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Topological Soliton Configurations in 8-Spinor Nonlinear Model

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We study the structure of the charged topological solitons in the lepton sector of the nonlinear 8-spinor model, at small distances the closed-string approximation being used. The mass, the spin and the magnetic moment of the soliton configuration with the unit leptonic number are estimated. The model is based on the well-known 8-spinor identity suggested by the Italian geometer Brioschi. Due to the identity the Dirac current appears to be time-like 4-vector that permits one to introduce the special form of the Higgs potential depending on the current squared. Within the framework of this model the natural classification of leptons and baryons can be realized via the Higgs mechanism. Concentrating on the lepton sector we study the simplest soliton configuration endowed with the unit Hopf index playing the role of the lepton number. Investigating the behavior of solutions at large and small distances we obtain the numerical estimate of physical characteristics of the topological soliton. The special symmetry group is used in our calculation, the combined rotations in ordinary and isotopic spaces being considered. The corresponding equivariant spinor fields involve phase functions linear with respect to azimuthal and toroidal angles. This property permits one to find explicit value of the topological invariant for the axially-symmetric configuration and to investigate the dependence of the physical characteristics on topology.

Key words and phrases: 8-spinor, topological charge, solitons.

1. Introduction

In our previous paper [1] the nonlinear 8-spinor model was suggested for the unified description of leptons and baryons as topological solitons in Faddeev [2] and Skyrme [3] models respectively. This unification is based on the special 8-spinor identity discovered by the Italian geometer Brioschi [4]:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + \mathbf{v}^2 + \mathbf{a}^2, \tag{1}$$

where the following quadratic spinor quantities are introduced:

$$s = \bar{\Psi}\Psi, \qquad p = \imath\bar{\Psi}\gamma_5\Psi, \quad \mathbf{v} = \bar{\Psi}\lambda\Psi,$$
$$\mathbf{a} = \imath\bar{\Psi}\gamma_5\lambda\Psi, \quad j_\mu = \bar{\Psi}\gamma_\mu\Psi, \quad \tilde{j}_\mu = \bar{\Psi}\gamma_\mu\gamma_5\Psi,$$

with $\bar{\Psi} = \Psi^+\gamma_0$ and λ standing for Pauli matrices in the flavor (isotopic) space. Here the diagonal (Weyl) representation for $\gamma_5 = \gamma_5^+$ is used and γ_μ, $\mu = 0, 1, 2, 3$, designate the unitary Dirac matrices acting on Minkowski spinor indices.

Taking into account the time-like character of the 4-vector j_μ, the topological distinction between leptons and baryons can be realized via the Higgs mechanism, the special form of the Higgs potential being used:

$$V = \frac{\sigma^2}{8}(j_\mu j^\mu - \varkappa_0^2)^2, \tag{2}$$

with σ and $\varkappa_0$ being some constant parameters. If one searches for localiRyb8zed soliton-like configurations in the model, one finds the natural boundary condition at space infinity:

$$\lim_{|\mathbf{r}|\to\infty} j_\mu j^\mu = \varkappa_0^2. \tag{3}$$

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As follows from the identity (1), the condition (3) determines the fixed (vacuum) point on the surface S^8. Using (3) and the well-known property of homotopic groups of spheres: $\pi_3(S^n) = 0$ for $n \geqslant 4$, one concludes that the two phases with nontrivial topological charges may exist in the model in question. The first one corresponds to the choice $\pi_3(S^3) = \mathbb{Z}$ (Skyrme Model) and the second one to the choice $\pi_3(S^2) = \mathbb{Z}$ (Faddeev Model).

For example, if the vacuum state Ψ_0 defines $s(\Psi_0) \neq 0$, then the configurations characterized by the chiral invariant $s^2 + \mathbf{a}^2$ determining sphere S^3 as the field manifold are possible, that corresponds to Skyrme Model phase. On the contrary, if only $v_3(\Psi_0) \neq 0$, then the $SO(3)$ invariant $\mathbf{v}^2$ determines the S^2 field manifold, that corresponds to Faddeev Model phase.

