δε. During unloading on curve BC the material behaves elastically with the same modulus of elasticity E as during loading on curve OA.

The stress reduction from B to C is thus $\Delta\varepsilon = E\Delta\varepsilon$. If the total strain is reduced further until point D is reached, the elastic strain component returns to null and only the plastic strain component remains. If the total strain is reduced still further, the stress becomes negative and the plastic strain remains constant. When the stress reaches the negative yield stress, the material becomes plastic and further reduction of the total strain leads to plastic strain increments. Since these plastic strain increments are negative, they reduce the plastic strains that were accumulated on curve AB [4].

Strain hardening. If the total strain of a specimen is increased so that its state enters the strain hardening zone of the diagram in figure 1, the strain first increases and then decreases until the material ruptures. At the same time, the area of the cross-section of the steel specimen is reduced. The phenomenon is called necking. Necking reduces the force that acts on the specimen.

Idealized stress-strain diagram. The observed stress-strain curve of steel in figure 1 is replaced by the ideal elastic-plastic stress-strain curve shown in figure 3. The upper and lower yield points are replaced by a common yield point which is characterized by the yield stress σ_y. The value of the yield strain $\varepsilon_y = \sigma_y / E$ at the yield point follows from the known value of the modulus of elasticity E. It is assumed that the amount of plastic strain which can occur is not limited, and that strain hardening does not occur. The stress-strain curves in tension and in compression are assumed to be similar. These assumptions greatly simplify the structural models of trusses.

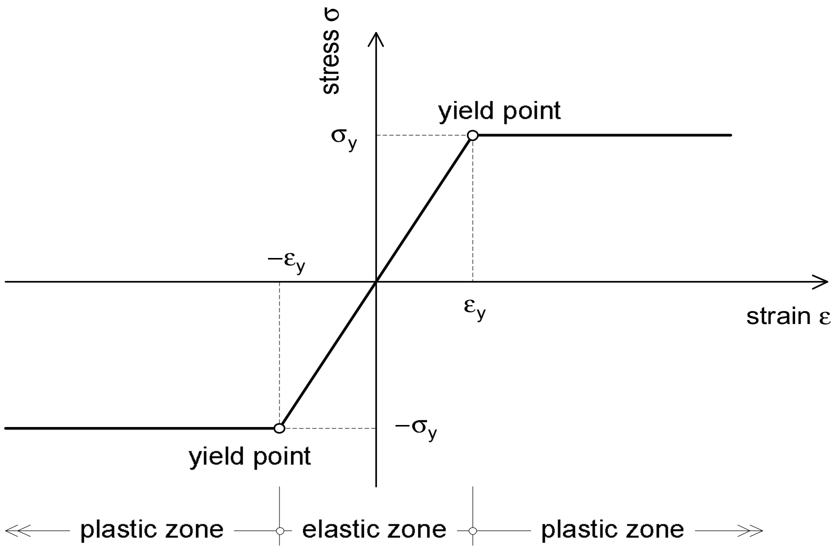


Fig. 3. Idealized stress-strain diagram of mild steel

Alternating plasticity. Let the load cycle shown in figure 4 start at the origin O of the stress-strain diagram. After the yield point B has been reached, the total strain is increased by a plastic strain increment ε_p to point C. The material is then unloaded elastically until the yield stress $-\sigma_y$ is reached at point D. The absolute value of the reduction in total strain from point D to point E equals the absolute value of the increase in total strain from point B to point C. The plastic strain is therefore reduced by $-\varepsilon_p$ so that the total plastic strain at point E is null. The stress is then increased until the material is free of stress, elastic strain and plastic strain at point O. The strain increments are repeated in subsequent load cycles. After a number of cycles which depends on the magnitude of the plastic strain increments, the steel may fail in fatigue. The phenomenon is called low cycle fatigue. It is not treated in this thesis [6].