In view of these topological arguments, using the analogy with Skyrme (or Faddeev) Model, we suggested in [1] the following Lagrangian density for the effective 8-spinor field model:

$$\mathcal{L} = \frac{1}{2\lambda^2}\overline{D_\mu\Psi}\gamma^\nu j_\nu D^\mu\Psi + \frac{\epsilon^2}{4}f_{\mu\nu}f^{\mu\nu} - V + \mathcal{L}_{\text{em}}, \tag{4}$$

where $f_{\mu\nu}$ stands for the antisymmetric tensor of Faddeev–Skyrme type:

$$f_{\mu\nu} = (\bar\Psi\gamma^\alpha D_{[\mu}\Psi)(\overline{D_{\nu]}\Psi}\gamma_\alpha\Psi), \tag{5}$$

with λ and ϵ being constant parameters of the model. It should be stressed that the first term in (4) generalizes the σ-model term in Skyrme Model and includes the projector $P = \gamma^0\gamma^\nu j_\nu$ on the positive energy states. The second term in (4) gives the generalization of Skyrme (or Faddeev) term. Ryb8 Here the interaction of the spinor field with the electromagnetic one is introduced via the extension of the derivative:

$$D_\mu\Psi = \partial_\mu\Psi - \imath e_0\Gamma_e A_\mu\Psi, \tag{6}$$

where A_μ stands for the vector-potential of the electromagnetic field and $\Gamma_e = \frac{1}{2}(\lambda_3 - 1)$ in (6) stands for the electric charge operator, e_0 being the corresponding coupling constant.

The electromagnetic part of the Lagrangian density was investigated in [5] and corresponds to the Mie generalized electrodynamics:

$$\mathcal{L}_{\text{em}} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}[1 + G(I)] - \frac{1}{8\pi}H(I), \tag{7}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $G(I)$, $H(I)$ are some functions of the Mie invariant $I = A_\mu A^\mu$. As was shown in [5], the model (7), for the power series representation of these functions:

$$G(I) = \alpha_1 I, \quad H(I) = \sum_{n=2}^{\infty}\beta_n I^n,$$

admits the existence of static soliton-like configurations with fixed electric charge and positive energy.

2. Structure of Lagrangian in Leptonic Sector

In the lepton sector the 8-spinor Ψ is invariant under the space reflection [1] $\Psi \to \gamma_0\Psi$ and therefore it reduces to 4-spinor $\varphi = \text{col}(\varphi_1, \varphi_2)$, with φ_1, φ_2 being 2-spinors. As was shown by Faddeev [2], the configurations endowed with the nontrivial Hopf index Q_H are similar to closed twisted strings. The appropriate description of these configurations, which are also typical for the Kerr solution in General Relativity [6], can be obtained by using the toroidal coordinates $x \in [0, \infty)$, $y \in [-\pi, \pi]$, $\phi \in [0, 2\pi]$

with the metric
$$ds^2 = dt^2 - a^2 e^{2\alpha}(dx^2 + dy^2 + \sinh^2 x d\phi^2), \qquad (8)$$
where $e^{-\alpha} = \cosh x - \cos y$ and a is the length parameter (the closure radius of the string).

The model admits the following infinitesimal spinor transformations defining the correspondent groups:

$$G_1 : \delta\Psi = \imath\delta\alpha\Psi;$$
$$G_2 : \phi \to \phi + \delta\phi, \quad \delta\Psi = \imath\delta\phi J_3\Psi;$$
$$G_3 : y \to y + \delta y, \quad \delta\Psi = \imath\delta y\, p_y\Psi;$$

where the following operators are introduced:

$$J_3 = -\imath\partial_\phi + \frac{1}{2}\sigma_3, \quad p_y = -\imath\partial_y,$$

the latter group being effective in the asymptotic region ($x \gg 1$). Let us construct the following group of combined transformations:

$$G = \mathrm{diag}(G_1 \otimes G_2) \otimes \mathrm{diag}(G_1 \otimes G_3), \qquad (9)$$

with the corresponding equivariant field having the form:

$$\varphi_1 = \mathrm{col}\,(u_1, v_1 \exp[\imath(\phi - n_1 y)]), \quad \varphi_2 = \mathrm{col}\,(v_2 \exp[-\imath(\phi - n_2 y)], u_2), \qquad (10)$$

where u_i, v_i do not depend on the azimuthal angle ϕ, and n_1, n_2 are some integers.