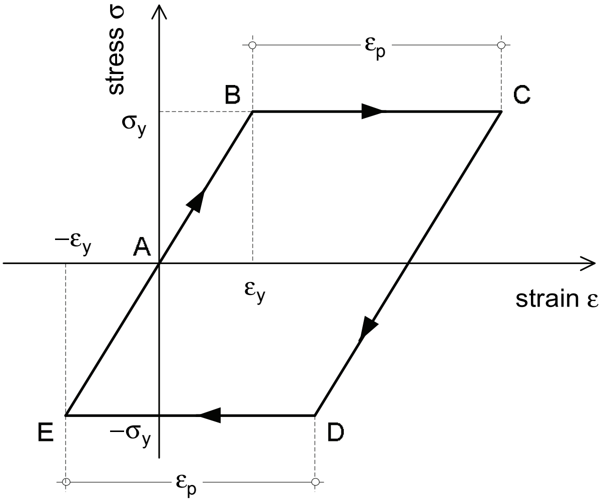


Fig. 4. Example of alternating plasticity

Ratcheting. Let the load cycles in figure 5 start at the origin O of the stress-strain diagram. The stress is increased until the yield stress is reached at point A_1. A total strain increment ε_p is applied at constant stress σ_y to change from state A_1 to state B_1. Since the stress is constant during this step, the elastic strain is also constant and the total strain increment is exclusively a plastic strain increment. The stress is reduced from σ_y to $-\sigma_y$ change the state from B_1 to C_1 at constant plastic strain. The material yields at point C_1. A total strain increment $-0.25\varepsilon_p$ is applied at constant stress $-\sigma_y$ to change from state C_1 to state D_1. Since the stress is constant during this step, the elastic strain is also constant and the total strain increment is exclusively a plastic strain increment. The plastic strain at point D_1 is $\varepsilon_p - 0.25\varepsilon_p = 0.75\varepsilon_p$. The stress is increased from $-\sigma_y$ to σ_y change the state from D_1 to A_2 at constant plastic strain. This completes the first load cycle.

The second load cycle from point A_2 to point D_2 is similar to the first cycle from point A_1 to point D_1 except that all plastic strain increments are added to the plastic strain $0.75\varepsilon_p$ which exists at point A_2. The plastic strain at the end of load cycle 2 at point A_3 is $1.50\varepsilon_p$. In the third load cycle, the plastic strain increases to $2.25\varepsilon_p$. As the number of load cycles increases, the plastic strain increases without limit. This un-limited growth of the plastic strain in a sequence of loads cycles is called ratcheting. The ratcheting phenomenon is treated in this thesis [6; 8].