Now we investigate the structure of topological solitons in the model in question using the perturbation method with respect to electromagnetic coupling constant e_0. In the first approximation we neglect the electromagnetic field and find the following action functional using the substitution (10):

$$\mathcal{A} = -2\pi a^3 \int\limits_0^\infty dx \int\limits_{-\pi}^{\pi} dy \Bigg\{ \frac{2e^\gamma}{\lambda^2}|\psi|^2\big[(\partial_1\psi)^2 + (\partial_2\psi)^2 + n_1^2|v_1|^2 + n_2^2|v_2|^2 +$$
$$+ 2n_2\mathrm{Im}(v_2^*\partial_2 v_2) + 2n_1\mathrm{Im}(v_1^*\partial_2 v_1) + e^{2(\alpha-\gamma)}\left(|v_1|^2 + |v_2|^2\right)\big] +$$
$$+ \frac{8\epsilon^2}{a^4}e^{\gamma-2\alpha}\Bigg[\Big(2\mathrm{Im}[(\psi^+\partial_1\psi)(\partial_2\psi^+\psi)] - \partial_1|\psi|^2(n_1|v_1|^2 - n_2|v_2|^2)^2\Big)^2 +$$
$$+ e^{2(\alpha-\gamma)}\left[\left(\partial_1|\psi|^2\right)^2 + \left(\partial_2|\psi|^2\right)^2\right]\left(|v_1|^2 - |v_2|^2\right)^2\Bigg] +$$
$$+ 2\sigma^2 e^{2\alpha+\gamma}\left(|\psi|^2 - \frac{\varkappa_0^2}{4}\right)^2\Bigg\}, \qquad (11)$$

where $e^\gamma = \sinh x\, e^\alpha$ and $\psi = \mathrm{col}(u_1, v_1, v_2, u_2)$.

Now it is worth-while to study the structure of the Hopf invariant Q_H playing the role of the lepton charge in our model.

3. Structure of Hopf Invariant

Hopf invariant Q_H as the generator of the homotopic group $\pi_3(S^2)$ classifies the mappings $\mathbf{n} : \mathbb{R}^3 \to S^2$, where the sphere S^2 is given by the unit vector $\mathbf{n} = \mathbf{v}/|\mathbf{v}|$.

Q_H can be represented by the Whitehead integral [7], [8]:

$$Q_H = \frac{1}{(4\pi)^2} \int \mathrm{d}^3x\,(\mathbf{c}\,\mathrm{rot}\,\mathbf{c}), \tag{12}$$

where the vector $\mathbf{c}$ is defined by the relation

$$\partial_i c_k - \partial_k c_i = \epsilon^{abc} \partial_i n^a \partial_k n^b n^c. \tag{13}$$

Hopf suggested an elegant method of calculating the integral (12) via the inverse Hopf mapping $S^3 \to S^2$. To this end let us introduce the auxiliary 2-spinor

$$\chi = \mathrm{col}\big(\cos A + \imath \sin A \cos B, \sin A \sin B \exp(\imath C)\big), \tag{14}$$

with A, B, C being angular coordinates on S^3. Then the following relations hold:

$$\mathrm{rot}\,\mathbf{c} = -2\imath[\nabla\chi^+\nabla\chi], \quad \mathbf{c} = \mathrm{Im}(\chi^+\nabla\chi), \quad \mathbf{n} = (\chi^+\lambda\chi). \tag{15}$$

Inserting (15) into (12), one finds that

$$Q_H = \frac{1}{(2\pi)^2} \int \mathrm{d}^3x\, \sin^2 A \sin B([\nabla A \nabla B]\nabla C) = \deg(S^3 \to S^3), \tag{16}$$

the latter formula expressing the identity of the homotopic groups $\pi_3(S^2) = \pi_3(S^3) = \mathbb{Z}$. If one introduces the new variables β and ρ by putting

$$\sin A \sin B = \sin(\beta/2), \quad \tan A \cos B = \tan \rho,$$

one can get the more compact expression for the Hopf invariant:

$$Q_H = -\frac{1}{8\pi^2} \int \mathrm{d}^3x\, \sin\beta([\nabla\beta\nabla\rho]\nabla C). \tag{17}$$