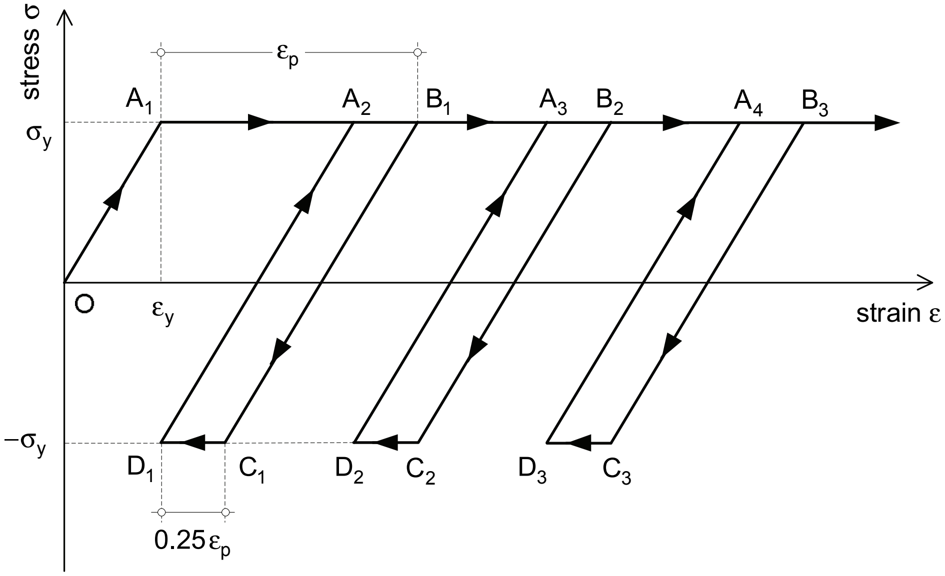


Fig. 5. Example of ratcheting

Shakedown. The plastic strain at a material point does not necessarily grow without limit under cyclic loads. The behavior at the material point depends on the total strain which is applied while the material is in the plastic state, and therefore on the overall behavior of the structure containing the material point [8]. A behavior of a structure, which undergoes plastic strains in the first few load cycles and then deforms completely elastic is called shakedown. Figure 6 shows the stress-strain diagram at a material point during shakedown. The path from point O to point C_2 corresponds to the path in figure 5. At point C_2 the shakedown is completed. In subsequent load cycles the state of the material point changes elastically between points B_2 and C_2.

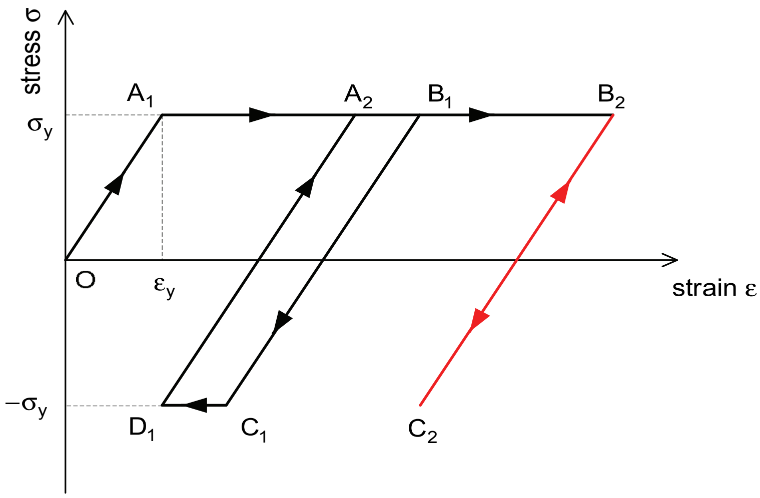


Fig. 6. Example of shakedown behavior

CONVENTIONAL METHODS OF ELASTIC-PLASTIC ANALYSIS

Types of Elastic-Plastic Analysis. Consider the model of a mild steel structure. Let the load which is applied to the structure be specified as the product of a load pattern and a load factor λ. Let the loading be quasi-static, but assume that the load factor λ(t) varies with pseudo-time t. The properties of function λ(t) determine the required type of elastic-plastic analysis.

Limit analysis. If the load factor λ(t) increases monotonically until the maximum load, which the structure can carry, is reached, a limit analysis is required. The purpose of the limit analysis is to find the collapse mechanism of the structure, which is associated with the smallest value of the load factor λ. The collapse mechanism is formed because some parts of the structure yield plastically. This type of elastic-plastic analysis is called limit analysis.

Shakedown analysis. If the load factor λ(t) varies cyclically with pseudo-time, the load pattern specifies the variation of the load pattern during one period of pseudo-time. Each load cycle lasts one period of pseudo-time. One of two subtypes of elastic-plastic behavior is observed [7].

In one subtype, the structure undergoes plastic deformation during the first cycles of loading but does not form a collapse mechanism. The behavior of the structure then becomes entirely elastic so that there are no further plastic strains. This subtype of behavior is called shakedown of the structure. The purpose of a shakedown analysis is to determine the greatest load factor for which shakedown occurs.

In the other subtype, the plastic strains in the structure continue to increase as it is being subjected to an increasing number of load cycles, but no collapse mechanism is formed. This type of behavior is called ratcheting of the structure. The deformation of the structure tends to become excessive due to the ratcheting so that the structure is no longer serviceable.