Taking into account that $n_1 + \imath n_2 = \sin\beta \exp(\imath\gamma)$, one finds the relation

$$\gamma = C - \rho \tag{18}$$

and the final expression for the Hopf invariant:

$$Q_H = \frac{1}{8\pi^2} \int \mathrm{d}^3x\, \sin\beta([\nabla\beta\nabla\gamma]\nabla C). \tag{19}$$

In our case the angular coordinates β, γ can be found from (10) and the definition of the vector $\mathbf{v} = \bar{\Psi}\lambda\Psi$ stemming the relation

$$\gamma = \mu(x, y) - \phi. \tag{20}$$

In view of (18) and (20) one can choose in (19) for the axially-symmetric configurations $C = -\phi$, that is

$$Q_H = -\frac{1}{8\pi^2} \int \mathrm{d}^3x\, \sin\beta([\nabla\beta\nabla\gamma]\nabla\phi) = -\frac{1}{4\pi} \int_0^\infty \mathrm{d}x \int_{-\pi}^{\pi} \mathrm{d}y\, \sin\beta(\beta_x\mu_y - \beta_y\mu_x). \tag{21}$$

Taking into account (10) and (20), one finds the following value of the leptonic charge L for our configuration (10):

$$Q_H = L = n_1 + n_2, \tag{22}$$

with the boundary condition for β being imposed:

$$\cos\beta(x=0) = 1, \quad \cos\beta(x=\infty) = -1.$$

4. Configuration with Unit Leptonic Charge

Let us consider the simplest state with the unit leptonic charge $L = n_1 = 1$ when $v_1 \neq 0$, $v_2 = 0$. In this case the following approximation is appropriate: $A_1 = A_2 = 0$, $A_0 \neq 0$, $A_3 \neq 0$, that permits using the substitution:

$$u_1 = \sqrt{R}\sin\Theta\sin\Phi_1, \quad v_1 = \sqrt{R}\sin\Theta\cos\Phi_1, \quad u_2 = \sqrt{R}\cos\Theta. \tag{23}$$

Inserting (23) into the Lagrangian (4), one gets the following action functional:

$$\mathcal{A} = 2\pi a \int\limits_0^\infty \mathrm{d}x \int\limits_{-\pi}^{\pi} \mathrm{d}y\, e^\gamma \Biggl\{ \frac{2}{\lambda^2}\biggl[R^2 e^{2\alpha} a^2 e_0^2 I \cos^2\Theta - \frac{1}{4}(\partial R)^2 -$$
$$- R^2(\partial\Theta)^2 - R^2\sin^2\Theta\left((\partial\Phi_1)^2 + \cos^2\Phi_1\coth^2 x\right)\biggr] +$$
$$+ 8\epsilon^2 R^2\biggl[(\partial R)^2\left(e_0^2 A_0^2\cos^4\Theta - \frac{e_{-2\gamma}}{a^2}(\sin^2\Theta\cos^2\Phi_1 - e_0 A_3\cos^2\Theta)^2\right) -$$
$$- \frac{e^{-2\alpha}}{a^2}\sin^4\Theta\cos^4\Phi_1(\partial_1 R)^2\biggr] - 2\sigma^2 a^2 e^{2\alpha}\left(R^2 - \frac{\varkappa_0^2}{4}\right)^2 -$$
$$- \frac{e^{2\alpha}a^2}{8\pi}H(I) + \frac{1}{8\pi}\big(1 + G(I)\big)\left[(\partial A_0)^2 - \frac{e^{-2\gamma}}{a^2}(\partial A_3)^2\right]\Biggr\},$$

where the denotations are used:

$$e^\gamma = \sinh x\, e^\alpha, \quad I = A_0^2 - \frac{e^{-2\gamma}}{a^2}A_3^2, \quad (\partial K)^2 \equiv (\partial_1 K)^2 + (\partial_2 K)^2.$$

Let us first study the behavior of functions R, Θ, Φ_1, A_0, A_3 as solutions to the equations of motion at $x \to \infty$:

$$\begin{gathered} R = R_0 + R_1\frac{1}{2}(1-\tanh x), \quad \Theta = \Theta_0 + \Theta_1\frac{1}{2}(1-\tanh x), \\ \Phi_1 = \frac{\pi}{2} - 2\arctan e^{-x}, \\ A_0 = A_{00} - A_{01}\frac{1}{2}(1-\tanh x), \quad A_3 = A_{30} - A_{31}\frac{1}{2}(1-\tanh x), \end{gathered} \tag{24}$$

with the following constraints being imposed:

$$R_1 = 8R_0\sin^2\Theta_0, \quad \Theta_1 = \sin 2\Theta_0, \quad A_{01} = S_0 A_{00}, \quad A_{31} = S_0 A_{30}, \tag{25}$$

where we use the denotations:

$$S_0 = \frac{a^2}{1 + G(I_0)}\left[H'(I_0) - \frac{16\pi e_0^2}{\lambda^2}R_0^2\cos^2\Theta_0\right], \quad I_0 = A_{00}^2 - \frac{1}{a^2}A_{30}^2$$

and introduce the small parameter

$$\zeta = a\sigma\lambda\varkappa_0 \ll 1, \tag{26}$$

neglecting the corresponding terms in the first approximation.

The behavior of fields at large distances ($x \to 0$, $y \to 0$) can be derived from linear equations of motion valid in the vicinity of the vacuum state Ψ_0, for which $u_1 = \sqrt{\varkappa_0/2}$:

$$\begin{gathered} R = \frac{\varkappa_0}{2}(1 - ze^{-\alpha}), \quad \Theta = \frac{\pi}{2} - u_0 \tanh x e^{-\alpha/2}, \quad \Phi_1 = \frac{\pi}{2} - k \tanh x e^{-\alpha/2}, \\ A_0 = \frac{q}{a\sqrt{2}} e^{-\alpha/2}, \quad A_3 = A \tanh^2 x e^{\alpha/2}, \end{gathered} \tag{27}$$

with q being electric charge of the particle-soliton and the parameter A proportional to its magnetic moment.

Now we intend to estimate the mass of the soliton through smooth matching the functions (24) and (27) at some point with the coordinates $x = x_0$ and $y = \pi/2$. From this condition one derives the numerical values $x_0 = \log(\sqrt{2} + \sqrt{3})$, $S_0 = 2$ and all other parameters: $\Theta_0 \approx 0.347826$; $k \approx 0.57277$; $z \approx 0.05694$; $R_0 \approx 0.25991\varkappa_0$; $R_1 \approx 0.241575\varkappa_0$. To estimate the energy $\mathcal{E}$, we divide the interval $[0, \infty)$ into two parts: $[0, x_0]$ and $[x_0, \infty)$ and calculate the corresponding integrals in (11) using the trial functions (27) and (24) respectively. The resulting energy $\mathcal{E} = -\mathcal{A}$, electromagnetic contribution being omitted, reads:

$$\mathcal{E} = 2\pi^2 a^3 \varkappa_0^2 \left(\frac{\alpha_0}{a^2\lambda^2} + \beta_0 \frac{\epsilon^2}{a^4} \varkappa_0^2 + \gamma_0 \sigma^2 \varkappa_0^2 \right), \tag{28}$$

where $\alpha_0 = 0.209223$; $\beta_0 = 8.06583 \cdot 10^{-3}$; $\gamma_0 = 1.205782$.

Minimization of the energy (28) with respect to the radius a gives

$$a^2 = \frac{1}{6\sigma^2 \varkappa_0^2 \gamma_0^2} \left[-\frac{\alpha_0}{\lambda^2} + \left(\frac{\alpha_0^2}{\lambda^4} + 12\sigma^2 \varkappa_0^2 \epsilon^2 \beta_0 \gamma_0 \right)^{1/2} \right].$$

Thus, the mass of the particle-soliton is given by

$$m = \frac{2\pi a \varkappa_0^2}{\lambda^2} \left(2\alpha_0 + M_0^2 a^2 \gamma_0 \right), \quad M_0 \equiv 2\sigma\lambda\varkappa_0.$$

5. Spin and Magnetic Moment of Soliton

Now we calculate the spin of the particle-soliton using the well-known expression for the z-projection of the angular momentum:

$$S = \int \mathrm{d}^3 x \, 2\mathrm{Re} \left(\frac{\partial \mathcal{L}}{\partial(\partial_t \Psi)} \imath J_3 \Psi \right) = -2\frac{e_0}{\lambda^2} \int \mathrm{d}^3 x \, |\psi|^2 |u_2|^2. \tag{29}$$

Introducing the density of the electric charge

$$\rho_e = -\frac{\partial \mathcal{L}}{\partial A_0}, \tag{30}$$

one finds from (29) and (30) that $S = \dfrac{1}{2e_0} \int \mathrm{d}^3 x \, \rho_e = \dfrac{q}{2e_0}$. Therefore for the standard choice $q = e_0$ one gets $S = 1/2$.