Elastic analysis. The theoretical basis of limit analysis is presented in section 1.2.2. The theoretical basis for shakedown analysis is presented in section 1.2.3. Both types of analysis may be compared to elastic analysis. The state of an elastic body with given geometry, material, loads and supports is described with 15 independent variables: 3 displacement coordinates, 6 strain coordinates and 6 stress coordinates. The variation of the variables with location and time must satisfy 15 independent governing equations: 3 equilibrium equations, 6 strain-displacement relations and 6 constitutive laws relating strain and stress coordinates. For admissible boundary conditions such as loads and prescribed displacements, the 15 variables are uniquely determined as functions of location and time by the 15 equations, unless there is bifurcation as determined in a stability analysis.

Rectangular Column. The structure rectangular column is a space regular truss as shown in Fig. 7 is used to illustrate the concepts of the nonlinear analysis and shakedown for space trusses. The bars of the truss have equal cross-sectional area A, modulus of elasticity E. The supports A, B, C, and D of the truss are pinned. Here in this section the truss is analyzed with both commercial and research programs which have been developed in this work. The nonlinear materially and geometrically behaviors with nonlinear research program which has been developed in chapter 3 is compared with ANSYS program. The flow diagrams related to the limit analysis and shakedown analysis presented in chapter 4 and according to these diagram two home-made programs have been developed in Java platform. Finally, the result of shakedown analysis is presented and compared with both two other programs.

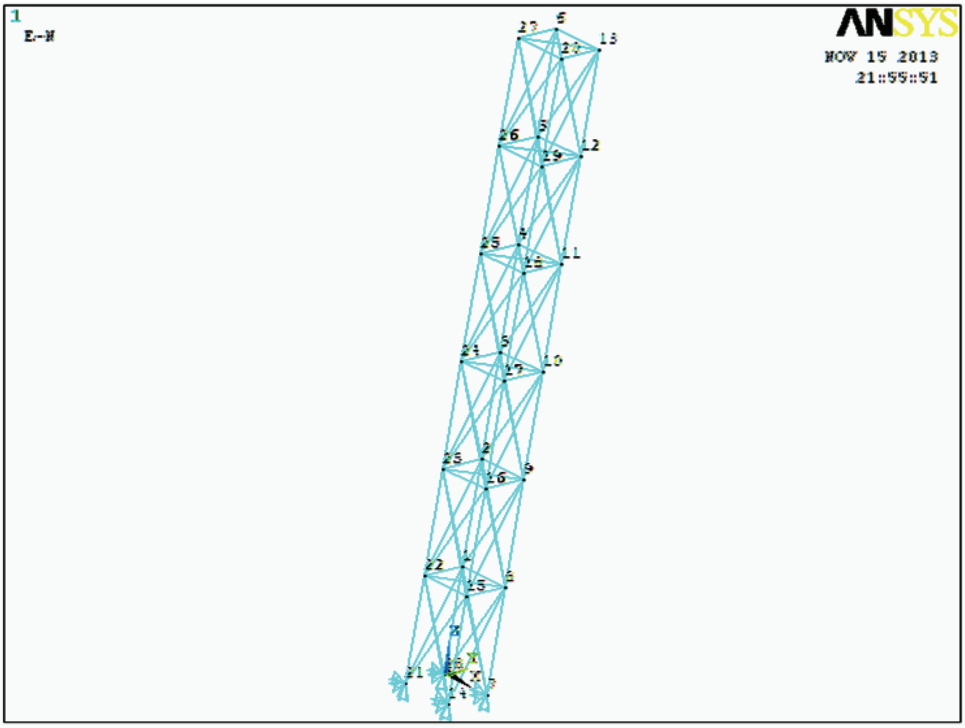


Fig. 7. Model of the Rectangular Column Truss

Limit Analysis for Rectangular Column. The structure rectangular column is analyzed here with the program, which has been developed according to the workflow of the limit analysis.

Analysis the rectangular column with ANSYS package. In the ANSYS producer, the procedure for obtaining the post critical and non-linear behaviors of the structure consist three methods: arc-length method and displacement control and force control. According to the response and behavior of the structure, using the nonlinear technique, the model can include features such as initial imperfections, plastic behavior, gaps, and large-deflection response. In addition, using deflection-controlled loading, we can even track the post-buckled performance of structure, which been used by A. Heidari for this structure in another work where the structure buckles into the configurations, such as snap-through, snap-back, and even looping in the load-deflection behavior of the structure [5].

For analyzing these space trusses, the element link 180 is used. Link 180 is a spar and used in a variety of engineering applications. This element also can be used to model trusses, sagging cables, links, springs, etc. This 3-D spar element is a uniaxial tension-compression element with three degrees of freedom at each node as translations in the nodal x, y, and z directions as in a pin-jointed structure, no bending of the element is considered. Plasticity, creep, rotation, large deflection, and large strain capabilities are included very well with the characters of this element. By default, LINK180 includes stress stiffness terms in any analysis and also the elasticity, isotropic hardening plasticity, kinematic hardening plasticity, Hill anisotropic plasticity, Chaboche nonlinear hardening plasticity, and creep are supported.

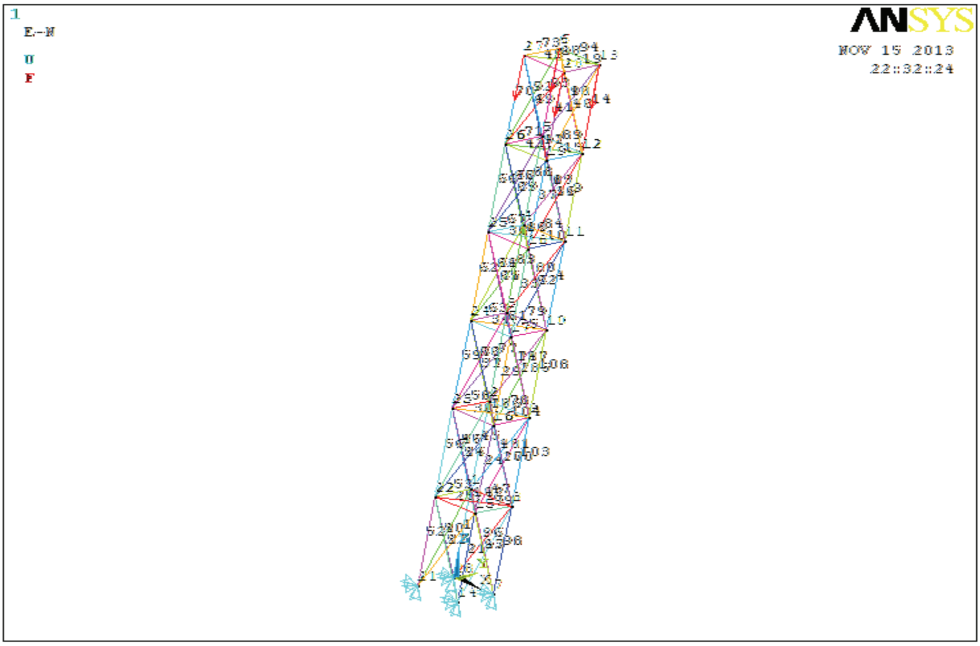


Fig. 8. Model of the rectangular column in ANSYS

The material has been used to analysis the truss is bilinear structural kinematic hardening with modulus of elasticity equal to 2.1 × 10^8 kg/m^2 and yield stress equal to 2.4 × × 10^5 kg/m^2 and behavior of the material is in Fig. 9.