To calculate the magnetic moment $\mathbf{m}$ of the particle-soliton, we use the classical electrodynamics formula for the vector-potential of the point-like magnetic moment:

$$\mathbf{A} = \frac{1}{r^3}[\mathbf{mr}].$$

Comparing this expression with the azimuthal component of the vector-potential

$$A_\phi = \frac{e^{-\gamma}}{a} A_3 = \frac{|\mathbf{m}|}{r^2} \sin\vartheta = \frac{aA\sqrt{2}}{r^2} \sin\vartheta,$$

one gets the relation

$$|\mathbf{m}| = a\sqrt{2}A. \tag{31}$$

On the other hand, in the first approximation with respect to e_0^2 one finds

$$I_0 \approx (\beta_2 a^2 - \alpha_1)^{-1} = A_{00}^2 - \frac{1}{a^2} A_{30}^2,$$

where for $q = e_0$ one obtains that $A_{00} = 3^{3/4} e_0/(2a)$, hence

$$A_{30} = \left[3^{3/2} \frac{e_0^2}{4} - (\beta_2 - \alpha_1/a^2)^{-1} \right]^{1/2}.$$

However, the smooth matching of A_3 gives the relation

$$A = \sqrt{\cosh x_0} \coth x_0 A_{30} = 3^{1/4} \sqrt{3/2} A_{30}.$$

Inserting this value of the constant A into (31), one can calculate the magnetic moment of the particle-soliton:

$$|\mathbf{m}| = 3^{3/4} \frac{a}{2} \left[3^{3/2} e_0^2 - 4(\beta_2 - \alpha_1/a^2)^{-1} \right]^{1/2}.$$

6. Conclusion

Using some ideas of Mie [9], the effective 8-spinor model unifying the models by Skyrme and Faddeev was suggested [5] permitting, via the mechanism of spontaneous symmetry breaking, to describe the particles as topological solitons. We consider the leptonic sector of the model in question and study the structure of axially-symmetric configuration with the unit lepton charge. Using the behavior of fields at large and small distances, one can estimate the mass, the spin and the magnetic moment of the particle-soliton. For the natural choice of the electric charge of the particle $q = e_0$ the spin proves to be 1/2.

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Топологические солитонные конфигурации в 8-спинорной нелинейной модели

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Изучается структура заряженных топологических солитонов в лептонном секторе нелинейной 8-спинорной модели, когда на малых расстояниях используется приближение замкнутых струн. Оцениваются масса, спин и магнитный момент солитонной конфигурации с единичным лептонным числом. Модель основана на хорошо известном 8-спинорном тождестве, предложенном итальянским геометром Бриоски. В силу этого тождества дираковский ток оказывается временно-подобным 4-вектором, что позволяет ввести специальную форму потенциала Хиггса, зависящего от квадрата тока. В рамках этой модели может быть реализована естественная классификация лептонов и барионов благодаря механизму Хиггса. Ограничившись лептонным сектором, мы изучаем простейшую солитонную конфигурацию, наделённую единичным индексом Хопфа, который играет роль лептонного числа. Исследуя поведение решений на больших и малых расстояниях, мы получаем численную оценку физических характеристик топологического солитона. В наших расчётах используется специальная группа симметрий, включающая комбинированные вращения в обычном и изотопическом пространствах. Соответствующие эквивариантные спинорные поля включают фазовые функции, линейно зависящие от азимутального и тороидального углов. Это свойство позволяет найти явное значение топологического инварианта для аксиально-симметрической конфигурации и исследовать зависимость физических характеристик от топологии.

Ключевые слова: 8-спинор, топологический заряд, солитоны.