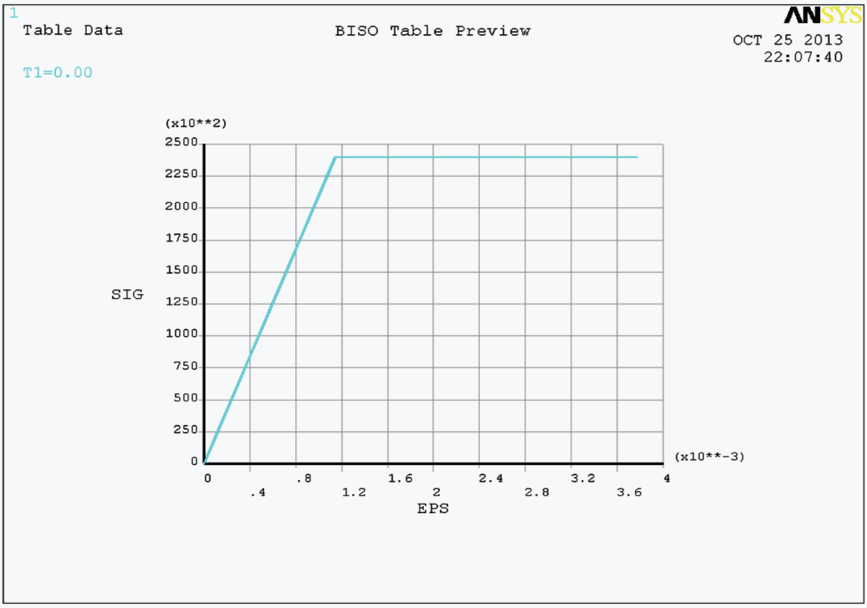


Fig. 9. Nonlinear material for rectangular column

It is necessary to mention that Arc-Length method with maximum multiplier 25 and minimum multiplier 0.0001 has been used and time at the end of load with 12 steps introduced 1.

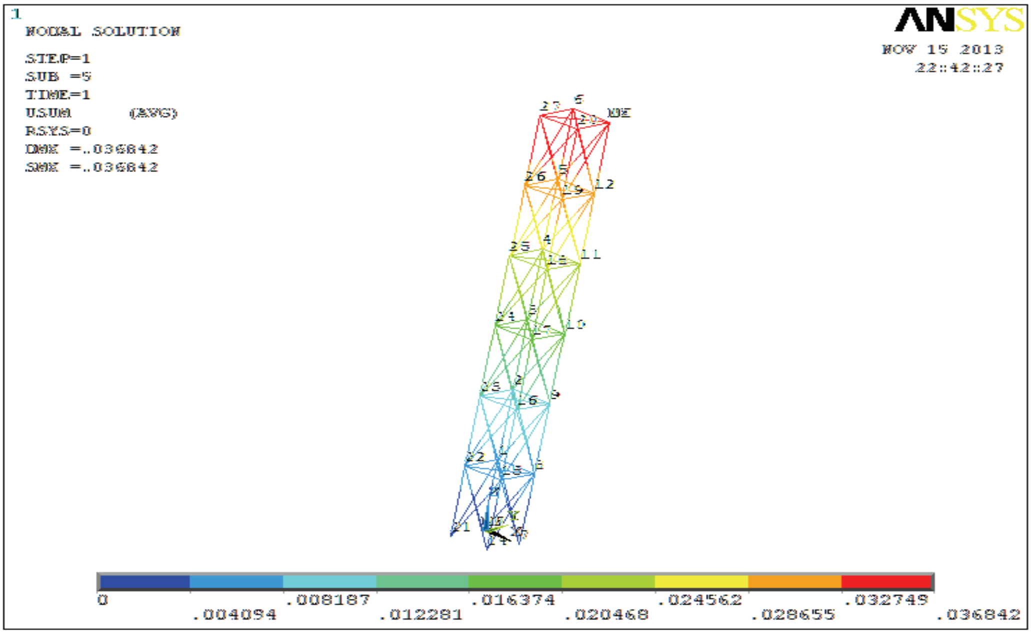
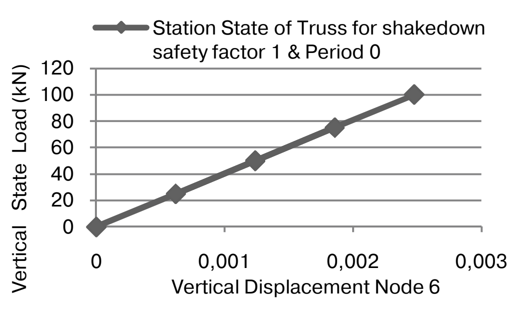
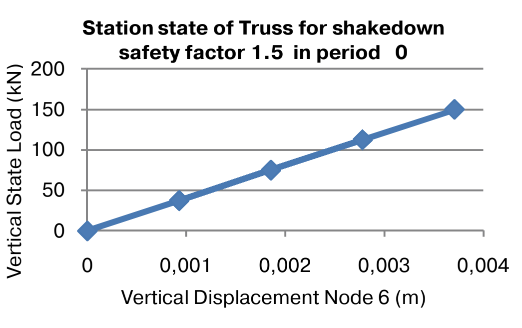
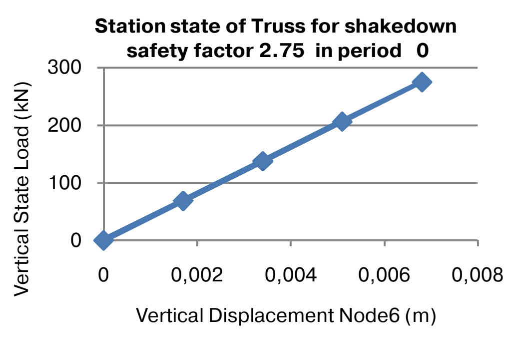


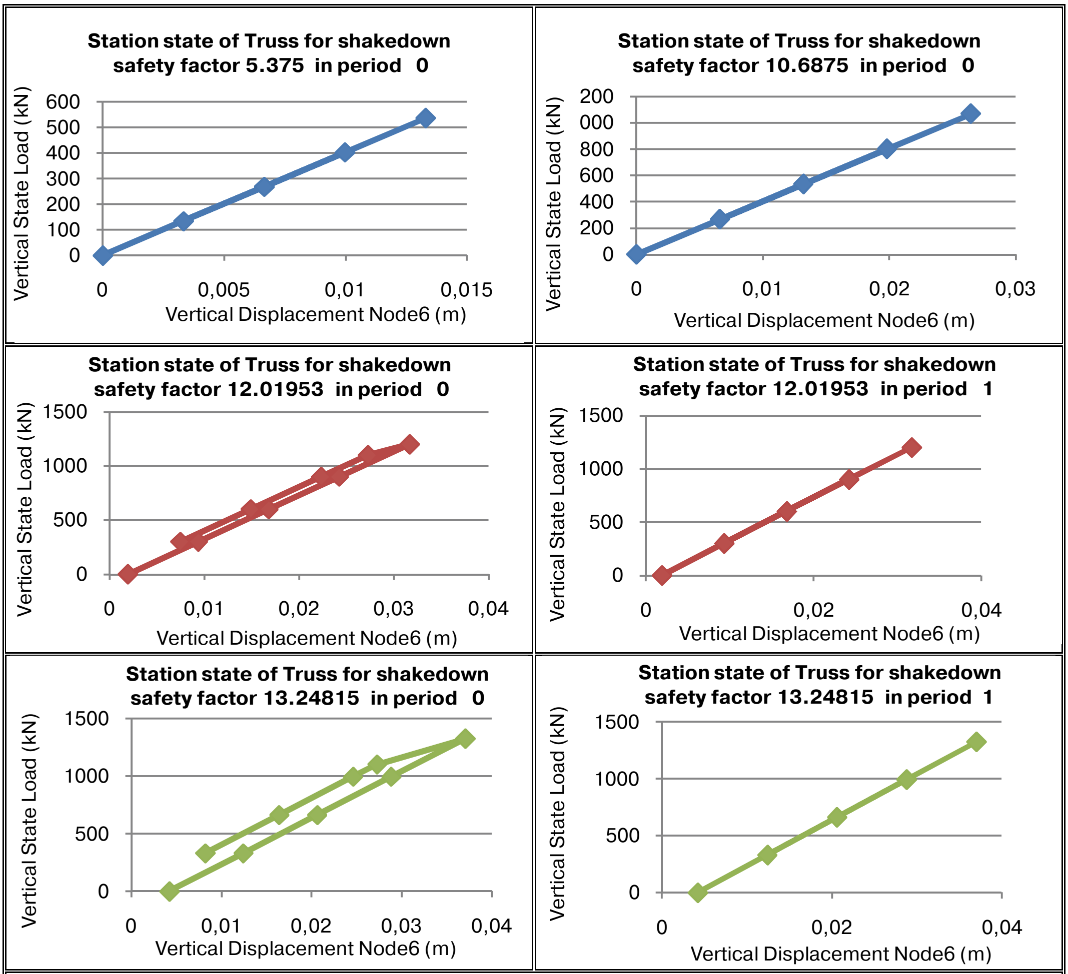
Fig. 10. Displacement contour at the end of nonlinear Analysis

THE RESULT OF THE LIMIT AND SHAKEDOWN ANALYSIS OF TRUSS RECTANGULAR COLUMN









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Shakedown for safety factor 13.248148
The shakedown safety factor lies in the range [13.248148, 13.248798]
End of Shakedown Analysis
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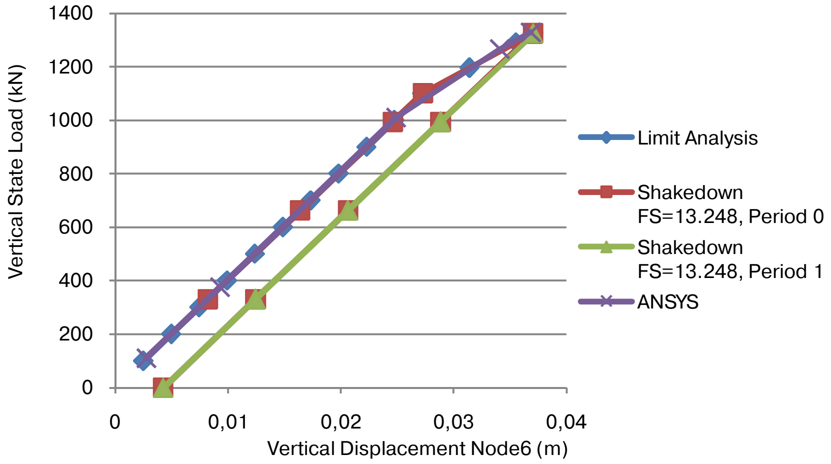

Fig. 11. Shakedown Analysis in comparison with Limit Analysis

Conclusions. The shakedown safety factor lies in the range [13.248148, 13.248798]. This safety factor has been obtain in limit analysis as 13.249 which is closed to shakedown analysis.

The technique has been developed in this research to obtain the limit load and real capacity of truss Rectangular Column worked very well and yielded satisfactory results when compared to the results of the Rectangular Column in ANSYS Package.

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АНАЛИЗ ПРИСПОСОБЛЯЕМОСТИ ФЕРМ И СРАВНЕНИЕ РЕЗУЛЬТАТОВ С ФУНДАМЕНТАЛЬНОЙ ТЕОРИЕЙ УПРУГОПЛАСТИЧЕСКОГО АНАЛИЗА ВКЛЮЧЕННОЙ В ИССЛЕДОВАНУЮ ПРОГРАММУ И АНСИС

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В работе приводятся результаты исследования в области нелинейного расчета пространственных стальных конструкций с учетом приспособляемости. Изложены теоретические основы и традиционные методы расчета на приспособляемость. Для иллюстрации разработанного метода приведен пример нелинейного расчета пространственной решетчатой колонны и представлены диаграммы пластической работы конструкции, демонстрирующие явление приспособляемости.

Ключевые слова: анализ приспособляемости, предельный анализ, пластичность, геометрический и физический нелиный, ферма.